

CONTRIBUTION

TITLE: **New proposal of Turbo codes for ADSL modems**

SOURCE: **VOCAL Technologies Ltd***

PROJECT: **T1E1.4, ADSL Enhancements**

ABSTRACT

This paper describes a new technique for coding and decoding signals that use turbo codes with QAM modulation for error correction.

* Contact: Juan Alberto Torres, jatorres@vocal.com,
Frederic Hirzel, fhirzel@vocal.com
Victor Demjanenko, victord@vocal.com
VOCAL Technologies Ltd.
200 John James Audubon Parkway
Buffalo, NY 14228
Phone: (716) 688-4675
Fax: (716) 639 0713

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1. INTRODUCTION.

Turbo codes present a new and very powerful error control technique, which allows communication very close to the channel capacity. Since its discovery in 1993, a lot of research has been done in the application of turbo codes in deep space communications, mobile satellite/cellular communications, microwave links, paging, in OFDM and CDMA architectures. Turbo codes have outperformed all previously known coding schemes regardless of the targeted channel. The extra coding gain offered by turbo codes can be used either to minimize bandwidth or to reduce power requirements in the link budget. Standards based in turbo codes have already been defined or are currently under investigation. Here are some examples:

- Inmarsat's new multimedia service is based on turbo codes and 16QAM that allows the user to communicate with existing Inmarsat 3 spot beam satellites from a notebook-sized terminal at 64 kbit/s.
- The Third Generation Partnership Project (3GPP) proposal for IMT-2000 includes turbo codes in the multiplexing and channel coding specification. The IMT-2000 represents the third generation mobile radio systems worldwide standard. The 3GPP objective is to harmonize similar standards proposals from Europe, Japan, Korea and the United States.
- NASA's next-generation deep-space transponder will support turbo codes and implementation of turbo decoders in the Deep Space Network is planned by 2003.
- The new standard of the Consultative Committee for Space Data Systems (CCSDS) is based on turbo codes. The new standard outperforms by 1.5 to 2.8 dB the old CCSDS standard based on concatenated convolutional code and Reed-Solomon code.
- The new European Digital Video Broadcasting (DVB) standard has also adopted turbo codes for the return channel over satellite applications.

In this paper we present a new perspective of the use of turbo codes for systems with different order of QAM modulations working at the same time. This perspective that avoids the most difficult part of turbo coding: the computing requirements. The way to avoid these computing requirements is to treat the QAM signal like two AM modulations (one in the I direction and one in the Q direction) and use the probabilities of the I and Q AM values as an input to the turbo code process.

Papers presented so far for ADSL modems using turbo codes give details of only 16QAM, and the computational effort is important.

Another innovation of the technique proposed here is the mapping used for the modulated signal, providing the most protected bits for the information bits and the least protected bits for the parity bits. In this paper we present the different possible puncturing for each order of a QAM modulated signal and we choose the one that provides better performance.

This is the first investigation in the performance of turbo codes for all constellation sizes, up to 1024 QAM. The recommended solutions achieve a target BER very close to the channel capacity for the respective spectral efficiency response. The turbo code schemes proposed here are more power efficient than the trellis coded modulation schemes used traditionally with V.32.bis or V.34 standards. The larger the constellation is the more improvements that this technique provides (i.e. for 16QAM the probabilities to compute in a classical turbo code are 16 points with 16 elements each point, with this technique the probabilities to compute are 8 (4 in each dimension) with 4 elements each, so if we use 10 iterations the computational savings is more

than $10 \cdot (16 \cdot 16 / 8 \cdot 4) = 80$ (almost two orders of magnitude); for 256QAM the computational savings is more than $10 \cdot (256 \cdot 256) / (32 \cdot 16) = 1280$ (over three orders of magnitude).

In the case of the G.992.1 this technique allow the system to work at the information rate bit of 400 kbps with E_b/N_0 below 2 dB (assuming 4QAM and spectral efficiency of 1bit/s/Hz 4 ksymbols/s and 100 tones).

We propose the use of square QAM constellations with different puncturing values as function of the signal-to-noise ratio. The square constellations allow for use of very efficient blind equalization techniques an effective maintaining I and Q independent. Taking into account the values of b_i and g_i we decide which constellation to use and with which parity.

2. CAPACITY BOUNDS

The minimum E_b/N_0 values to achieve the Shannon bound, 4QAM, 8QAM, 16QAM, 32QAM, 64QAM, 128QAM, 256QAM, 512QAM and 1024QAM bounds for spectral efficiencies from 2/3 up to 7 bits/s/Hz respectively are as in Table 1 for a BER= 10^{-5} .

Table 1. Shannon and QAM bounds.

Spectral efficiency η [bit/s/Hz]	Shannon bound [dB]	4 QAM bound [dB]	16 QAM bound [dB]	64 QAM bound [dB]	256 QAM bound [dB]	1024 QAM bound [dB]
2/3	-0.5	0.3	-0.4	-0.4	-0.4	-0.4
1	0	1.0	0.1	0.1	0.1	0.1
2	1.75	∞	2.1	2.09	2.09	2.09
3	3.7	-	4.6	4.3	4.3	4.3
4	5.6	-	∞	6.6	6.6	6.6
5	7.9	-	-	9.1	9.0	9.0
6	10.3	-	-	∞	11.7	11.7
7	12.6	-	-	-	14.5	14.5

The conversion from E_s/N_0 to E_b/N_0 is performed using the following relation

$$E_b/N_0[\text{dB}] = E_s/N_0[\text{dB}] - 10 \log_{10}(\eta) [\text{dB}] \quad (1)$$

where η is the number of information bits per symbol.

The required C/N_0 given a certain E_b/N_0 can be found using the following relation:

$$C/N_0 [\text{dB-Hz}] = E_b/N_0 [\text{dB}] + 10 \log_{10}(R_b) [\text{dB-Hz}] \quad (2)$$

where R_b is the information bit rate.

For a D-dimension modulation the following formulae are used:

$$SNR = \frac{E [|a_k^2|]}{E [|w_k^2|]} = \frac{E [|a_k^2|]}{D \sigma_N^2} = \frac{E_{av}}{D \sigma_N^2} \quad (3)$$

$$SNR = \frac{E_s}{D \frac{N_0}{2}} = \frac{\eta E_b}{D \frac{N_0}{2}} \quad (4)$$

where σ_N^2 is the noise variance in each of the D dimension and η is the number of information bits per symbol. From the above relations:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} \tag{5}$$

3. CODING

The proposed coding scheme is shown in Figure 1. The two systematic recursive codes (SRC) used are identical and are defined in Figure 2. The code is described by the generating polynomials 35o and 23o.

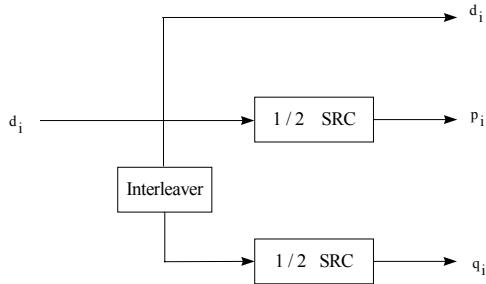


Figure 1

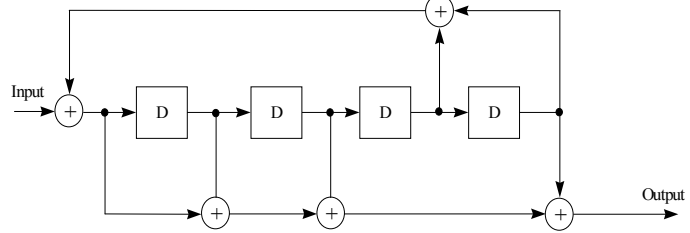


Figure 2

4. MODULATION FOR 1 BIT/S/HZ SPECTRAL EFFICIENCY.

The scheme proposed in this case combines a rate 1/2 coding scheme with 4QAM.

4.1 Puncturing

In order to obtain a code rate of 1/2, every other bit of the parity bits p and q from Figure 1 are punctured. The puncturing pattern is given in Table 2.

Table 2. Puncturing and Mapping for Rate 1/2 4QAM.

Information bit (d)	d ₁	d ₂
Parity bit (p)	p ₁	-
Parity bit (q)	-	q ₂
2AM symbol (I)	(u ₁) = (d ₁)	(u ₁) = (d ₂)
2AM symbol (Q)	(u ₂) = (p ₁)	(u ₂) = (q ₂)
4QAM symbol (I, Q)	(I,Q) = (u ₁ , u ₂) = (d ₁ , p ₁)	(I,Q) = (u ₁ , u ₂) = (d ₂ , q ₂)

4.2. Modulation

A 4QAM scheme is shown in Figure 3. An equivalent 2AM modulation is shown in Figure 4.

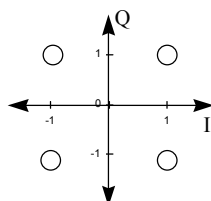


Figure 3

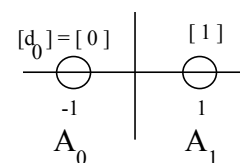


Figure 4

At time k, the symbol $u^k = (u_1^k, u_2^k)$ is sent through the channel and the point r^k in two dimensional space is received. For a 4QAM constellation with points at -A and A, The E_{av} is:

$$E_{av} = \frac{4(A^2 + A^2)}{4} = 2A^2 \tag{6}$$

For a rate 2/4 code and 4QAM, the noise variance is:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right) = 2 A^2 \left(\frac{2 \times 1 \times E_b}{N_0} \right)^{-1} = A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (7)$$

4.3 Bit probabilities.

For an AWGN channel the following expressions need to be evaluated:

$$LLR(u_i^k) = \log \left(\frac{\sum_{i=1}^1 \exp \left\{ -\frac{1}{2\sigma_N^2} (I^k - a_{1i}^k)^2 \right\} J}{\sum_{i=1}^1 \exp \left\{ -\frac{1}{2\sigma_N^2} (I^k - a_{0i}^k)^2 \right\} J} \right) = \log \left(\frac{\exp \left\{ -\frac{1}{2\sigma_N^2} (I^k - A_1)^2 \right\} J}{\exp \left\{ -\frac{1}{2\sigma_N^2} (I^k - A_0)^2 \right\} J} \right) \quad (8)$$

$$LLR(u_2^k) = \log \left(\frac{\sum_{i=1}^1 \exp \left\{ -\frac{1}{2\sigma_N^2} (Q^k - a_{1i}^k)^2 \right\} J}{\sum_{i=1}^1 \exp \left\{ -\frac{1}{2\sigma_N^2} (Q^k - a_{0i}^k)^2 \right\} J} \right) = \log \left(\frac{\exp \left\{ -\frac{1}{2\sigma_N^2} (Q^k - B_1)^2 \right\} J}{\exp \left\{ -\frac{1}{2\sigma_N^2} (Q^k - B_0)^2 \right\} J} \right) \quad (9)$$

The above LLRs are used as inputs to the turbo encoder. There is no need to compute the 4 LLRs for all symbols because I and Q signals are treated independently. Also the simulations for the 2 bit-LLR values are reduced to one term each. Due to the puncturing, with one in two parity bits being transmitted, the expected performance will be lower when compared with the non-punctured scheme.

4.4 Simulations Results.

Figure 5 shows the performance of a turbo code using a 1024 bit S-type interleaver. The target BER of 10^{-7} for a 1,024 information bit interleaver can be achieved at $E_b/N_0 = 2.1$ dB

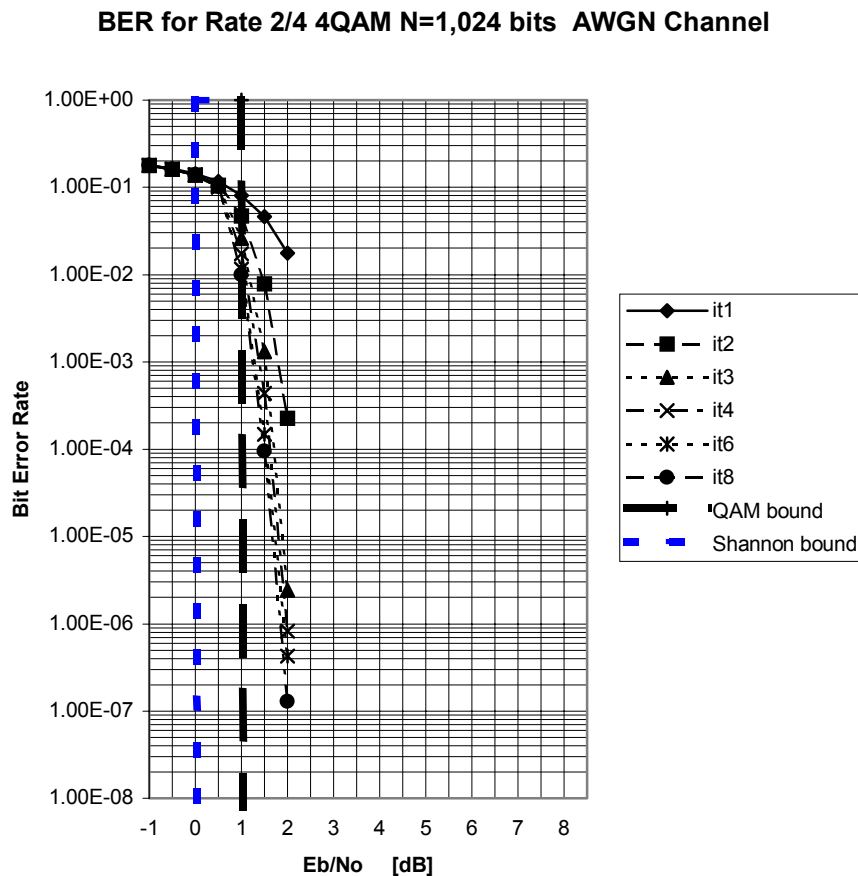


Figure 5

5. CODING AND MODULATION FOR 2 BIT/S/HZ SPECTRAL EFFICIENCY.

The investigated combines 2/4 coding scheme with 16QAM.

5.1 Puncturing.

In order to obtain a rate 2/4 code, every other bit of the parity bits p and q from Figure 1 are punctured. The puncturing patten is given in Table 3.

Table 3. Puncturing and Mapping for Rate 2/4 16QAM.

Information bit (d)	d ₁	d ₂
parity bit (p)	p ₁	-
parity bit (q)	-	q ₂
4AM symbol (I)	(u ₁ , u ₂) = (d ₁ , p ₁)	
4AM symbol (Q)	(u ₃ , u ₄) = (d ₂ , p ₂)	
16 QAM symbol (I, Q)	(I, Q) = (u ₁ , u ₂ , u ₃ , u ₄) = (d ₁ , p ₁ , d ₂ , p ₂)	

5.2 Modulation

At time k, the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k)$ is sent through the channel and the point r^k in two dimensional space is received. For a 16QAM constellation with points at -3A, -A, A and 3A, The E_{av} is:

$$E_{av} = \frac{4(A^2 + A^2) + 8(A^2 + 9A^2) + 4(9A^2 + 9A^2)}{16} = 10A^2 \quad (10)$$

For a rate 1/2 code and 16QAM, the noise variance in each dimension is:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right) = 10A^2 \left(\frac{2 \times 2 \times E_b}{N_0} \right)^{-1} = 2.5A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (11)$$

It is assumed that the time k, u_1^k and u_2^k modulate the I component and u_3^k and u_4^k modulate the Q component for a 16QAM scheme. The symbol u^k symbol has the following mapping: $u^k = (u_1^k, u_2^k, u_3^k, u_4^k) = (d^i, p^i, d^{i+1}, q^{i+1})$. The parity bits are mapped to the least protected bits of the QAM symbol. Note that k denotes the symbol time index and i the information bit time index. This means a puncturing of one in two parity bits. Considering two independent Gaussian noise sources with identical variance σ_N^2 , the LLR can be determined independently for each I and Q.

At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the 4 bit-LLR values. The mapping of the information bit is made to the most protected bit in each dimension (u_1^k for the I signal and u_3^k for the Q signal). In order to estimate the performance of this scheme, rate 1/2 turbo code and 4AM modulation is used, instead of 16QAM modulation.

5.3 Bit probabilities.

For an AWGN channel the following expressions need to be evaluated:

$$LLR(u_i^k) = \log \left(\frac{\sum_{i=1}^2 \exp\left[-\frac{1}{2\sigma_N^2} (I^k - a_{1i}^k)^2\right] J}{\sum_{i=1}^2 \exp\left[-\frac{1}{2\sigma_N^2} (I^k - a_{0i}^k)^2\right] J} \right) = \log \left(\frac{\exp\left[-\frac{1}{2\sigma_N^2} (I^k - A_2)^2\right] J + \exp\left[-\frac{1}{2\sigma_N^2} (I^k - A_3)^2\right] J}{\exp\left[-\frac{1}{2\sigma_N^2} (I^k - A_0)^2\right] J + \exp\left[-\frac{1}{2\sigma_N^2} (I^k - A_1)^2\right] J} \right) \quad (12)$$

$$LLR(u_2^k) = \log \left(\frac{\sum_{i=1}^2 \exp\left[-\frac{1}{2\sigma_N^2} (I^k - d_{1i}^k)^2\right] J}{\sum_{i=1}^2 \exp\left[-\frac{1}{2\sigma_N^2} (I^k - d_{0i}^k)^2\right] J} \right) = \log \left(\frac{\exp\left[-\frac{1}{2\sigma_N^2} (I^k - A_1)^2\right] J + \exp\left[-\frac{1}{2\sigma_N^2} (I^k - A_3)^2\right] J}{\exp\left[-\frac{1}{2\sigma_N^2} (I^k - A_0)^2\right] J + \exp\left[-\frac{1}{2\sigma_N^2} (I^k - A_2)^2\right] J} \right) \quad (13)$$

$$LLR(u_3^k) = \log \left(\frac{\sum_{i=1}^2 \exp\left[-\frac{1}{2\sigma_N^2} (Q^k - d_{1i}^k)^2\right] J}{\sum_{i=1}^2 \exp\left[-\frac{1}{2\sigma_N^2} (Q^k - d_{0i}^k)^2\right] J} \right) = \log \left(\frac{\exp\left[-\frac{1}{2\sigma_N^2} (Q^k - B_2)^2\right] J + \exp\left[-\frac{1}{2\sigma_N^2} (Q^k - B_3)^2\right] J}{\exp\left[-\frac{1}{2\sigma_N^2} (Q^k - B_0)^2\right] J + \exp\left[-\frac{1}{2\sigma_N^2} (Q^k - B_1)^2\right] J} \right) \quad (14)$$

$$LLR(u_4^k) = \log \left(\frac{\sum_{i=1}^2 \exp\left[-\frac{1}{2\sigma_N^2} (Q^k - d_{1i}^k)^2\right] J}{\sum_{i=1}^2 \exp\left[-\frac{1}{2\sigma_N^2} (Q^k - d_{0i}^k)^2\right] J} \right) = \log \left(\frac{\exp\left[-\frac{1}{2\sigma_N^2} (Q^k - B_1)^2\right] J + \exp\left[-\frac{1}{2\sigma_N^2} (Q^k - B_3)^2\right] J}{\exp\left[-\frac{1}{2\sigma_N^2} (Q^k - B_0)^2\right] J + \exp\left[-\frac{1}{2\sigma_N^2} (Q^k - B_2)^2\right] J} \right) \quad (15)$$

The above LLRs are used as inputs to the turbo encoder. There is no need to compute the 16LLRs for all symbols because I and Q signals are treated independently. Also the simulations for the 4 bit-LLR values are reduced to 2 terms each. Due to the puncturing, with one in two parity bits being transmitted, the expected performance will be lower when compared with the non punctured scheme.

5.4 Simulations Results.

The rate 2/4 16QAM scheme described in this chapter achieves the target BER at less than 1.5 dB from capacity. The implementation in hardware is feasible and it can be used at very high data rates. The target BER of 10^{-7} can be achieved at $E_b/N_0 = 4.5$ dB for N=1024 information bits, as shown in Figure 6.

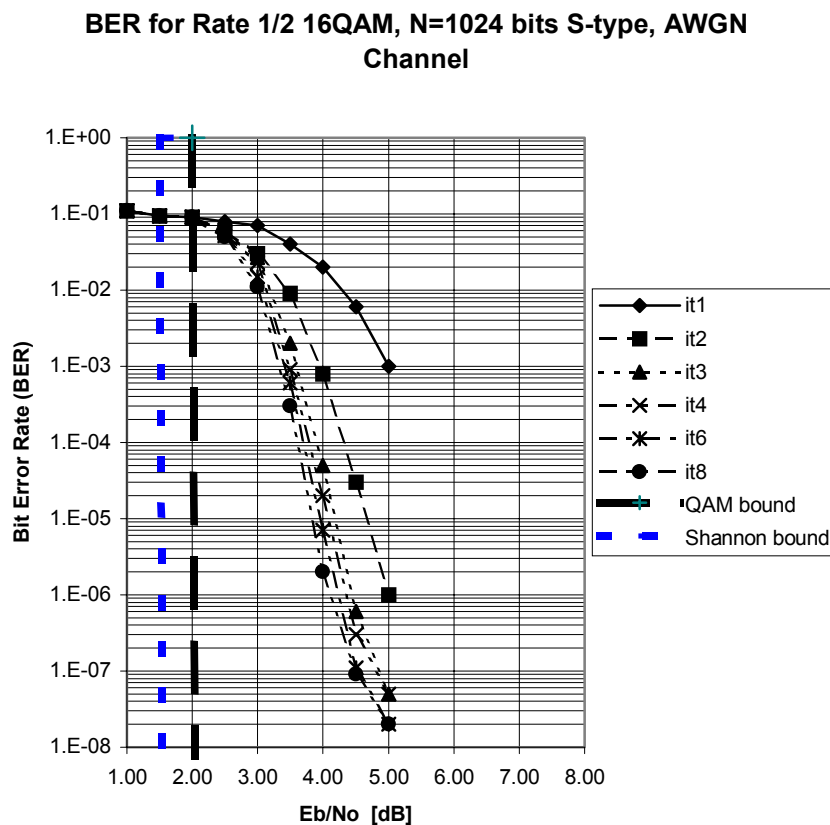


Figure 6

6. CODING AND MODULATION FOR 3 BIT/S/HZ SPECTRAL EFFICIENCY.

Three options are investigated in this section. The first scheme combines a rate 3/4 coding scheme with 16QAM. The second scheme combines a rate 3/4 coding scheme with 64QAM.

6.1 Option 1 Rate 3/4 Turbo code and 16QAM

6.1.1 Puncturing.

In order to obtain a rate 3/4 code, the puncturing pattern given in Table 4 is used.

Table 4. Puncturing and Mapping for Rate 3/4 16QAM

Information bit (d)	d ₁	d ₂	d ₃	d ₄	d ₅	d ₆
parity bit (p)	-	p ₂	-	-	-	-
parity bit (q)	-	-	-	-	q ₅	-
4AM symbol (I)	(d ₁ , d ₂)			(d ₄ , d ₅)		
4AM symbol (Q)	(d ₃ , p ₂)			(d ₆ , p ₅)		
16 QAM symbol (I,Q)	(I,Q) = (d ₁ , d ₂ , d ₃ , p ₂)			(I,Q) = (d ₄ , d ₅ , d ₆ , q ₅)		

6.1.2 Modulation

It is assumed that at time k, u₁^k and u₂^k modulate the I component and u₃^k and u₄^k modulate the Q component of a 16QAM scheme. In order to estimate the performance of this scheme, rate 3/4 turbo codes and 4 AM modulation are used. For a rate 3/4 code and 16QAM, noise variance is:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} = 10 A^2 \left(\frac{2x3x E_b}{N_0} \right)^{-1} = \frac{10}{6} A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (16)$$

The puncturing and mapping scheme is shown in Table 4 for 6 consecutive information bits that are encoded into 8 coded bits, therefore two 16QAM symbols.

6.1.3 Bit Probabilities

For each received symbol, the bit probabilities are computed as described in equations (12)-(15).

6.1.4 Simulation Results

Figure 7 shows the simulation results for 6,144 information bits with S-type interleaver. A BER of 10⁻⁷ can be achieved after 8 iterations at E_b/N₀ = 5.75 dB for N = 6144 information bits.

6.2 Option 2 Rate 3/6 Turbo code and 64QAM

6.2.1 Puncturing.

In order to obtain a rate 3/6 code, the puncturing patten used is given in Table 5.

Table 5. Puncturing and Mapping for Rate 3/6 64QAM

Information bit (d)	d ₁	d ₂	d ₃	d ₄	d ₅	d ₆
parity bit (p)	p ₁	-	p ₃	-	p ₅	-
parity bit (q)	-	Q ₂	-	q ₄	-	q ₆
8AM symbol (I)	(d ₁ , d ₂ , p ₁)			(d ₄ , d ₅ , q ₄)		
8AM symbol (Q)	(d ₃ , p ₃ , q ₂)			(d ₆ , p ₅ , q ₆)		
64 QAM symbol (I,Q)	(I,Q)=(d ₁ ,d ₂ , p ₁ ,d ₃ , p ₃ ,q ₂)			(I,Q)=(d ₄ ,d ₅ , q ₄ ,d ₆ , p ₅ ,q ₆)		

**BER for Rate 3/4 16QAM, N=6,144 bits S-type,
AWGN Channel**

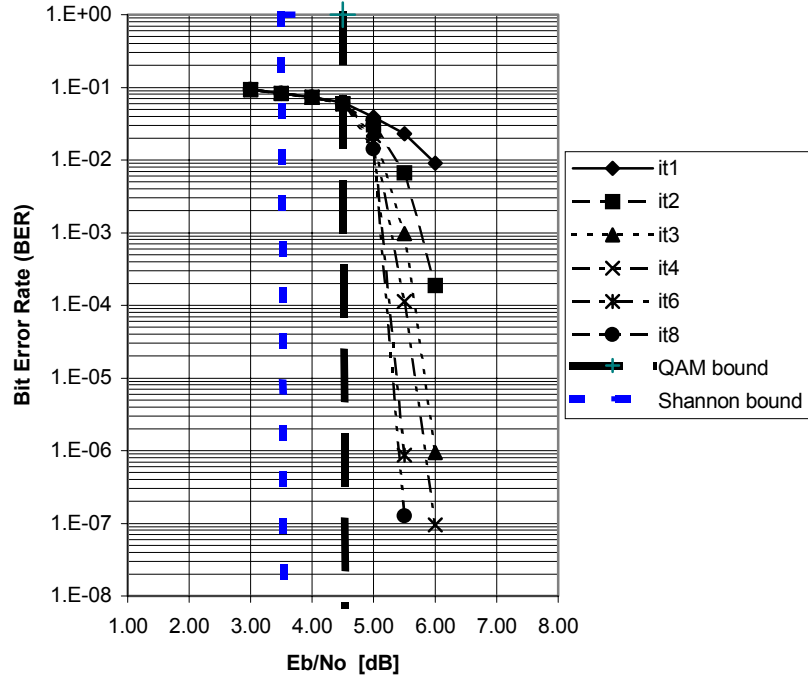


Figure 7

6.2.2 Modulation

At time k , the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k)$, is sent through the channel and the point r^k in two dimensional space is received. It is assumed that at time k u_1^k, u_2^k and u_3^k modulate the I component and u_4^k, u_5^k and u_6^k modulate the Q component of a 64QAM scheme. For 64QAM constellations with points at $-7A, -5A, -3A, -A, A, 3A, 5A, 7A$ The E_{av} is:

$$E_{av} = (8(49+25+9+1)+8(25+49+49+9+49+1)+8(25+9+25+1)+8(9+1)) A^2 / 64 = 42 A^2 \quad (17)$$

For a rate 3/6 code and 64QAM, the noise variance in each dimension is:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} = 42 A^2 \left(\frac{2x3x E_b}{N_0} \right)^{-1} = 7 A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (18)$$

In order to estimate the performance of this scheme, when rate 3/6 turbo codes and 8AM modulation is used. The puncturing and mapping scheme is shown in Table 5 for 6 consecutive information bits that are encoded into 12 coded bits, therefore two 64QAM symbols.

6.2.3 Bit Probabilities

For an AWGN channel, the following expressions need to be evaluated for the I dimension:

$$LLR(u_i^k) = \log \left(\frac{\sum_{l=1}^4 \exp[-\frac{I}{2\sigma_N^2} (I^k - a_{l,i}^k)^2] J}{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2} (I^k - a_{0,i}^k)^2] J} \right) = \quad (19)$$

$$= \log \left(\frac{\exp[-\frac{I}{2\sigma_N^2} (I^k - A_4)^2] J + \exp[-\frac{I}{2\sigma_N^2} (I^k - A_5)^2] J + \exp[-\frac{I}{2\sigma_N^2} (I^k - A_6)^2] J + \exp[-\frac{I}{2\sigma_N^2} (I^k - A_7)^2] J}{\exp[-\frac{I}{2\sigma_N^2} (I^k - A_0)^2] J + \exp[-\frac{I}{2\sigma_N^2} (I^k - A_1)^2] J + \exp[-\frac{I}{2\sigma_N^2} (I^k - A_2)^2] J + \exp[-\frac{I}{2\sigma_N^2} (I^k - A_3)^2] J} \right)$$

$$LLR(u_2^k) = \log \left(\frac{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{1,i}^k)^2] J}{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{0,i}^k)^2] J} \right) =$$

$$= \log \left(\frac{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_2)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_3)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_6)^2] J + \exp[-\frac{I}{2\sigma_n^2}(I^k - A_7)^2] J}{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_0)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_1)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_4)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_5)^2] J} \right)$$

$$LLR(u_3^k) = \log \left(\frac{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{1,i}^k)^2] J}{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{0,i}^k)^2] J} \right) =$$

$$= \log \left(\frac{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_1)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_5)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_3)^2] J + \exp[-\frac{I}{2\sigma_n^2}(I^k - A_7)^2] J}{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_0)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_4)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_2)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_6)^2] J} \right)$$

An identical computation effort is required for the Q dimension, the I^k being replaced with the Q^k demodulated value in order to evaluate $LLR(u_4^k)$, $LLR(u_5^k)$ and $LLR(u_6^k)$.

6.2.4 Simulation Results

Figure 8 shows the simulation results for 6,144 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at $E_b/N_0 = 6.1$ dB for $N = 6,144$ information bits. This result is 0.5 dB worse than the performance of the rate 3/4 16QAM scheme.

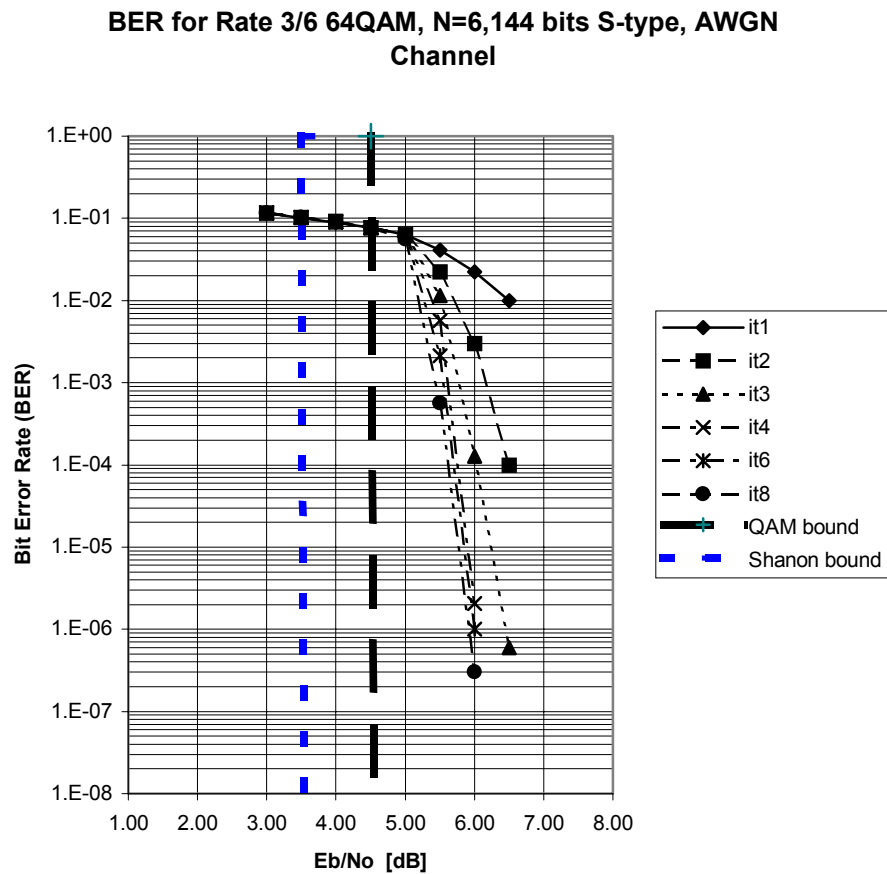


Figure 8

7. CODING AND MODULATION FOR 4 BIT/S/HZ SPECTRAL EFFICIENCY.

This section investigated four schemes for transmission of 4 information bits in a 64 QAM symbol. The mapping used for 64QAM constellation has a very significant impact on the performance of these schemes. The first scheme uses independent I and Q mapping and also uses Gray mapping in each dimension. The second scheme uses independent I and Q mapping, but natural mapping in each dimension. The third scheme used a conventional trellis coded modulation approach based on Ungerboeck set partitioning of the 64QAM set. This partitioning technique splits the constellation in sub-constellations with increased Euclidian distance. In these schemes all the information bits are coded. The fourth scheme used the same conventional trellis coded modulation approach based on Ungerboeck set partitioning of the 64QAM set. However, only two information bits are encoded by a rate 1/2 code. The four coded bits select a subpartition of four points. The other two information bits, which are sent uncoded, identify the transmitted point.

7.1 Option 1 – Rate 4/6 64QAM with independent I and Q and with Gray Mapping.

7.1.1 Puncturing

In order to obtain a rate 4/6 code, the puncturing patten used is shown in Table 6.

Table 6. Puncturing and Mapping for Rate 4/6 64QAM Option 1

Information bit (d)	d ₁	d ₂	d ₃	d ₄
parity bit (p)	p ₁	-	-	-
parity bit (q)	-	-	q ₃	-
8AM symbol (I)	(d ₁ , d ₂ , p ₁)			
8AM symbol (Q)	(d ₃ , d ₄ , q ₃)			
64 QAM symbol (I, Q)	(I, Q)=(d ₁ , d ₂ , p ₁ , d ₃ , d ₄ , q ₃)			

7.1.2 Modulation

Gray mapping was used in each dimension. Four information bits are required to be sent using a 64QAM constellation. For a rate 4/6 code and 64QAM, the noise variance in each dimension is

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} = 42 A^2 \left(\frac{2 \times 4 \times E_b}{N_0} \right)^{-1} = 5.25 A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (22)$$

The puncturing and mapping scheme is shown in Table 6 for 4 consecutive information bits that are encoded into 6 coded bits, therefore one 64QAM symbol. The turbo encoder with the puncturing presented in Table 6 is a rate 4/6 turbo code which in conjunction with 64QAM gives a spectral efficiency of 4 bits/s/Hz. Considering two independent Gaussian noises with identical variance σ_N^2 , the LLR can be determined independently for each I and Q. It is assumed that at time k u_1^k , u_2^k and u_3^k modulate the I component and u_4^k , u_5^k and u_6^k modulate the Q component of the 64QAM scheme. At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the LLR values.

7.1.3 Bit Probabilities

From each received symbol, the bit probabilities are computed as follows:

$$LLR(u_1^k) = \log \left(\frac{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{1,i}^k)^2] J}{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{0,i}^k)^2] J} \right) = \tag{23}$$

$$= \log \left(\frac{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_4)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_5)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_6)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_7)^2] J}{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_0)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_1)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_2)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_3)^2] J} \right)$$

$$LLR(u_2^k) = \log \left(\frac{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{1,i}^k)^2] J}{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{0,i}^k)^2] J} \right) = \tag{24}$$

$$= \log \left(\frac{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_2)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_3)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_6)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_7)^2] J}{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_0)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_1)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_4)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_5)^2] J} \right)$$

$$LLR(u_3^k) = \log \left(\frac{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{1,i}^k)^2] J}{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2}(I^k - a_{0,i}^k)^2] J} \right) = \tag{25}$$

$$= \log \left(\frac{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_1)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_5)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_3)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_7)^2] J}{\exp[-\frac{I}{2\sigma_N^2}(I^k - A_0)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_4)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_2)^2] J + \exp[-\frac{I}{2\sigma_N^2}(I^k - A_6)^2] J} \right)$$

For I dimension. An identical computation effort is required for the Q dimension, the I^k being replaced with the Q^k demodulated value in order to evaluate $LLR(u_4^k)$, $LLR(u_5^k)$ and $LLR(u_6^k)$.

7.1.4 Simulation Results

Figure 9 shows the simulation results for 4,096 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at $E_b/N_0 = 8.3$ dB.

7.2 Option 2 – Rate 4/6 64QAM with independent I and Q and Natural Mapping.

7.2.1 Puncturing

In order to obtain a rate 4/6 code, the puncturing patten used is shown in Table 7.

Table 7. Puncturing and Mapping for Rate 4/6 64QAM Option 2

Information bit (d)	d ₁	d ₂	d ₃	d ₄
parity bit (p)	p ₁	-	-	-
parity bit (q)	-	-	q ₃	-
8AM symbol (I)	(d ₁ , d ₂ , p ₁)			
8AM symbol (Q)	(d ₃ , d ₄ , q ₃)			
64 QAM symbol (I, Q)	(I, Q)=(d ₁ , d ₂ , p ₁ , d ₃ , d ₄ , q ₃)			

**BER for Rate 4/6 64QAM N=4,096 bits AWGN Channel ,
Gray Mapping**

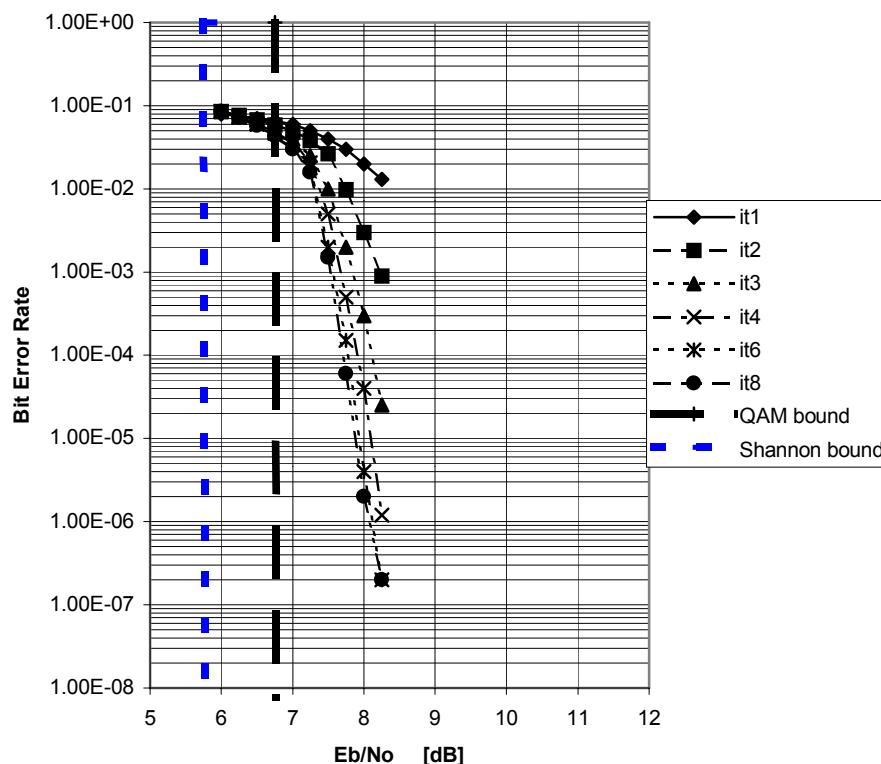


Figure 9.

7.2.2 Modulation

Natural mapping is used in each dimension. Four information bits are required to be sent using a 64QAM constellation. This is equivalent to a rate 2/3 coding scheme. For a rate 4/6 code and 64QAM, the noise variance in each dimension is

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} = 42 A^2 \left(\frac{2x4x E_b}{N_0} \right)^{-1} = 5.25 A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (26)$$

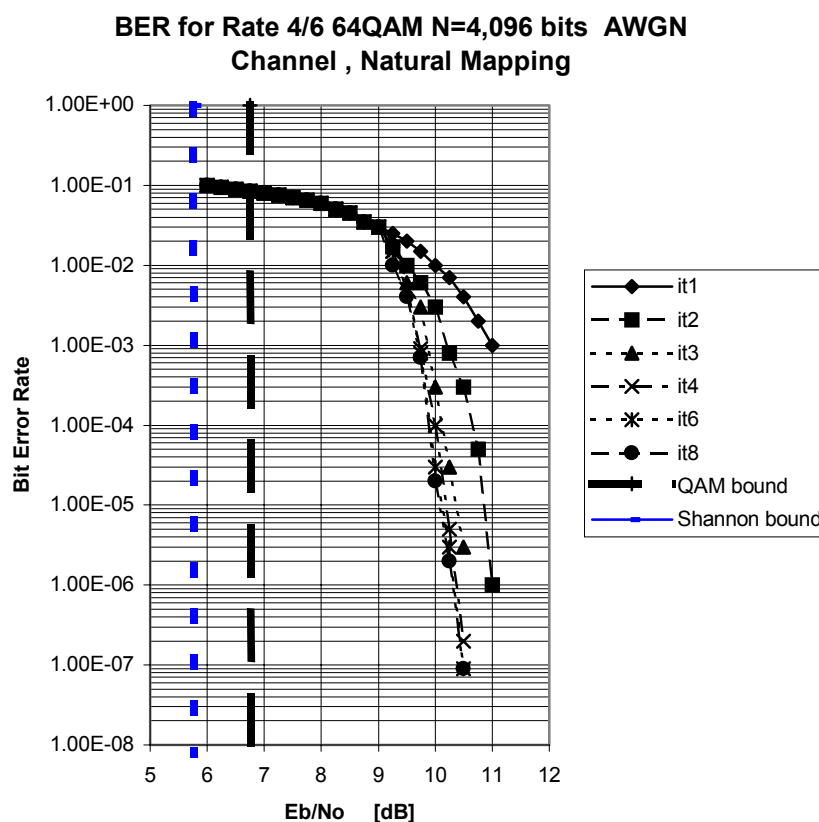
The puncturing and mapping scheme is shown in Table 7 for 4 consecutive information bits that are encoded into 6 coded bits, therefore one 64QAM symbol. The turbo encoder with the puncturing presented in Table 7 is a rate 4/6 turbo code which in conjunction with 64QAM gives a spectral efficiency of 4 bits/s/Hz. Considering two independent Gaussian noises with identical variance σ_N^2 , the LLR can be determined independently for each I and Q. It is assumed that at time k u_1^k , u_2^k and u_3^k modulate the I component and u_4^k , u_5^k and u_6^k modulate the Q component of the 64QAM scheme. At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the LLR values.

7.2.3 Bit Probabilities

From each received symbol, the bit probabilities are computed as described in equations (23) (24) and (25) for I dimension. An identical computation effort is required for the Q dimension, the I^k being replaced with the Q^k demodulated value in order to evaluate $LLR(u_4^k)$, $LLR(u_5^k)$ and $LLR(u_6^k)$.

7.2.4 Simulation Results

Figure 10 shows the simulation results for 4,096 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at $E_b/N_0 = 10.5$ dB.



8. CODING AND MODULATION FOR 5 BIT/S/HZ SPECTRAL EFFICIENCY.

This section investigated a rate 5/8 coding scheme with 256QAM.

8.1 Puncturing

In order to obtain a rate 5/8 code, the puncturing patten used is shown in Table 8.

Table 8. Puncturing and Mapping for Rate 5/8 256QAM

Information bit (d)	d ₁	d ₂	d ₃	d ₄	d ₅	d ₆	d ₇	d ₈	d ₉	d ₁₀
parity bit (p)	p ₁	-	-	-	p ₅	-	-	p ₈	-	-
parity bit (q)	-	-	q ₃	-		q ₆	-	-	-	q ₁₀
16AM symbol (I)	(d ₁ , d ₂ , d ₃ , p ₁)					(d ₆ , d ₇ , d ₈ , q ₆)				
16AM symbol (Q)	(d ₄ , d ₅ , q ₃ , p ₅)					(d ₉ , d ₁₀ , p ₈ , q ₁₀)				
256 QAM symbol (I, Q)	(d ₁ , d ₂ , d ₃ , p ₁ , d ₄ , d ₅ , q ₃ , p ₅)					(d ₆ , d ₇ , d ₈ , q ₆ , d ₉ , d ₁₀ , p ₈ , q ₁₀)				

8.2 Modulation

For a 256QAM constellation with points at $-15A, -13A, -11A, -9A, -7A, -5A, -3A, -A, A, 3A, 5A, 7A, 9A, 11A, 13A, 15A$. E_{av} is:

$$E_{av} = 170 A^2 \quad (27)$$

It is assumed that at time k the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k)$ is sent though the channel. It is assumed that at time k the symbol u_1^k, u_2^k, u_3^k and u_4^k modulate the I component and u_5^k, u_6^k, u_7^k and u_8^k modulate the Q component of a 256QAM scheme.

For a rate 5/8 code and 256QAM, the noise variance is:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} = 170 A^2 \left(\frac{2 \times 5 \times E_b}{N_0} \right)^{-1} = 17 A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (28)$$

In order to study the performance of this scheme, a rate 5/8 turbo code and a 16AM is used. The 256QAM scheme will achieve a similar performance in terms of bit error rate (BER) at twice the spectral efficiency, assuming an ideal demodulator. The puncturing and mapping scheme shown in Table 8 is for 10 consecutive information bits that are coded into 16 encoded bits, therefore, one 256QAM symbol. The turbo encoder is a rate 5/8 turbo code, which in conjunction with 256QAM, gives a spectral efficiency of 5 bits/s/Hz.

8.3 Bit Probabilities

The 16AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k, u_4^k)$, where u_1^k is the most significant bit and u_4^k is the least significant bit. The following set can be defined.

1. bit-1-is-0 = { $A_0, A_1, A_2, A_3, A_4, A_5, A_6, A_7$ }
2. bit-1-is-1 = { $A_8, A_9, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{15}$ }
3. bit-2-is-0 = { $A_0, A_1, A_2, A_3, A_8, A_9, A_{10}, A_{11}$ }
4. bit-2-is-1 = { $A_4, A_5, A_6, A_7, A_{12}, A_{13}, A_{14}, A_{15}$ }
5. bit-3-is-0 = { $A_0, A_1, A_4, A_5, A_8, A_9, A_{12}, A_{13}$ }
6. bit-3-is-1 = { $A_2, A_3, A_6, A_7, A_{10}, A_{11}, A_{14}, A_{15}$ }
7. bit-4-is-0 = { $A_0, A_2, A_4, A_6, A_8, A_{10}, A_{12}, A_{14}$ }
8. bit-4-is-1 = { $A_1, A_3, A_5, A_7, A_9, A_{11}, A_{13}, A_{15}$ }

From each received symbol, R^k , the bit probabilities are computed as follows:

$$LLR(u_1^k) = \log \left(\frac{\sum_{A_i \in \text{bit-1-is-1}} \exp \left(-\frac{1}{2\sigma_N^2} \| R^k - A_i \|^2 \right)}{\sum_{A_j \in \text{bit-1-is-0}} \exp \left(-\frac{1}{2\sigma_N^2} \| R^k - A_j \|^2 \right)} \right) \quad (29)$$

$$LLR(u_2^k) = \log \left(\frac{\sum_{A_i \in \text{bit-2-is-1}} \exp \left(-\frac{1}{2\sigma_N^2} \| R^k - A_i \|^2 \right)}{\sum_{A_j \in \text{bit-2-is-0}} \exp \left(-\frac{1}{2\sigma_N^2} \| R^k - A_j \|^2 \right)} \right) \quad (30)$$

$$LLR(u_3^k) = \log \left(\frac{\sum_{A_i \in \text{bit-3-is-1}} \exp \left(-\frac{1}{2\sigma_N^2} \| R^k - A_i \|^2 \right)}{\sum_{A_j \in \text{bit-3-is-0}} \exp \left(-\frac{1}{2\sigma_N^2} \| R^k - A_j \|^2 \right)} \right) \quad (31)$$

$$LLR(u_4^k) = \log \left(\frac{\sum_{A_i \in \text{bit-4-is-1}} \exp \left(-\frac{1}{2\sigma_N^2} \| R^k - A_i \|^2 \right)}{\sum_{A_j \in \text{bit-4-is-0}} \exp \left(-\frac{1}{2\sigma_N^2} \| R^k - A_j \|^2 \right)} \right) \quad (32)$$

8.4 Simulation Results

Figure 11 shows the simulation results for 5,120 information bits (1,204QAM symbols) with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at $E_b/N_0 = 11.8$ dB.

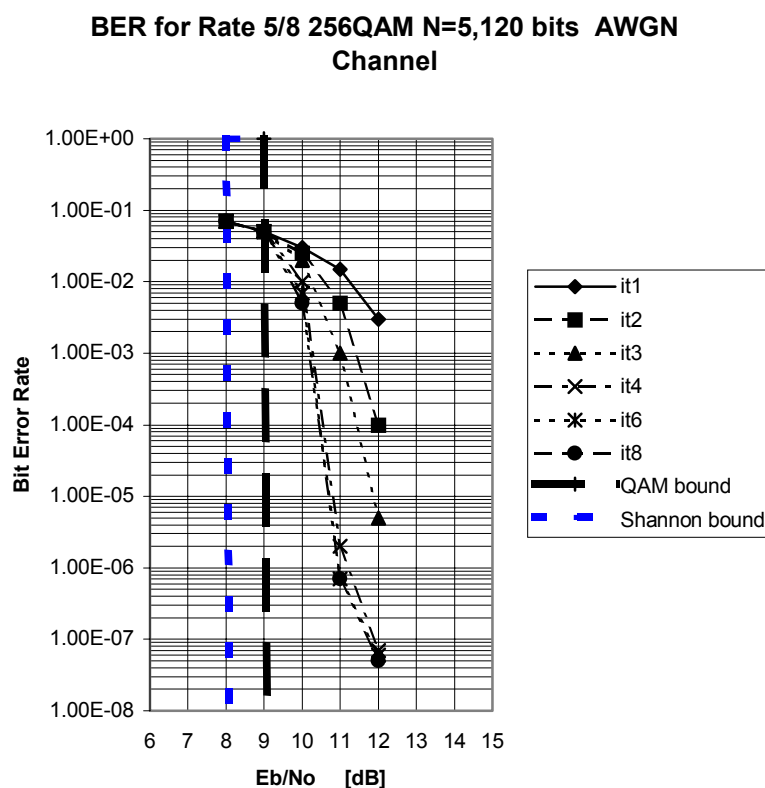


Figure 11

9. CODING AND MODULATION FOR 6 BIT/HZ SPECTRAL EFFICIENCY

This section investigates a rate 6/8 coding scheme with 256QAM.

9.1 Puncturing

In order to obtain a rate 6/8 code, the puncturing pattern used is shown in Table 9.

Table 9. Puncturing and Mapping for Rate 6/8 256QAM Option 1

Information bit (d)	d ₁	d ₂	d ₃	d ₄	d ₅	d ₆
parity bit (p)	p ₁	-	-	-	-	-
parity bit (q)	-	-	-	q ₄	-	-
16AM symbol (I)	(d ₁ , d ₂ , d ₃ , p ₁)					
16AM symbol (Q)	(d ₄ , d ₅ , d ₆ , q ₄)					
256QAM symbol (I, Q)	(I, Q) = (d ₁ , d ₂ , d ₃ , p ₁ , d ₄ , d ₅ , d ₆ , q ₄)					

9.2 Modulation

It is assumed that at time k the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k)$ is sent though the channel. It is assumed that at time k the symbol u_1^k, u_2^k, u_3^k and u_4^k modulate the I component and u_5^k, u_6^k, u_7^k and u_8^k modulate the Q component of a 256QAM scheme.

For a rate 6/8 code and 256QAM, the noise variance is:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} = 170 A^2 \left(\frac{2 \times 6 \times E_b}{N_0} \right)^{-1} = 14.16 A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (33)$$

The puncturing and mapping scheme shown in Table 11 is for 6 consecutive information bits that are coded into 8 coded bits, therefore, one 256QAM symbol. The turbo encoder is a rate 6/8 turbo code, which in conjunction with 256QAM, gives a spectral efficiency of 6 bits/s/Hz.

9.3 Bit Probabilities

The 16AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k, u_4^k)$, where u_1^k is the most significant bit and u_4^k is the least significant bit. The following set can be defined.

1. bit-1-is-0 = { $A_0, A_1, A_2, A_3, A_4, A_5, A_6, A_7$ }
2. bit-1-is-1 = { $A_8, A_9, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{15}$ }
3. bit-2-is-0 = { $A_0, A_1, A_2, A_3, A_8, A_9, A_{10}, A_{11}$ }
4. bit-2-is-1 = { $A_4, A_5, A_6, A_7, A_{12}, A_{13}, A_{14}, A_{15}$ }
5. bit-3-is-0 = { $A_0, A_1, A_4, A_5, A_8, A_9, A_{12}, A_{13}$ }
6. bit-3-is-1 = { $A_2, A_3, A_6, A_7, A_{10}, A_{11}, A_{14}, A_{15}$ }
7. bit-4-is-0 = { $A_0, A_2, A_4, A_6, A_8, A_{10}, A_{12}, A_{14}$ }
8. bit-4-is-1 = { $A_1, A_3, A_5, A_7, A_9, A_{11}, A_{13}, A_{15}$ }

From each received symbol, R^k , the bit probabilities are computed as equations (29) to (32).

9.4 Simulation Results

Figure 12 shows the simulation results for 6,144 information bits (1,204QAM symbols) with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at $E_b/N_0 = 14.2$ dB.

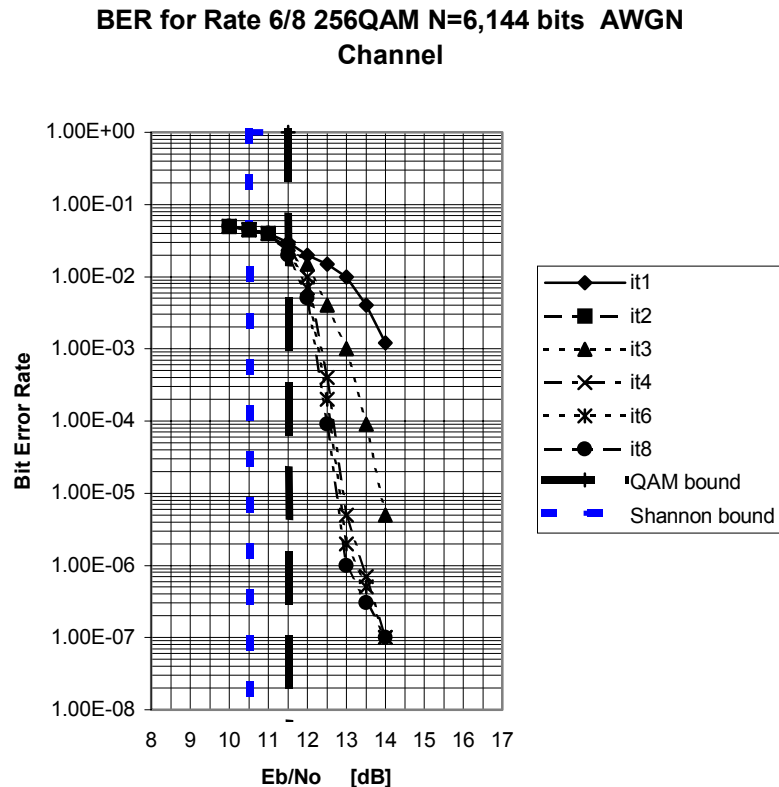


Figure 12

10. CODING AND MODULATION FOR 7 BIT/HZ SPECTRAL EFFICIENCY

This section investigated one scheme that use independent I and Q modulation. The scheme combines a rate 7/10 coding scheme with 1024QAM.

10.1 Puncturing

In order to obtain a rate 7/10 code, the puncturing patten used is shown in Table 12.

Table 12. Puncturing and Mapping for Rate 7/10 1024QAM

Information bit (d)	d ₁	d ₂	d ₃	d ₄	d ₅	d ₆	d ₇	d ₈	d ₉	d ₁₀	d ₁₁	d ₁₂	d ₁₃	d ₁₄
parity bit (p)	p ₁	-	-	-	-	p ₆	-	-	-	-	p ₁₁	-	-	-
parity bit (q)	-	-	q ₃	-	-	-	-	q ₈	-	-	-	-	q ₁₃	-
32AM symbol (I)	(d ₁ , d ₂ , d ₃ , p ₁ , q ₃)							(d ₈ , d ₉ , d ₁₀ , d ₁₁ , q ₈)						
32AM symbol (Q)	(d ₄ , d ₅ , d ₆ , d ₇ , p ₆)							(d ₁₂ , d ₁₃ , d ₁₄ , p ₁₁ , q ₁₃)						
1024QAM symbol	(d ₁ ,d ₂ , d ₃ , p ₁ , q ₃ , d ₄ ,d ₅ , d ₆ ,d ₇ ,p ₆)							(d ₈ ,d ₉ ,d ₁₀ ,d ₁₁ ,q ₈ ,d ₁₂ ,d ₁₃ ,d ₁₄ ,p ₁₁ ,q ₁₃)						

10.2 Modulation

It is assumed that at time k the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k, u_9^k, u_{10}^k)$ is sent though the channel. It is assumed that at time k the symbol $u_1^k, u_2^k, u_3^k, u_4^k$ and u_5^k modulate the I component and $u_6^k, u_7^k, u_8^k, u_9^k$ and u_{10}^k modulate the Q component of a 1024QAM scheme. For a 1024QAM constellation with points at -31A, -29A, -27A, -25A, -23A, -21A, -19A, -17A, -15A, -13A, -11A, -9A, -7A, -5A, -3A, -A, A, 3A, 5A, 7A, 9A, 11A, 13A, 15A, 17A, 19A, 21A, 23A, 25A, 27A, 29A, 31A. E_{av} is:

$$E_{av} = 682 A^2 \quad (34)$$

For a rate 7/10 code and 512QAM, the noise variance is:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} = 682 A^2 \left(\frac{2 \times 7 \times E_b}{N_0} \right)^{-1} = 48.7 A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (35)$$

The puncturing and mapping scheme shown in Table 12 is for 14 consecutive information bits that are coded into 20 coded bits, therefore, two 1024QAM symbols. The turbo encoder is a rate 7/10 turbo code, which in conjunction with 1024QAM, gives a spectral efficiency of 7 bits/s/Hz.

10.3 Bit Probabilities

The 32AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k)$, where u_1^k is the most significant bit and u_5^k is the least significant bit. The following set can be defined.

1. bit-1-is-0 = { A₀, A₁, A₂, A₃, A₄, A₅, A₆, A₇, A₈, A₉, A₁₀, A₁₁, A₁₂, A₁₃, A₁₄, A₁₅ }
2. bit-1-is-1 = { A₁₆, A₁₇, A₁₈, A₁₉, A₂₀, A₂₁, A₂₂, A₂₃, A₂₄, A₂₅, A₂₆, A₂₇, A₂₈, A₂₉, A₃₀, A₃₁ }
3. bit-2-is-0 = { A₀, A₁, A₂, A₃, A₄, A₅, A₆, A₇, A₁₆, A₁₇, A₁₈, A₁₉, A₂₀, A₂₁, A₂₂, A₂₃ }
4. bit-2-is-1 = { A₈, A₉, A₁₀, A₁₁, A₁₂, A₁₃, A₁₄, A₁₅, A₂₄, A₂₅, A₂₆, A₂₇, A₂₈, A₂₉, A₃₀, A₃₁ }
5. bit-3-is-0 = { A₀, A₁, A₂, A₃, A₈, A₉, A₁₀, A₁₁, A₁₆, A₁₇, A₁₈, A₁₉, A₂₄, A₂₅, A₂₆, A₂₇ }
6. bit-3-is-1 = { A₄, A₅, A₆, A₇, A₁₂, A₁₃, A₁₄, A₁₅, A₂₀, A₂₁, A₂₂, A₂₃, A₂₈, A₂₉, A₃₀, A₃₁ }
7. bit-4-is-0 = { A₀, A₁, A₄, A₅, A₈, A₉, A₁₂, A₁₃, A₁₆, A₁₇, A₂₀, A₂₁, A₂₄, A₂₅, A₂₈, A₂₉ }
8. bit-4-is-1 = { A₂, A₃, A₆, A₇, A₁₀, A₁₁, A₁₄, A₁₅, A₁₈, A₁₉, A₂₂, A₂₃, A₂₆, A₂₇, A₃₀, A₃₁ }
9. bit-5-is-0 = { A₀, A₂, A₄, A₆, A₈, A₁₀, A₁₂, A₁₄, A₁₆, A₁₈, A₂₀, A₂₂, A₂₄, A₂₆, A₂₈, A₃₀ }
10. bit-5-is-1 = { A₁, A₃, A₅, A₇, A₉, A₁₁, A₁₃, A₁₅, A₁₇, A₁₉, A₂₁, A₂₃, A₂₅, A₂₇, A₂₉, A₃₁ }

From each received symbol, R^k , the bit probabilities are computed as follows:

$$LLR(u_1^k) = \log \left(\frac{\sum_{A_i \in \text{bit-1-i s-1}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_i\|\right)}{\sum_{A_j \in \text{bit-1-i s-0}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_j\|\right)} \right) \tag{36}$$

$$LLR(u_2^k) = \log \left(\frac{\sum_{A_i \in \text{bit-2-i s-1}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_i\|\right)}{\sum_{A_j \in \text{bit-2-i s-0}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_j\|\right)} \right) \tag{37}$$

$$LLR(u_3^k) = \log \left(\frac{\sum_{A_i \in \text{bit-3-i s-1}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_i\|\right)}{\sum_{A_j \in \text{bit-3-i s-0}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_j\|\right)} \right) \tag{38}$$

$$LLR(u_4^k) = \log \left(\frac{\sum_{A_i \in \text{bit-4-i s-1}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_i\|\right)}{\sum_{A_j \in \text{bit-4-i s-0}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_j\|\right)} \right) \tag{39}$$

$$LLR(u_5^k) = \log \left(\frac{\sum_{A_i \in \text{bit-5-i s-1}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_i\|\right)}{\sum_{A_j \in \text{bit-5-i s-0}} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_j\|\right)} \right) \tag{40}$$

10.4 Simulation Results

Figure 13 shows the simulation results for 2,048 information bits (1,204QAM symbols) with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at $E_b/N_0 = 17$ dB.

**BER for Rate 7/10 1024QAM N=2,044 bits
AWGN Channel**

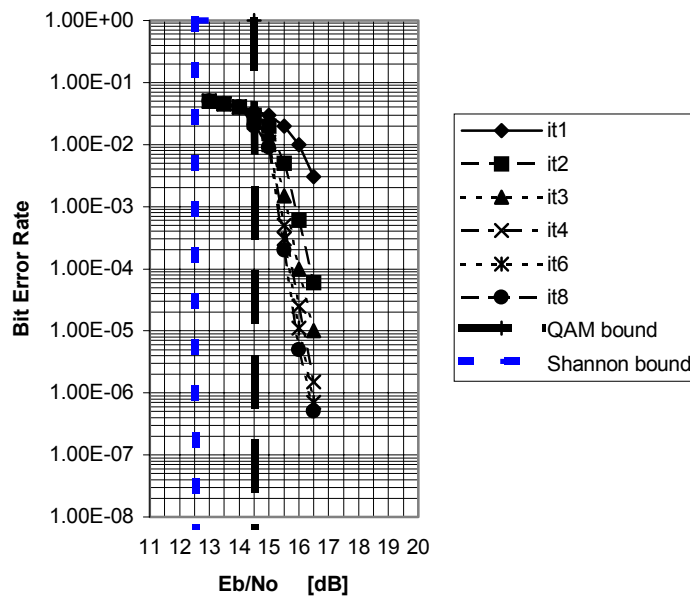


Figure 13

11. POWER VS. BANDWIDTH IN AN AWGN CHANNEL

This section gives an estimate of the trade off which can be achieved between minimum required E_b/N_0 and bandwidth efficiency. An information data rate of 2,048 Mbit/s and a maximum transmitter delay of 1 ms is considered. The corresponding interleaver size is 2,048 bits.

11.1 Channel model

All the simulations assumed the additive white Gaussian noise (AWGN) channel model, with independent I and Q signals.

11.2 Simulation Results

Simulations were run for bandwidth efficiencies from 1 to 7 bit/symbol using the recommended coding and modulation schemes. The results are shown in Figures 14 to 20.

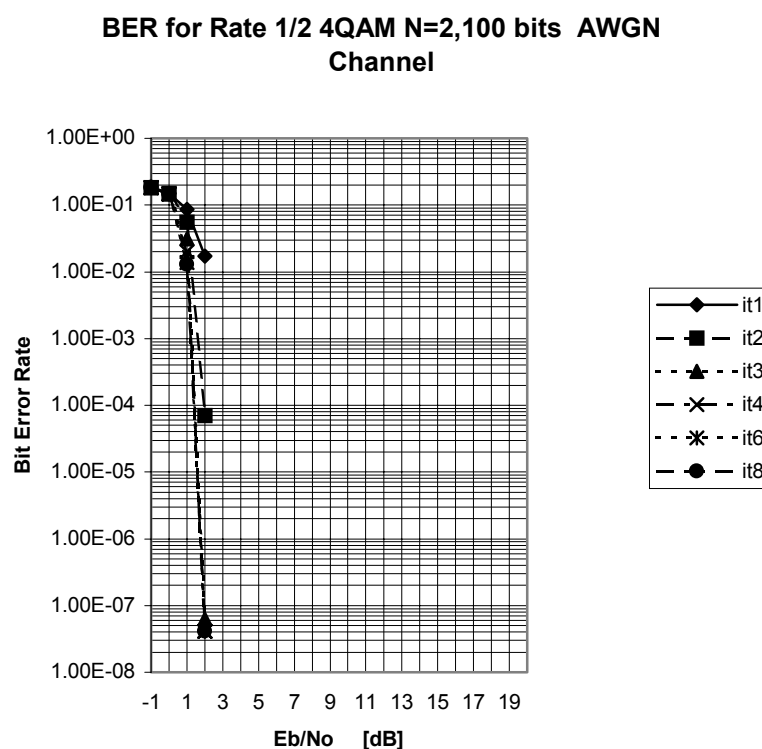


Figure 14

BER for Rate 2/4 16QAM N=2,100 bits AWGN Channel

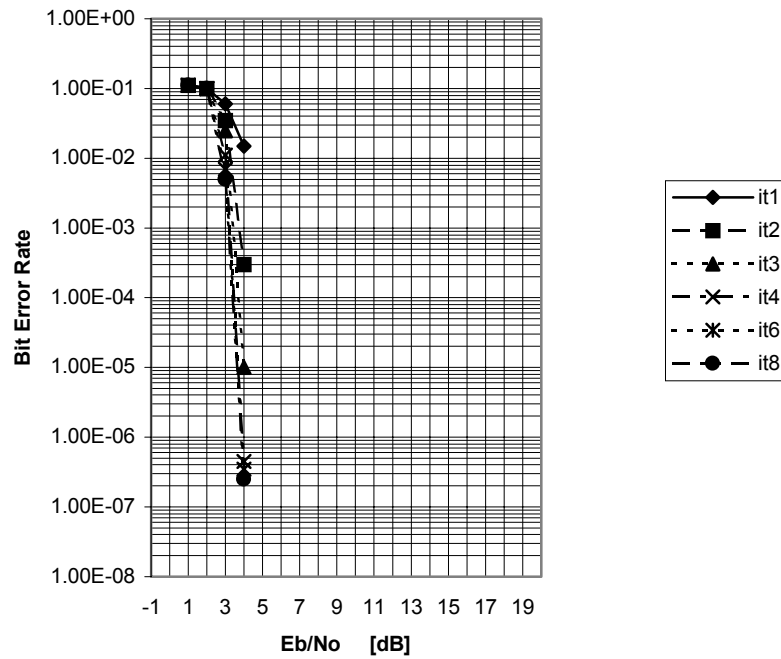


Figure 15

BER for Rate 3/4 16QAM N=2,100 bits AWGN Channel

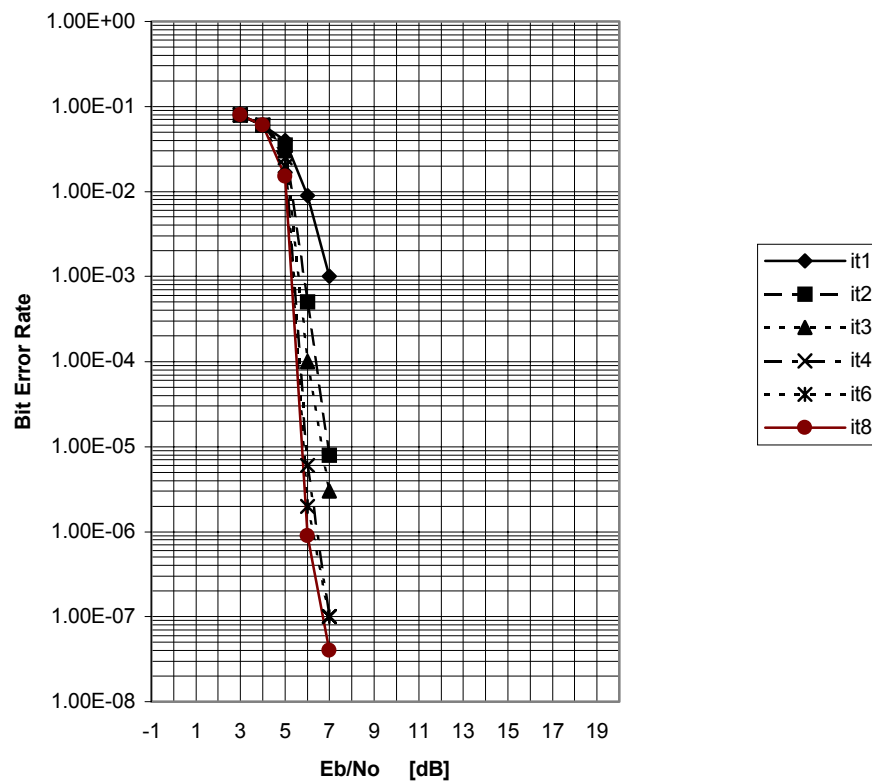


Figure 16

BER for Rate 4/6 64QAM N=2,100 bits AWGN Channel

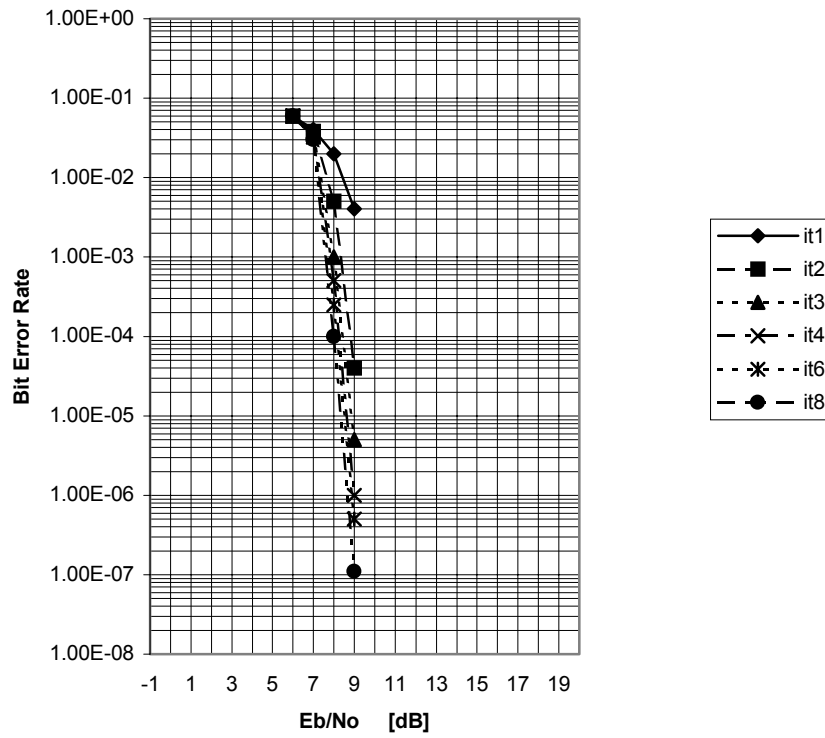


Figure 17

BER for Rate 5/8 256QAM N=2,100 bits AWGN Channel

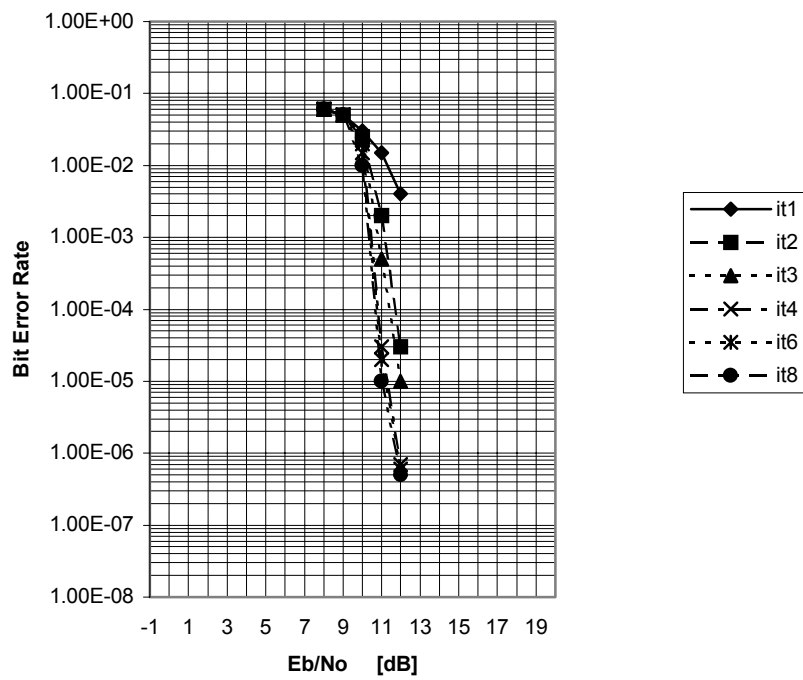


Figure 18

BER for Rate 6/8 256QAM N=2,100 bits AWGN Channel

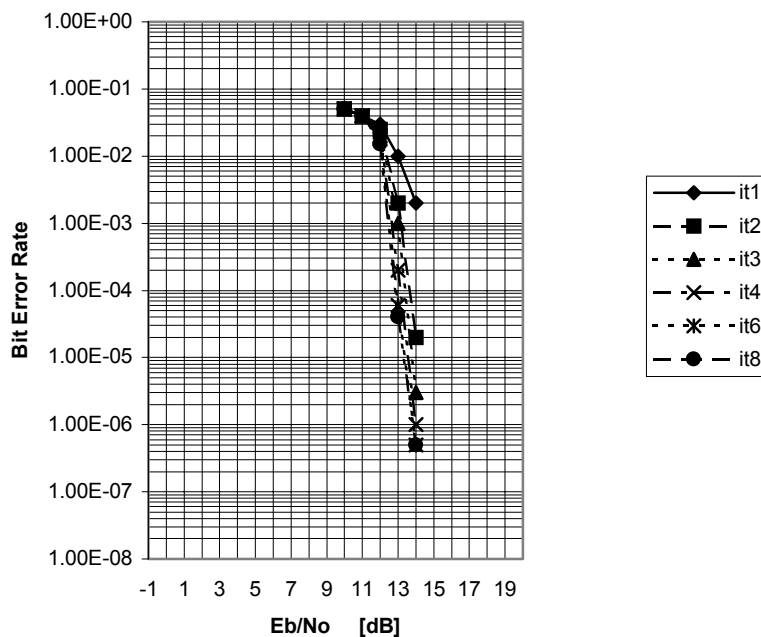


Figure 19

BER for Rate 7/10 1024QAM N=2,100 bits AWGN Channel

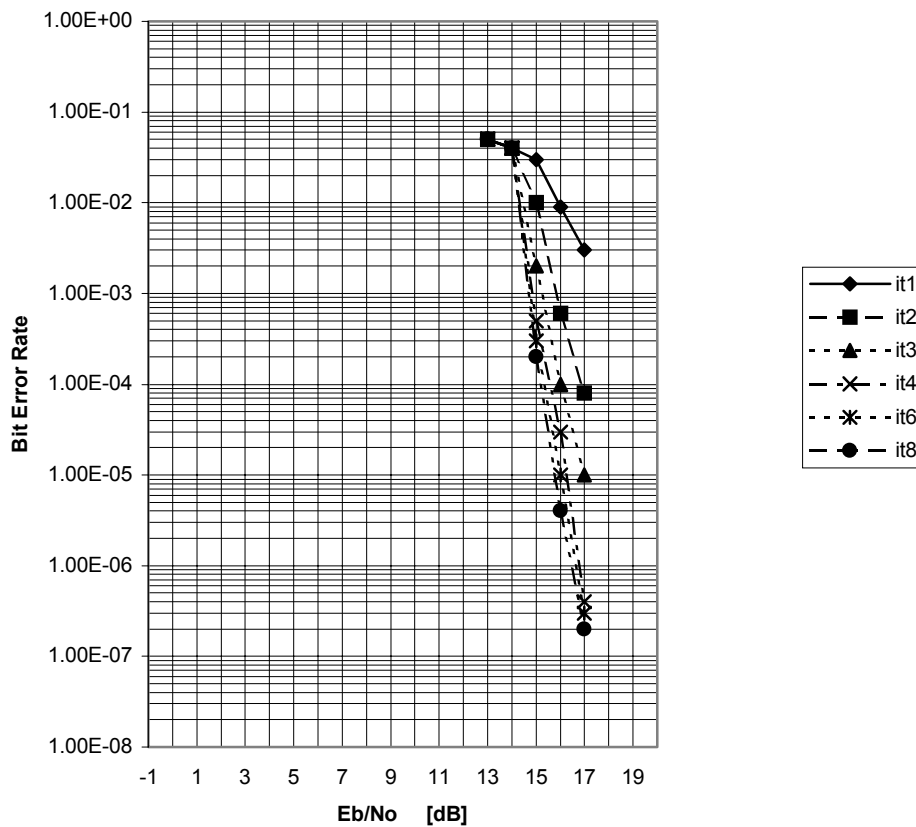


Figure 20

11.3 Conclusions

Table 11 summarizes the minimum E_b/N_0 required to achieve a BER of 10^{-7} .

Table 11 Minimum E_b/N_0 required to achieve a BER of 10^{-7}

Spectral efficiency η [bits/s/Hz]	Coding Rate And Modulation	Symbol Rate [ksym/s]	E_b/N_0 For BER = 10^{-7} [dB]
1	1/2 and 4QAM	2048	2.2
2	2/4 and 16QAM	1024	4.2
3	3/4 and 16QAM	682	6.5
4	4/6 and 64QAM	512	9.1
5	5/8 and 256QAM	408	12.3
6	6/8 and 256QAM	342	14.5
7	7/10 and 1024QAM	292	17.0

The results show the potential reduction in bandwidth for a given signal-to-noise ratio for a particular channel. If an E_b/N_0 of 17 dB or more is available, the symbol rate can be reduced from 1024 ksym/s to 292 ksym/s, a reduction of more than 70%.

12. CONCLUSIONS

Table 12 summarizes all simulation results from this study.

Table 12. Summary of Some Simulation Results.

Spectral efficiency η [bits/s/Hz]	Coding Rate	Modulation	Interleaver size [Information bits]	Required E_b/N_0 [dB]	QAM bound [dB]
1	1/2	4QAM	1,024	2.1	1.0
2	2/4	16QAM	512	5.3	2.1
2	2/4	16QAM	768	4.9	2.1
2	2/4	16QAM	1,024	4.5	2.1
2	2/4	16QAM	2,048	4.2	2.1
2	2/4	16QAM	32,728	2.9	2.1
3	3/4	16QAM	2,048	6.5	4.6
3	3/4	16QAM	4,096	5.8	4.6
3	3/6	64QAM	4,096	6.1	4.3
4	4/6	64QAM	2,048	9.1	6.6
4	4/6	64QAM-1	4,096	8.3	6.6
4	4/6	64QAM-2	4,096	10.5	6.6
5	5/6	64QAM	5,120	13.0	9.0
5	5/8	256QAM	2,048	12.3	9.0
5	5/8	256QAM	5,120	11.8	9.0
6	6/8	256QAM	2,048	14.5	11.7
6	6/8	256QAM	6,144	14.2	11.7
7	7/10	1024QAM	2,044	17.0	14.5

13. PROPOSAL

We propose that the T1E1.4 committee supports to include in the G.992.1.bis and G.992.2.bis this proposal for Forward Error Correction ADSL without Reed –Solomon codes. These results are so close to the QAM bounds that the Reed-Solomon is not needed. Instead of using Reed-Solomon as outer encoder we propose to use a longer interleaver in the turbo coder, up to 32768 information bits, that allow better results with the same global delay.