Using Type-2 Fuzzy Logic Systems to Infer the Operative Configuration of Distribution Networks

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Abstract— This work presents an approach aiming at inferring the operative configuration (OC) of distribution networks. In order to attain this objective, the OC problem is decomposed in two parts. In the first part, the available real-time data and expert knowledge are integrated by means of rule-based Type-2 Fuzzy Logic Systems. As a result, an approximate initial solution is obtained. In the second part, the initial solution is dynamically tuned using customer trouble calls. This is done by means of an approach based on fuzzy relational equations and fuzzy abductive inference. At every stage, all plausible solutions are calculated in order to obtain a complete outlook of possible options. The performance of the methodology is evaluated on a real distribution feeder and the results are presented.

***Index Terms*—Fuzzy sets, fuzzy logic, fuzzy systems, distribution networks, power flow analysis, state estimation.**

I. NOMENCLATURE

CTC: Customer Trouble Calls.
ERT: Extended Real-Time (minutes).
HV/MV SE: High Voltage/Medium Voltage Substation.
QS: Quality of Service.
RT: Real-Time (seconds).
SCADA: Supervisory Control and Data Acquisition.

II. INTRODUCTION

THE increasing levels of QS demanded by regulatory agencies and customers, have encouraged Latin American distribution utilities to improve their outage management and operation strategies as a means to reduce economic penalties. In this context, the identification of the OC plays an important role because it is the starting point for distribution outage management and state estimation. The OC is defined as the state (open/close) of the protective and switching devices installed on the medium voltage distribution network [1]. Traditionally, the solution of the OC problem has depended mostly on the analysis of CTC and feeder inspections, because scarce RT data have been available at the distribution level [2]-[5]. However, CTC may not be received immediately afterwards the outages have occurred or may be related to low voltage (household) problems rather than MV outages [6]. Furthermore, various CTC or feeder inspections may be necessary to locate the operated protective devices; this implies additional requirements of personnel and resources. Other difficulties are caused by the heterogeneity, extension and dynamic nature of the MV network and its susceptibility to frequent natural phenomena. All these limitations tend to increase the duration of outages to levels that are incompatible with current QS regulations. In recent years, distribution SCADA has become accessible, at least at the HV/MV SE, supplying important RT data to several applications. However, the coverage of distribution SCADA is generally insufficient to obtain an accurate assessment of the OC using only RT data [6]. In this sense, it has been observed that a convenient strategy is to integrate the data provided by distribution SCADA and CTC [6]-[8], in this way their individual deficiencies are surpassed and their potentialities are fully exploited. Based on this analysis, in this work the OC problem is decomposed in two parts. In the first part, an approximate initial OC is obtained using the data supplied by SCADA and the available qualitative data and expert knowledge. Here, a novel RT model based on power flow analysis, fuzzy fault currents calculation and rule-based Type-2 Fuzzy Logic Systems (T2-FLS) is proposed. In the second part, the initial solution is dynamically tuned using the ERT data (CTC). Here, an ERT model based on fuzzy relational equations and fuzzy abductive inference is proposed.

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III. MATHEMATICAL MODEL

A radial MV feeder with low level of monitoring and control in RT is showed in Fig. 1. The protective devices (d_1-d_7) are installed along the feeder and the switching devices (d_8) may be installed along or at the end points of the feeder (tie-switches). Although some tie-switches may be supervised by SCADA, the common practice is to avoid their automatic operation, mostly due to personnel and operative security issues. Therefore the OC of one feeder is independent of the rest of the network. The changes on the OC can be classified as planned and unexpected. Unexpected changes modify the OC from a known initial state to an unknown final state and can be classified as permanent (e.g., fuse meltdown, recloser lockout, etc) and temporary (e.g., fast operations of a recloser). Only unexpected permanent changes are considered here because they have the greater negative impact on the QS.

A. RT Model

When a permanent fault occurs on a MV feeder, a fault current is detected and subsequently eliminated by the operation of the protective device located closer to the fault (recloser, sectionalizer or fuse). This operation modifies the OC and the total active power flow delivered to the feeder. Currently, many HV/MV SE have modern microprocessor-based overcurrent relays capable to record the magnitude of fault currents (I_{sc}) and classify the faults. The fault current data and the active power flow variations caused by the operation of protective devices (ΔP) can be reported in RT to the control center by means of SCADA. From their analysis it is possible to infer the new OC.

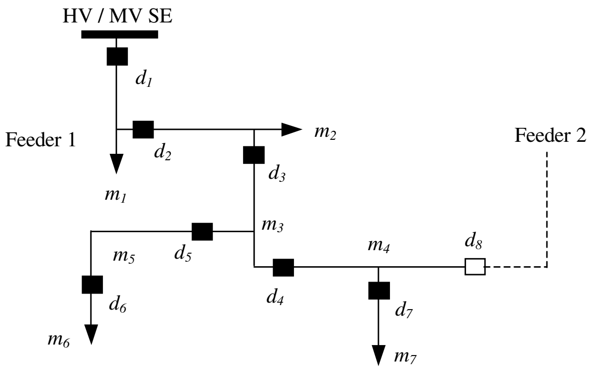

Fig.1. Example of a radial MV distribution feeder.

Because of the topological characteristics and logistical constrains of poorly monitored distribution feeders, often the RT data are not sufficient to calculate a definitive and indisputable solution [1]. In these situations it is only possible to reduce the solution space and to obtain a small group of candidate solutions. These results may be improved using the quantitative data obtained from the analysis of the history of operation of feeders (frequency of operation of protective devices, fault rates, etc). Unfortunately, in many utilities these data are scarce and/or uncertain. However, expert operators and crews successfully use linguistic expressions and rules to transmit their knowledge and to compensate the lack of quantitative data. Generally, these qualitative data are abundant but also uncertain, because of the nature of human language, however they are extremely useful and therefore continuously used in daily labors. From this analysis, it is observed the necessity to integrate quantitative and qualitative data in order to compensate their individual deficiencies. To attain this goal, this work proposes the rule-based Type-2 Fuzzy Logic System (T2-FLS) of Fig. 2, to merge both types of data, quantitative (numerical) and qualitative (linguistic).

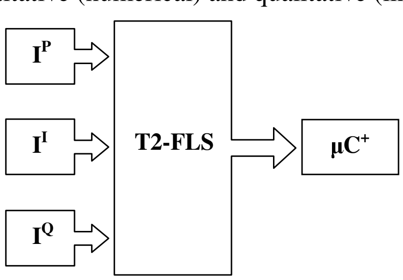

Fig. 2. Scheme of the T2-FLS proposed for the calculation of the OC.

Every input of the T2-FLS of Fig. 2 is an n-dimensional vector, where n is the total number of protective devices of the feeder. The ith element of every vector is the value of the corresponding operation index for the ith device (1)-(3):

$$\mathbf{I^P} = \begin{bmatrix} I_1^P & I_2^P & \dots & I_n^P \end{bmatrix} \quad (1)$$

$$\mathbf{I^I} = \begin{bmatrix} I_1^I & I_2^I & \dots & I_n^I \end{bmatrix} \quad (2)$$

$$\mathbf{I^Q} = \begin{bmatrix} I_1^Q & I_2^Q & \dots & I_n^Q \end{bmatrix} \quad (3)$$

The vector $\mathbf{I^P}$ contains the operation indexes obtained from the analysis of ΔP. $\mathbf{I^I}$ contains the operation indexes obtained from the analysis of I_{sc}. $\mathbf{I^Q}$ contains the operation indexes obtained from the analysis of quantitative and qualitative data related to the QS. Here are considered the external factors that affect the operation of the feeder and the characteristics of the zones protected by every device. The universe of discourse of I_i^P, I_i^I and I_i^Q is the interval [0, 1]; the closer the index to one, the higher its influence on the output. The output of the T2-FLS ($\mathbf{\mu C^+}$) is also an n-dimensional vector, which contains the final operation indexes of the protective devices of every feeder (4), the universe of discourse of μc_i^+ is [0, 1].

$$\mathbf{\mu C^+} = \begin{bmatrix} \mu c_1^+ & \mu c_2^+ & \dots & \mu c_n^+ \end{bmatrix} \quad (4)$$

From the RT perspective, μc_i^+ represents the certainty of operation of the ith protective device. Those devices whose μc_i^+ satisfy a decision condition are selected as candidates for operation. These results may serve as a rough guide for the distribution operator, in order to take some initial outage management decisions while additional ERT data are received. The T2-FLS integrates the information supplied by individual indexes using a rule base, which models the expert knowledge of operators. The procedures of calculation of the operation indexes are explained in section IV.

Before continuing, a brief introduction of the generalities of type-2 fuzzy sets and T2-FLS is presented. A thoroughly description of their mathematical theory and applications can be found in [9]. A conventional type-1 fuzzy set A defined on the universe of discourse X, handles the uncertainties about the meaning of words using a two-dimensional membership function (type-1 membership function) which is totally crisp. This is showed in (5), where $\mu_A(x)$ is the *grade of the membership function* (a crisp number) for a generic element $x \in X$ and the symbol $\int$ denotes union over all admissible x.

$$A = \int_{x \in X} \mu_A(x)/x \quad (5)$$

A type-2 fuzzy set $\tilde{A}$ models the uncertainties about the meaning of words using a three-dimensional membership function (type-2 membership function) which itself is fuzzy. This is showed in (6), (7) where $\mu_{\tilde{A}}(x)$ is the *secondary membership function* (a type-1 fuzzy set) for a generic element $x \in X$. The domain (J_x) and amplitude ($f_x(u)$) of $\mu_{\tilde{A}}(x)$ are the *primary membership* of x and *secondary grade*, respectively.

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x)/x \tag{6}$$

$$\mu_{\tilde{A}}(x) = \int_{u \in J_x} f_x(u)/u, \quad J_x \subseteq [0,1] \tag{7}$$

The union $(\bigcup)$ of all primary memberships of $\tilde{A}$ is a bounded region called *fingerprint of uncertainty* (FOU) (8). The upper and lower bounds of the FOU are called the *upper and lower membership functions* of $\tilde{A}$, respectively (9), (10). The type-2 membership function provides additional degrees of freedom to capture more information about the represented linguistic term. Type-2 fuzzy sets are useful in circumstances where it is difficult to determine the "correct" type-1 membership function for a type-1 fuzzy set.

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x \tag{8}$$

$$\bar{\mu}_{\tilde{A}}(x) = \overline{FOU(\tilde{A})} = \bigcup_{x \in X} \overline{J_x} \quad \forall x \in X \tag{9}$$

$$\underline{\mu}_{\tilde{A}}(x) = \underline{FOU(\tilde{A})} = \bigcup_{x \in X} \underline{J_x} \quad \forall x \in X \tag{10}$$

T2-FLS provide a robust mathematical framework to model the uncertainty of natural language and compute using linguistic terms and numerical values. T2-FLS can be broadly classified as of general and interval kind, the former use type-2 fuzzy sets whose secondary grades may take any value on the interval [0, 1] (*general type-2 fuzzy sets*) and the later use type-2 fuzzy sets whose secondary grades are equal to one (*interval type-2 fuzzy sets*). In this work only T2-FLS of interval kind are proposed because they are computationally more efficient than T2-FLS of general kind [9]. An interval T2-FLS is very similar to a conventional type-1 fuzzy logic system (T1-FLS), the main difference is that it uses interval type-2 fuzzy sets to model the uncertainties of the words of its rule base. Its inputs may be modeled as type-2 singletons, type-1 fuzzy sets or interval type-2 fuzzy sets, depending on the nature of their uncertainty. This is showed in Fig. 3, where the shaded area of the FOU models the uniformity of the secondary grades of an interval type-2 fuzzy set ($f_x(u) = 1, \forall u \in J_x$). The output of an interval T2-FLS is a crisp value (y_{iC}) and a type-1 interval ($y_i = [y_{iL}, y_{iU}]$) called type-reduced fuzzy set. The type-reduced fuzzy set (Fig. 4) is a measure of the uncertainty of the crisp output, in an analogous way to probability-based models where the standard deviation is a measure of the uncertainty of the mean.

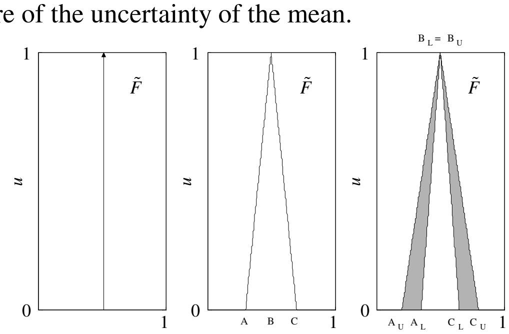

Fig. 3. Membership functions of type-2 singleton (left) and type-1 triangular fuzzy set (middle), FOU of interval type-2 triangular fuzzy set (right).

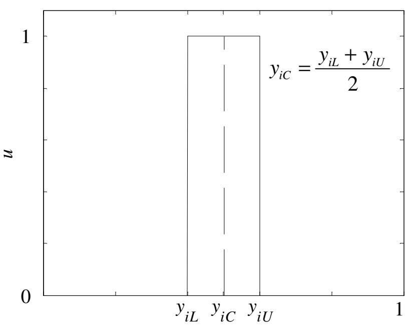

Fig. 4. Type reduced fuzzy set (solid line) and defuzzified crisp output of an interval T2-FLS (dash line).

B. ERT Model

From the ERT perspective, the identification of the OC is a diagnostic problem in which, given a set of *observed manifestations* (CTC), the objective is to identify the set of *disorders* (operated protective devices) that cause them. In order to solve this problem, it is necessary to consider the additional information available (RT results, cause-effect relationship among protective devices and customer groups, etc). In Fig. 1 it is observed that the protective devices (d_1-d_7) and the protected zones or groups of customers (m_1-m_7) are related by means of a cause-effect relationship. This means that the operation of a protective device causes outages on the zones situated downstream from its location. Therefore it also causes the CTC received from these zones. This relationship can be represented through the fuzzy relational model of diagnostic proposed in [10], [11]. This model has the advantages to handle uncertainty in a non-probabilistic way and to represent the certainty (instead of the intensity) of presence of binary manifestations. These are important features because in many distribution networks there are not enough data to calculate with accuracy the prior probabilities of faults used by other diagnostic models [10]. Additionally, it is necessary to handle the uncertainty related to CTC [6]. In this model, the disorders are the state of the n protective devices and the manifestations are the state of the n protected zones (groups of customers) of the feeder. The disorders are modeled by a crisp set **D** (11), where d_i represents the ith protective device. The manifestations are modeled by the crisp set **M** (12), where m_j represents the jth zone or group of customers of the feeder.

$$\mathbf{D} = \{d_1 \quad d_2 \quad \ldots \quad d_n\} \tag{11}$$

$$\mathbf{M} = \{m_1 \quad m_2 \quad \ldots \quad m_n\} \tag{12}$$

The "more or less" certainly present and "more or less" certainly absent manifestations are modeled by the fuzzy sets $\boldsymbol{M^+}$ (13) and $\boldsymbol{M^-}$ (14), respectively. Here, the degrees of membership of the fuzzy sets $\boldsymbol{M^+}$ and $\boldsymbol{M^-}$ model the certainty of presence and absence of a binary manifestation [10]. These sets satisfy $\boldsymbol{M^+} \cap \boldsymbol{M^-} = \varnothing$, which means that it is not possible to be certain both of the presence and of the absence of the same manifestation simultaneously. $\boldsymbol{\bar{M}^-}$ models the "more or less" possibly present manifestations (15).

$$\boldsymbol{M^+} = \frac{\mu^+_{m_1}}{m_1} + \frac{\mu^+_{m_2}}{m_2} + \cdots + \frac{\mu^+_{m_n}}{m_n} \quad (13)$$

$$\boldsymbol{M^-} = \frac{\mu^-_{m_1}}{m_1} + \frac{\mu^-_{m_2}}{m_2} + \cdots + \frac{\mu^-_{m_n}}{m_n} \quad (14)$$

$$\boldsymbol{\bar{M}^-} = \frac{1-\mu^-_{m_1}}{m_1} + \frac{1-\mu^-_{m_2}}{m_2} + \cdots + \frac{1-\mu^-_{m_n}}{m_n} \quad (15)$$

It is observed that the pair $\boldsymbol{M^+}$ and $\boldsymbol{\bar{M}^-}$ satisfies the usual duality between what is "more or less" certain, i.e., necessarily true, and what is "more or less" possibly true. Here the meaning of presence or absence of a manifestation is related to the presence or absence of CTC, which confirm that an outage affects all the customers of zone m_j. In this way the uncertainty associated to CTC is considered, this is necessary because the CTC received may be related to a low voltage problem instead that a MV outage. In this work, only outages caused by the operation of MV protective devices (reclosers, sectionalizers, branch fuses) are analyzed. The levels of certainty of (13), (14) are chosen by the operators based on their criteria and they depend mostly on the type (residential, business, industrial, etc) and amount of customers who have called and on the description of the suspected problem (if available). It may be convenient to assign linguistic qualifiers to these certainty levels in order to facilitate the communication among operators, crews and customers. In this work, the qualifiers of Table I are used [11].

TABLE I

Linguistic Qualifier	$\mu^+_{m_j}$	$\mu^-_{m_j}$
Certain	1.00	0.00
Almost certain	0.70	0.00
Likely	0.30	0.00
Unknown	0.00	0.00

The cause-effect relationship between disorders and manifestations is modeled by matrices $\mathbf{\mu C^+}$ (16) and $\mathbf{\mu C^-}$ (17). The elements of $\mathbf{\mu C^+}$ are calculated by means of the T2-FLS of Fig. 2 and they model the relationship among all the disorders and their "more or less" certain manifestations. The elements of $\mathbf{\mu C^-}$ models the relationship among the disorders and their "more or less" impossible manifestations. Here μc_{ij}^+ and μc_{ij}^- express the level of certainty of presence and absence of manifestation m_j when disorder d_i alone is present.

$$\mathbf{\mu C^+} = \begin{bmatrix} \mu c^+_{11} & \mu c^+_{12} & \cdots & \mu c^+_{1n} \\ \mu c^+_{21} & \mu c^+_{22} & \cdots & \mu c^+_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu c^+_{n1} & \mu c^+_{n2} & \cdots & \mu c^+_{nn} \end{bmatrix} \quad (16)$$

$$\mathbf{\mu C^-} = \begin{bmatrix} \mu c^-_{11} & \mu c^-_{12} & \cdots & \mu c^-_{1n} \\ \mu c^-_{21} & \mu c^-_{22} & \cdots & \mu c^-_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu c^-_{n1} & \mu c^-_{n2} & \cdots & \mu c^-_{nn} \end{bmatrix} \quad (17)$$

The matrix representation has been selected for convenience, in order to facilitate the explanation of the procedure of solution. Formally, as it is explained in [10], [11] the ith row of $\mathbf{\mu C^+}$ is the fuzzy set of manifestations which are "more or less" certainly present when disorder d_i alone is present ($\mathbf{M}(\boldsymbol{d_i})^+$). The ith row of $\mathbf{\mu C^-}$ is the fuzzy set of manifestations which are "more or less" certainly absent when disorder d_i alone is present ($\mathbf{M}(\boldsymbol{d_i})^-$). Here $\mathbf{\mu C^+}$ plays a double role, from the RT perspective μc_i^+ represents the certainty of operation of the ith protective device. However, from the ERT perspective μc_i^+ represents a causality relation between the operation of the ith protective device and the reception of CTC from its protected zones. The ERT interpretation is based on the following approximate reasoning: the more certain the operation of a protective device, the more certain the reception of CTC from its protected zones. Furthermore, the value of μc_{ij}^+ is the same for all the m zones located downstream of the ith protective device (18). This simplification is based on the following analysis: as the outage duration increases, CTC are going to be received from all the zones affected (but in an unknown order). It is then verified that a relation of necessity (or certainty) exists among the operation of ith protective device and the reception of CTC from the zones located downstream of this device. This relationship also justifies the adoption of the model proposed in [10], [11].

$$\mu c^+_{i1} = \mu c^+_{i2} = \ldots = \mu c^+_{im} = \mu c^+_{i} \quad i = 1,\ldots,n \quad (18)$$

It is interesting to observe that the values of μc_j^+ are dynamic and that they depend on the RT data. $\mathbf{\mu C^-}$ on the contrary is defined according to the initial configuration of the feeder, which is assumed as known. The initial values of μc_{ij}^- are one or zero, $\mu c_{ij}^- = 1$ if m_j is never present when disorder d_i alone is present and $\mu c_{ij}^- = 0$ in any other case, these values change only if reconfiguration is executed.

Once the CTC are received, the objective is to identify the protective devices that cause them. This problem is solved using the fuzzy relational model and a procedure based on the *Parsimonious Set Covering Theory* (PSCT), this procedure is explained in section IV.

IV. Methodology of Solution

A. Calculation of $\boldsymbol{I^P}$

A fault on the MV network causes the operation of a protective device and a variation on the active power flow delivered to the feeder (ΔP). If the power flows through the protective devices for the load condition previous to the fault are known (19), it is possible to identify the operated device comparing these power flows with ΔP. The device that more closely matches ΔP is the more likely operated.

$$\mathbf{P_d} = \begin{bmatrix} P_1 & P_2 & \ldots & P_n \end{bmatrix} \quad (19)$$

The power flows through the protective devices are calculated using the load allocation procedure proposed in [1]. This procedure iteratively tunes the nodal demands of the feeder; until the total power flows calculated match the power flows measured at the HV/MV SE. The relative uncertainty of the results is small for the protective devices located closer to the HV/MV SE and increases for those located far, this is expected because the results are tuned only with respect to the HV/MV SE measurements [12]. Evidently, the calculated values are only good approximations of the real power flows. An appropriate way to consider this uncertainty is modeling the calculated power flows as fuzzy numbers. Fuzzy numbers are normal and convex fuzzy sets used in connection with applications where an explicit representation of the ambiguity and uncertainty found in numerical data is desirable, their mathematical theory is explained in [13].

Once $\mathbf{P_d}$ is calculated, every P_i is modeled by an interval type-2 triangular fuzzy number $\tilde{P}_i$, whose support is assigned according to the physical location of the corresponding protective device, i.e., narrower for those located close to the HV/MV SE and wider for those located far. Interval type-2 triangular fuzzy numbers have been chosen because they are naturally associated to the intuitive meaning of "approximately P_i". Furthermore they are more flexible and general than type-1 fuzzy numbers, which have been used in related applications [12]. Once $\tilde{P}_i$ is calculated, I_i^P is obtained by means of (20)-(22), this procedure is showed graphically in Fig. 5. It is observed that I_i^P is a type-1 interval that represents the degree of similarity between P_i and ΔP.

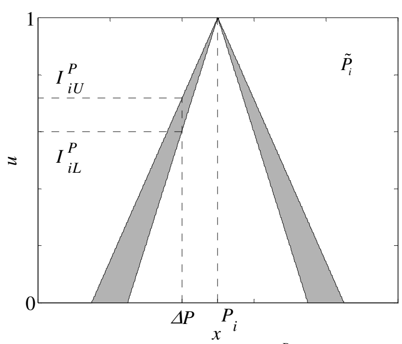

Fig. 5. Calculation of I_i^P.

$$I_i^P = \left[I_{iL}^P, I_{iU}^P \right] \tag{20}$$

$$I_{iL}^P = \underline{\mu}_{\tilde{P}_i}\left(\Delta P\right) \tag{21}$$

$$I_{iU}^P = \overline{\mu}_{\tilde{P}_i}\left(\Delta P\right) \tag{22}$$

B. Calculation of $\mathbf{I^I}$

The computation of $\mathbf{I^I}$ is based on the application of the fuzzy extension principle to traditional fault analysis [1]. The fuzzy extension principle is a mathematical tool which allows the calculation of the value of a function $y = f(x_1,x_2,\ldots,x_n)$ when the inputs (x_i) are fuzzy sets [13]. Using the extension principle, the concept of fuzzy fault currents (FFC) is introduced in order to consider the uncertainties associated to fault current calculation. In the FFC calculation, the input variables (x_i) may be deterministic and/or fuzzy, the function (f) is any of the classical fault equations and the output (y) is a FFC. The main advantage of the FFC concept is that provides a mathematical framework to compute with uncertain variables. This is very useful to consider the uncertainty associated to the value of the fault impedance (Z_f), which is the most uncertain and influential of all the variables involved in the calculation of fault currents in distribution feeders.

In the FFC, Z_f is considered as a pure resistance (R_f), which is modeled as an interval type-2 triangular fuzzy number, the other variables are considered as deterministic. The parameters of $\tilde{R}_f$ consider the usual range of values of the fault impedance, which can be obtained from the analysis of historical data of faults. The extension principle asserts that, for a function $y = f(x)$ that performs a one-to-one mapping between elements of x (of the universe X), onto elements y (of another universe Y), the images of A and $\tilde{A}$ under f are (23) and (24), where B and $\tilde{B}$ are type-1 and type-2 fuzzy sets.

$$B = f\left(A\right) = \int_{x \in X} \mu_A\left(x\right) / f\left(x\right) \tag{23}$$

$$\tilde{B} = f\left(\tilde{A}\right) = \int_{x \in X} \mu_{\tilde{A}}\left(x\right) / f\left(x\right) \tag{24}$$

Firstly, for the calculation of I_i^I, (23) is used to obtain FFC for k points along the feeder ($\tilde{I}_k$). In this step, $f(x)$ is a fault equation, e.g., (25), and $\tilde{A}$ is $\tilde{R}_f$. In (25), I_{fk} is the magnitude of the fault current for a single line-to-ground fault at point k and U_{pfk} is the pre-fault voltage. It is assumed that the positive and negative sequence impedances of the feeder are the same (R_k^+, X_k^+). Moreover, an approximate equation (based on the principle of superposition) is used to consider the contribution of the load current (26). The pre-fault load current ($\tilde{I}_{pf}$) also may be fuzzy, however in this work it is considered as crisp.

$$I_{fk} = 3U_{pfk} \Big/ \sqrt{\left(2R_k^+ + R_k^0 + 3R_f\right)^2 + \left(2X_k^+ + X_k^0\right)^2} \tag{25}$$

$$\tilde{I}_k = \tilde{I}_{fk} + \tilde{I}_{pf} \tag{26}$$

Afterwards, the index I_k^I is obtained by means of (27)-(29), this procedure is similar to the one showed in Fig. 5. Here I_k^I is the degree of similarity between I_k (the equivalent crisp fault current calculated at point k) and the magnitude of the fault current measured at the HV/MV SE (I_{sc}).

$$I_k^I = \left[I_{kL}^I, I_{kU}^I \right] \tag{27}$$

$$I_{kL}^I = \underline{\mu}_{\tilde{I}_k}\left(I_{sc}\right) \tag{28}$$

$$I_{kU}^P = \overline{\mu}_{\tilde{I}_k}\left(I_{sc}\right) \tag{29}$$

Finally, I_i^I is calculated by means of (30)-(32), here m is the number of points protected by the ith protective device. It is observed that I_i^I is also a type-1 interval.

$$I_i^I = \left[I_{iL}^I, I_{iU}^I \right] \tag{30}$$

$$I_{iL}^I = \max\left(I_{kL}^I\right), \quad k = 1,\ldots,m \tag{31}$$

$$I_{iU}^I = \max\left(I_{kU}^I\right), \quad k = 1,\ldots,m \tag{32}$$

C. Calculation of $\mathbf{I^Q}$

For the calculation of $\mathbf{I^Q}$, it is necessary to select additional influential variables in order to compensate the lack of RT data and to obtain an approximate idea of the actual performance of the feeder. These data may be quantitative and/or qualitative and generally are vague and uncertain. In this work three variables are selected, the frequency of operation of protective devices (33), the time elapsed since the last full maintenance was executed on the zone protected by every device (34) and the current weather conditions (35).

$$\mathbf{F^O} = \begin{bmatrix} F_1^O & F_2^O & \dots & F_n^O \end{bmatrix} \quad (33)$$

$$\mathbf{T^M} = \begin{bmatrix} T_1^M & T_2^M & \dots & T_n^M \end{bmatrix} \quad (34)$$

$$\mathbf{W} = \begin{bmatrix} W_1 & W_2 & \dots & W_n \end{bmatrix} \quad (35)$$

These variables are a good selection because they provide an approximate idea of how prone are the protective devices to operate. $\mathbf{I^Q}$ is calculated by means of the T2-FLS of Fig. 6, which uses 27 rules to model the expert knowledge of distribution operators. Every rule has three antecedents (F_i^O, T_i^M and W_i) and one consequent (I_i^Q). Every antecedent is described by three linguistic terms and the consequent by nine, these terms are modeled by interval type-2 triangular fuzzy sets. The rules relate the input and output variables using these linguistic terms, in this way the logical deductions made by expert operators are modeled. Some examples are:

If F_i^O is *Low* and T_i^M is *Short* and W_i is *Good* then I_i^Q is *Insignificant*
If F_i^O is *Medium* and T_i^M is *Long* and W_i is *Regular* then I_i^Q is *Medium*
If F_i^O is *High* and T_i^M is *Medium* and W_i is *Bad* then I_i^Q is *Very Big*

The universe of discourse of F_i^O, T_i^M and W_i is also [0, 1]; the closer the index to one, the higher its influence on the output. Two types of inputs are used, when quantitative data is available, the inputs are modeled as type-2 singletons. When qualitative data (linguistic terms) are available, the inputs are modeled as interval type-2 triangular fuzzy sets.

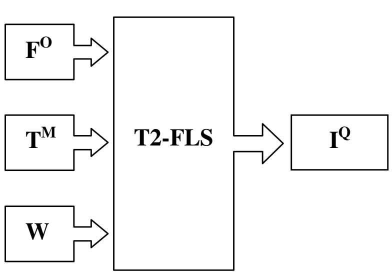

Fig. 6. Scheme of the T2-FLS proposed for the calculation of $\mathbf{I^Q}$.

The model is flexible enough to calculate when the inputs are all quantitative, all qualitative or a combination of both. I_i^Q is the type-reduced fuzzy set of this T2-FLS (36), it is obtained using the extended sup-★ composition (under maximum-product norms) and center of sets (COS) type-reduction [9]. These operations also are used for the T2-FLS of Fig. 2 (μc_i^+).

$$I_i^Q = \left[I_{iL}^Q, I_{iU}^Q \right] \quad (36)$$

D. Calculation of $\boldsymbol{\mu C^+}$

The T2-FLS of Fig. 2 also uses 27 rules to model the knowledge of operators. Every rule has three antecedents (I_i^P, I_i^I and I_i^Q) and one consequent (μc_i^+). Every antecedent is described by three linguistic terms and the consequent by nine. These terms are modeled by interval type-2 triangular fuzzy sets. The inputs of this T2-FLS are type-1 intervals (I_i^P, I_i^I and I_I^Q) and its type-reduced fuzzy set is μc_i^+ (37).

$$\mu c_i^+ = \left[\mu c_{iL}^+, \mu c_{iU}^+ \right] \quad (37)$$

The identification of the operated protective device is a binary decision problem; therefore it is necessary to select a decision condition. The devices that satisfy the decision condition are considered as candidates for operation. If only one device satisfies the decision condition, then the OC problem is solved using the RT data. On the contrary, it is necessary to use the ERT data to confirm which device has operated and to finish the inference. Based on the binary nature of the problem and the universe of discourse of the operation indexes, a decision value of 0.5 is defined [1], [6], [9]. The decision condition is defined using the type-reduced fuzzy set and the decision value, in this way the uncertainties of the output are considered and a *soft decision condition* is obtained. In this work (38) is used, because it represents a trade-off solution with respect to the degree of risk of the selection.

$$\mu c_{i\,C}^+ > 0.5, \text{ where } \mu c_{i\,C}^+ = (\mu c_{i\,L}^+ + \mu c_{i\,U}^+)/2 \quad (38)$$

E. Solution of the ERT model

The ERT model is solved using a procedure based on fuzzy abductive inference and PSCT, which is used as a computation algorithm to generate the solution hypotheses. First a crisp diagnosis problem is formulated, $\boldsymbol{\mu}\mathbf{C^+}$ and $\boldsymbol{M^+}$ are replaced by their crisp equivalents $\mathbf{C}$ (39) and $\mathbf{M^+}$ (40) [14].

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \quad (39)$$

$$\mathbf{M^+} = \left\{ m_1 \quad m_2 \quad \dots \quad m_k \right\} \quad (40)$$

These equivalents are obtained replacing the elements of $\boldsymbol{\mu}\mathbf{C^+}$ and $\boldsymbol{M^+}$ that are greater than zero by ones. The elements of $\boldsymbol{M^+}$ which are equal to zero are not taken into account, therefore $\mathbf{M^+}$ has only r elements, where $r \leq n$. The crisp problem is defined as the quad-tuple $\mathbf{P}$=<$\mathbf{D}$, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{M^+}$>, where $\mathbf{D}$ and $\mathbf{M}$ are (11) and (12), respectively. To solve this problem, it is necessary to explain the concepts of *effects*, *causes* and *covers* [14]. For any $d_i \in \mathbf{D}$ and $m_j \in \mathbf{M}$ in $\mathbf{P}$:

- effects(d_i) = $\{m_j \mid c_{ij} > 0\}$
- causes(m_j) = $\{d_i \mid c_{ij} > 0\}$
- For any $D_I \subseteq \mathbf{D}$, effects(D_I) = $\bigcup$effects(d_i), $d_i \in D_I$
- For any $M_J \subseteq \mathbf{M}$, causes(M_I) = $\bigcup$causes(m_j), $m_j \in M_J$
- The set D_I is a cover of M_J if $M_J \subseteq$ effects (D_I)

The premise of the PSCT is that a plausible diagnosis hypothesis of **P** must be a parsimonious cover of $\mathbf{M}^+$. The parsimonious covers are known as explanations, the most plausible explanation is the solution of **P**. A cover is parsimonious if satisfies the principle of parsimony or principle of simplicity (commonly known as Ocam's razor). This principle states that simple solutions are preferred over complex ones, this is intuitively satisfactory because it is well known that simple faults are more likely to occur than multiple faults. Up to date several different criteria have been proposed in the literature to evaluate the parsimony of a cover. In this work, the irredundancy criterion, which has been successfully used in many engineering applications [14], is selected. This criterion states that a cover D_I of $\mathbf{M}^+$ is an explanation if it has no proper subsets that also cover $\mathbf{M}^+$. The explanations of $\mathbf{M}^+$ are calculated using the interactive algorithm presented in [15]. Afterwards, the plausibility of every explanation is evaluated using (41), (42).

$$P(d_i) = \vee\left\{ \underset{j=1,n}{\vee}\left[\wedge\left(\mu c_{ij}^{+}, \mu_{m_j}^{+} \right) \right], \underset{j=1,n}{\vee}\left[\wedge\left(\mu c_{ij}^{-}, \mu_{m_j}^{-} \right) \right] \right\} \qquad (41)$$

$$P(\mathbf{D_s}) = \sum_{i=1}^{nd} \frac{P(d_i)}{nd} \qquad (42)$$

The first part of equation (41) evaluates the consistency among the "more or less" certain manifestations of d_i and the "more or less" certainly present manifestations. Its second part evaluates the consistency among the "more or less" impossible manifestations of d_i and the "more or less" certainly absent manifestations. The algebraic sum (43) and product (44) are selected to evaluate the t-conorm ($\vee$) and t-norm ($\wedge$) respectively. The algebraic sum is an adequate operator because it reflects the contribution of every component to their union. This is convenient, because the plausibility of d_i must increases, as more of its certain manifestations are present.

$$\mu_{A\vee B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\cdot\mu_B(x) \qquad (43)$$

$$\mu_{A\wedge B}(x) = \mu_A(x)\cdot\mu_B(x) \qquad (44)$$

Equation (42) evaluates the relative plausibility of the sth explanation and *nd* is the number of disorders of every explanation. It is observed that this equation "penalizes" multiple disorder explanations. This is intuitively satisfactory because these explanations are generally less plausible. The results of (42) are ordered and presented to the operator. The explanation that maximizes (42), is the solution of the OC in ERT. However, all the explanations are presented in order to provide a general outlook of the plausible solutions; in this way the operator can take an informed final decision.

V. Results

Two cases have been simulated on a real feeder of the city of San Juan, Argentina (Fig. 7). This feeder has total power flows of $P_m = 1222$ kW and $Q_m = 516$ kVAR, a voltage level of 13.2 kV, 172 nodes and 55 protective devices. Only the HV/MV SE is monitored in RT by SCADA. The results of the RT model for different single-phase power flow variations ΔP (kW) and single line-to-ground fault currents I_{sc} (A) are showed on Tables II and V, respectively. These values of ΔP are approximations of the crisp values of F2403 (61.19 kW) and F1369 (33.77 kW) calculated using the load allocation method proposed in [1]. The values of I_{sc} cover the possible fault currents for single line-to-ground faults located along the zone protected by these devices, for 10 Ω $\le R_f \le$ 20 Ω. The values of F_i^O, T_i^M and W_i for F2403 and F1369 are ($F_i^O = 0$, T_i^M = Medium and W_i = Regular).

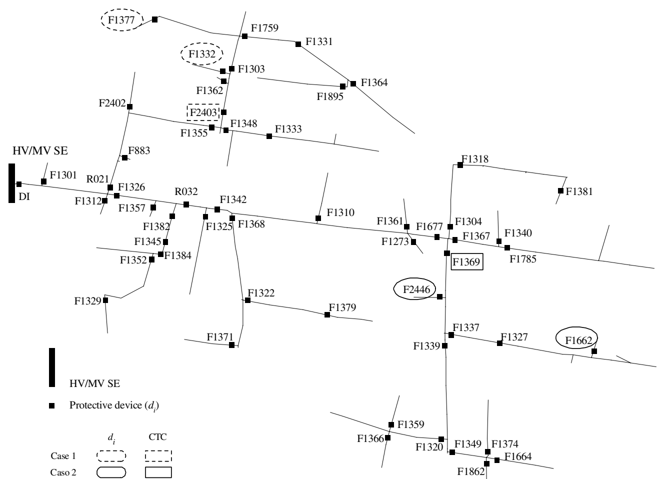

Fig. 7. Test feeder

In the first case, six fault situations have been simulated on the zone protected by fuses F2403. It is observed that the RT results are not conclusive enough to take a reliable decision. Therefore the CTC are used to finish the inference. For the first situation (ΔP = 57 kW, I_{sc} = 400 A), a CTC is received from the zone protected by F1377, the qualifier selected by the operator is "almost certain". The results of the ERT model are presented on Table III, it is observed that these results are more conclusive than those of Table II. Therefore F2403 is selected as the operated device. Afterwards a second CTC is received from the zone protected by F1332, the qualifier selected is "likely". Using both CTC, the results of Table IV are obtained. Predictably, the plausibility of F2403 has increased; these results reinforce the initial selection.

TABLE II

ΔP	I_{sc}	d_i	$I_{i\,L}^{P}$	$I_{i\,U}^{P}$	$I_{i\,L}^{I}$	$I_{i\,U}^{I}$	$I_{i\,L}^{Q}$	$I_{i\,U}^{Q}$	$\mu c_{i\,L}^{+}$	$\mu c_{i\,C}^{+}$	$\mu c_{i\,U}^{+}$
57	400	F1677	0.80	0.90	0.86	0.91	0.20	0.31	0.74	0.79	0.85
		F2403	0.66	0.83	0.85	0.90	0.20	0.31	0.69	0.74	0.78
	500	F2403	0.66	0.83	0.65	0.77	0.20	0.31	0.64	0.68	0.72
		F1677	0.80	0.90	0.22	0.48	0.20	0.31	0.55	0.59	0.64
	600	F2403	0.66	0.83	0.25	0.50	0.20	0.31	0.52	0.56	0.61
		F1677	0.80	0.90	0.00	0.20	0.20	0.31	0.50	0.54	0.58
65	400	F2403	0.69	0.84	0.85	0.90	0.20	0.31	0.70	0.75	0.80
		F1677	0.07	0.54	0.86	0.91	0.20	0.31	0.45	0.51	0.57
	500	F2403	0.69	0.84	0.65	0.77	0.20	0.31	0.65	0.69	0.73
		F1312	0.35	0.68	0.92	0.95	0.20	0.31	0.57	0.63	0.70
	600	F1312	0.35	0.68	0.70	0.80	0.20	0.31	0.51	0.57	0.63
		F2403	0.69	0.84	0.25	0.50	0.20	0.31	0.53	0.57	0.62

TABLE III

d_i	Explanations			
	D_1	**D_2**	**D_3**	**D_4**
F2403	1	0	0	0
F1355	0	1	0	0
F1303	0	0	1	0
F1355	0	0	0	0
$P(\mathbf{D_s})$	0.51	0.27	0.26	0.22

TABLE IV

d_i	Explanations			
	$\mathbf{D}_1$	$\mathbf{D}_2$	$\mathbf{D}_3$	$\mathbf{D}_4$
F2403	1	0	0	0
F1355	0	1	0	0
F1303	0	0	1	0
F1332	0	0	1	1
F1377	0	0	0	1
$P(\mathbf{D}_x)$	0.62	0.36	0.18	0.16

In the second case, six fault situations have been analysed. Also in this case the RT results are not conclusive to take a reliable decision. For the fourth situation (ΔP = 36 kW, I_{sc} = 300 A), a CTC is received from the zone protected by F1662, the qualifier selected is "almost certain". The results of the ERT model are presented on Table VI, these results are more conclusive than those of Table V. Therefore it is acceptable to affirm that F1369 is the operated device. If a second CTC is received from the zone protected by F1339 and the qualifier "almost certain" is selected, the results of Table VII are obtained. Here, the plausibility of F1369 has increased, confirming the initial selection.

TABLE V

ΔP	I_{sc}	d_i	$I_i^P{}_L$	$I_i^P{}_U$	$I_i^I{}_L$	$I_i^I{}_U$	$I_i^Q{}_L$	$I_i^Q{}_U$	$\mu c_i^+{}_L$	$\mu c_i^+{}_C$	$\mu c_i^+{}_U$
30	300	F1369	0.72	0.81	0.44	0.63	0.20	0.31	0.58	0.63	0.68
		F1339	0.00	0.00	0.99	0.99	0.34	0.49	0.36	0.38	0.40
	400	F1369	0.72	0.81	0.79	0.86	0.20	0.31	0.68	0.73	0.78
		F1303	0.24	0.62	0.97	0.98	0.25	0.37	0.54	0.59	0.64
	500	F1369	0.72	0.81	0.14	0.43	0.20	0.31	0.51	0.55	0.59
		F1303	0.24	0.62	0.40	0.60	0.25	0.37	0.39	0.45	0.52
36	300	F1369	0.84	0.89	0.44	0.63	0.20	0.31	0.62	0.67	0.73
		F1303	0.91	0.96	0.04	0.36	0.25	0.37	0.57	0.61	0.65
	400	F1303	0.91	0.96	0.97	0.98	0.25	0.37	0.84	0.89	0.93
		F1369	0.84	0.89	0.79	0.86	0.20	0.31	0.73	0.78	0.83
	500	F1303	0.91	0.96	0.40	0.60	0.25	0.37	0.64	0.70	0.76
		F1369	0.84	0.89	0.14	0.43	0.20	0.31	0.54	0.59	0.63

TABLE VI

d_i	Explanations						
	$\mathbf{D}_1$	$\mathbf{D}_2$	$\mathbf{D}_3$	$\mathbf{D}_4$	$\mathbf{D}_5$	$\mathbf{D}_6$	$\mathbf{D}_7$
F1369	1	0	0	0	0	0	0
F1337	0	1	0	0	0	0	0
F1677	0	0	1	0	0	0	0
F1327	0	0	0	1	0	0	0
F1342	0	0	0	0	1	0	0
F1326	0	0	0	0	0	1	0
F1662	0	0	0	0	0	0	1
$P(\mathbf{D}_x)$	0.46	0.22	0.13	0.12	0.08	0.06	0.06

TABLE VII

d_i	Explanations						
	$\mathbf{D}_1$	$\mathbf{D}_2$	$\mathbf{D}_3$	$\mathbf{D}_4$	$\mathbf{D}_5$	$\mathbf{D}_6$	$\mathbf{D}_7$
F1369	1	0	0	0	0	0	0
F1677	0	1	0	0	0	0	0
F1337	0	0	1	0	0	0	0
F1327	0	0	0	1	0	0	0
F1342	0	0	0	0	0	1	0
F1339	0	0	1	1	1	0	0
F1362	0	0	0	0	1	0	0
F1326	0	0	0	0	0	0	1
$P(\mathbf{D}_x)$	0.71	0.24	0.24	0.19	0.16	0.15	0.12

VI. CONCLUSIONS

In this work, an approach aiming at inferring the OC of distribution networks using RT and ERT data has been presented. The results demonstrate that the methodology can consider the uncertainties of different sources of data (quantitative and qualitative) and is able to consolidate them by means of T2-FLS. Type-2 fuzzy sets and T2-FLS bring flexibility and robustness to the RT model, making possible to consider the uncertainties associated to the inputs and meaning of words used in conventional rule-based T1-FLS. This facilitates the initial implementation of the RT model because the available uncertain data and expert knowledge can be used in a systematic way. The methodology provides a sound mathematical framework for the integration of RT data, CTC and the expert knowledge of the distribution operators. The results are dynamically improved as more data are received and are encouraging for a practical implementation.

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VIII. BIOGRAPHIES

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