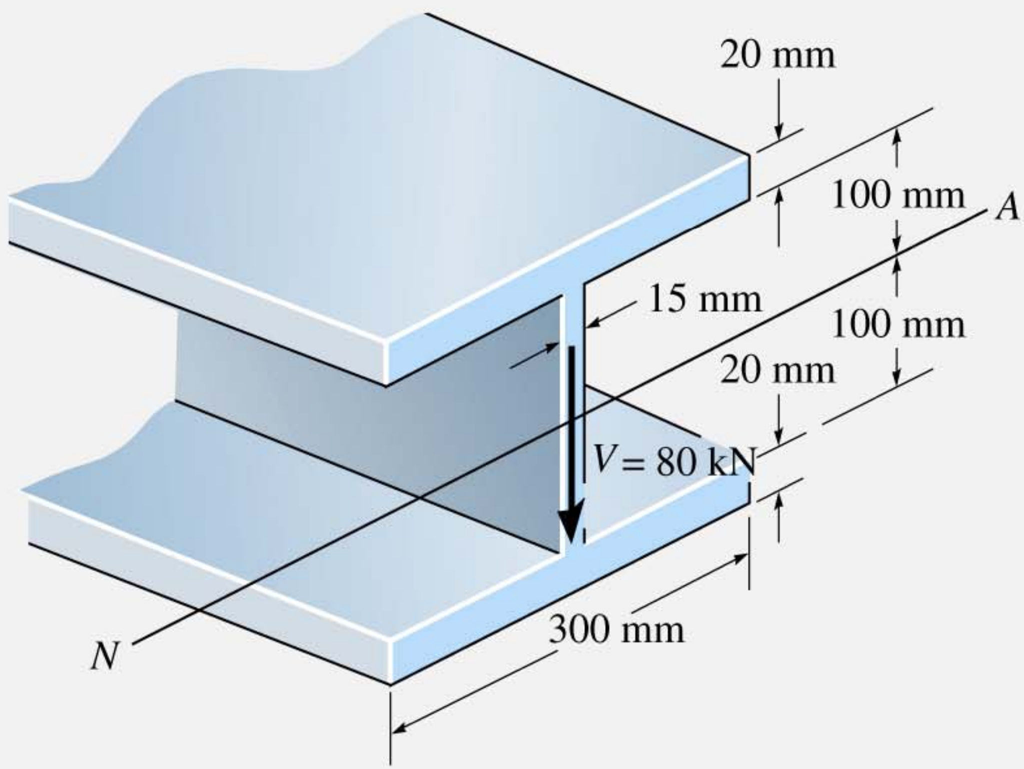
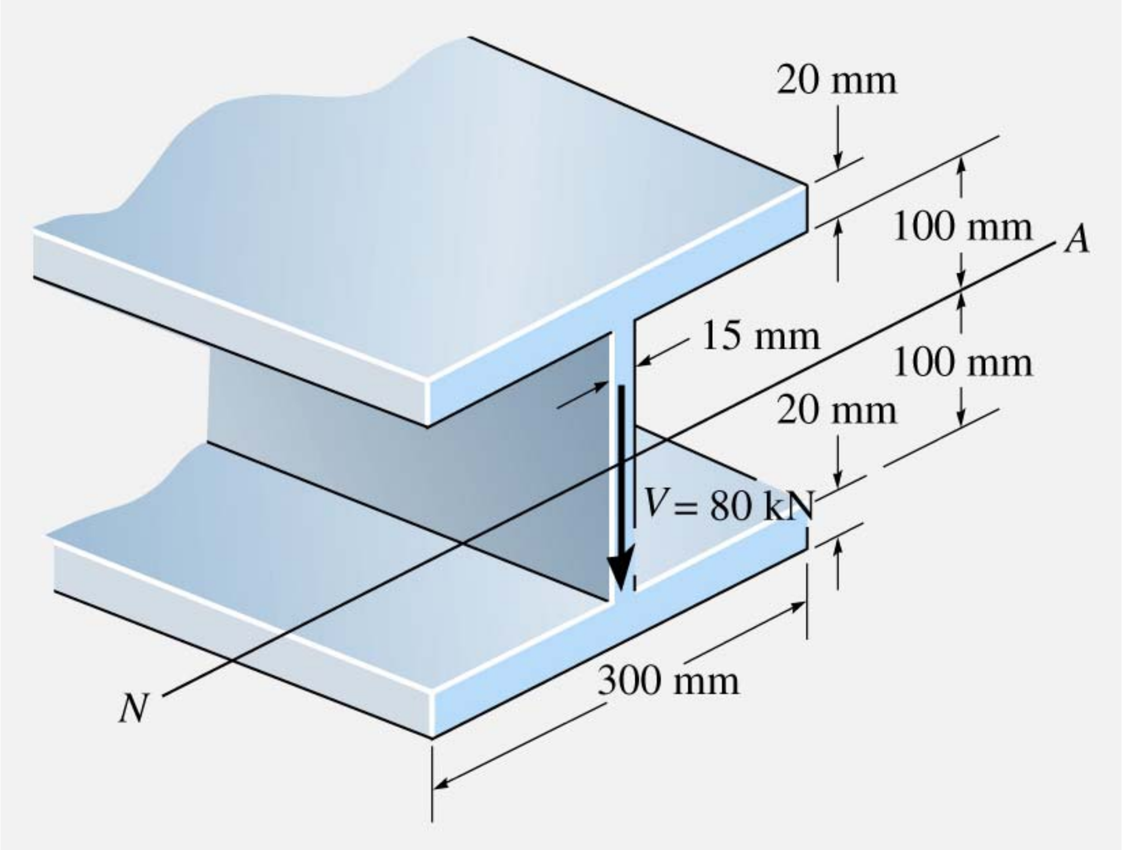
Examples on shear stress in beams

EXAMPLE 7–2

A steel wide-flange beam has the dimensions shown in Fig. 7–11*a*. If it is subjected to a shear of $V = 80$ kN, (*a*) plot the shear-stress distribution acting over the beam's cross-sectional area, and (*b*) determine the shear force resisted by the web.



(a)

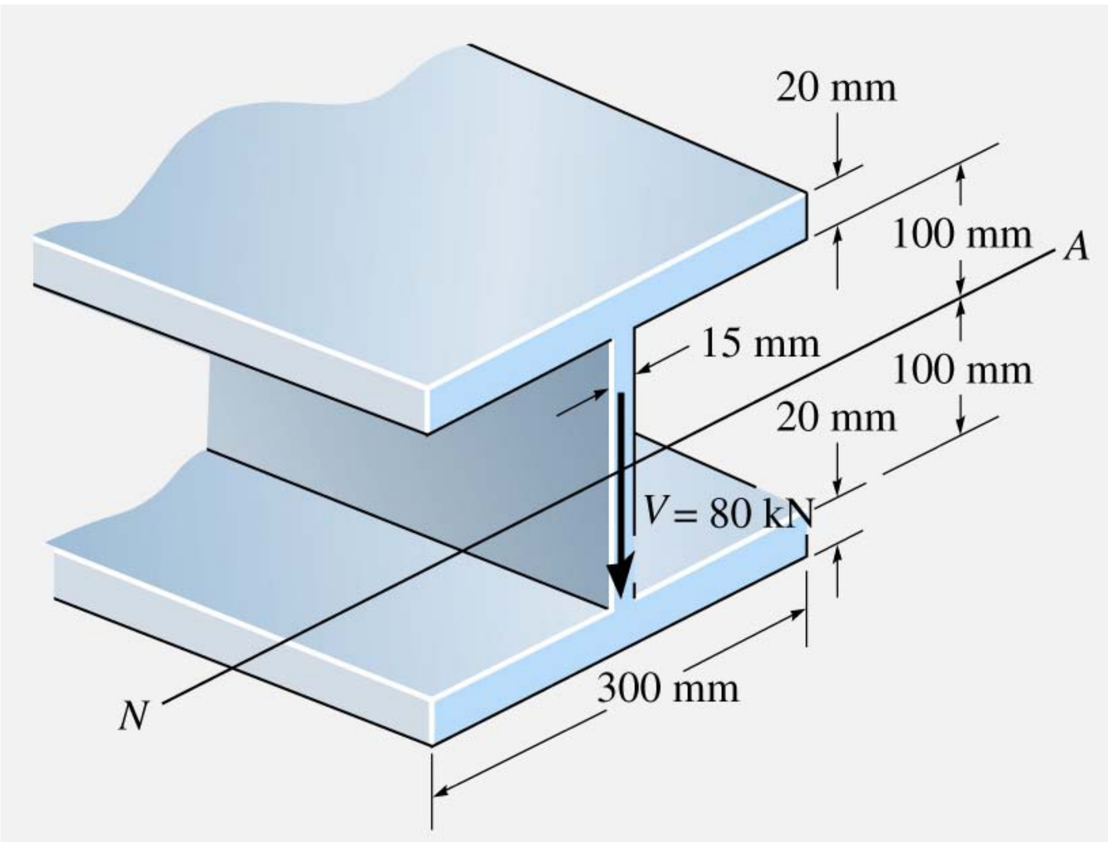


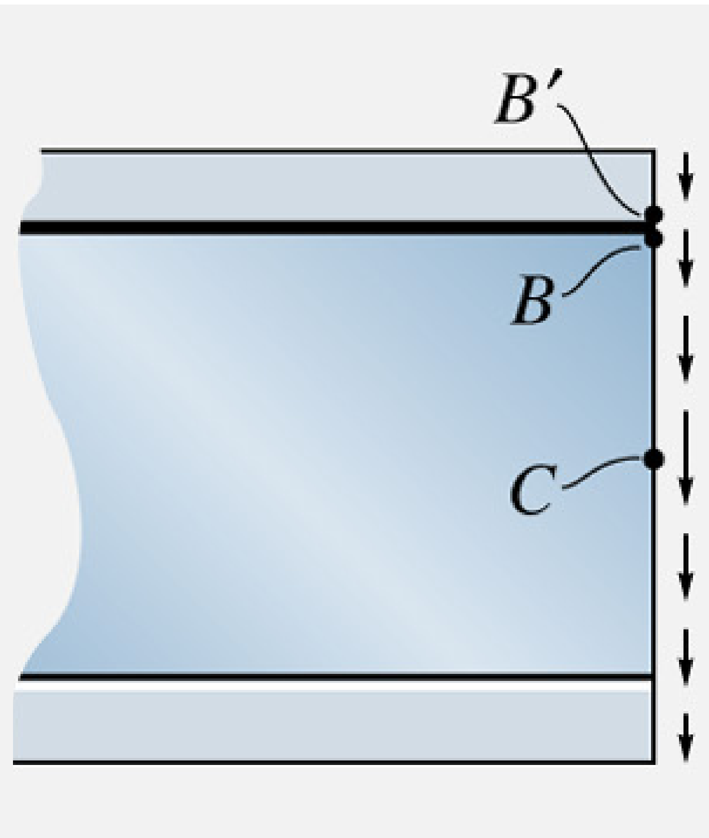
$$I = \left[\frac{1}{12}(0.015\text{ m})(0.200\text{ m})^3\right] + 2\left[\frac{1}{12}(0.300\text{ m})(0.02\text{ m})^3 + (0.300\text{ m})(0.02\text{ m})(0.110\text{ m})^2\right] = 155.6(10^{-6})\text{ m}^4$$

Moment of inertia of the cross section area

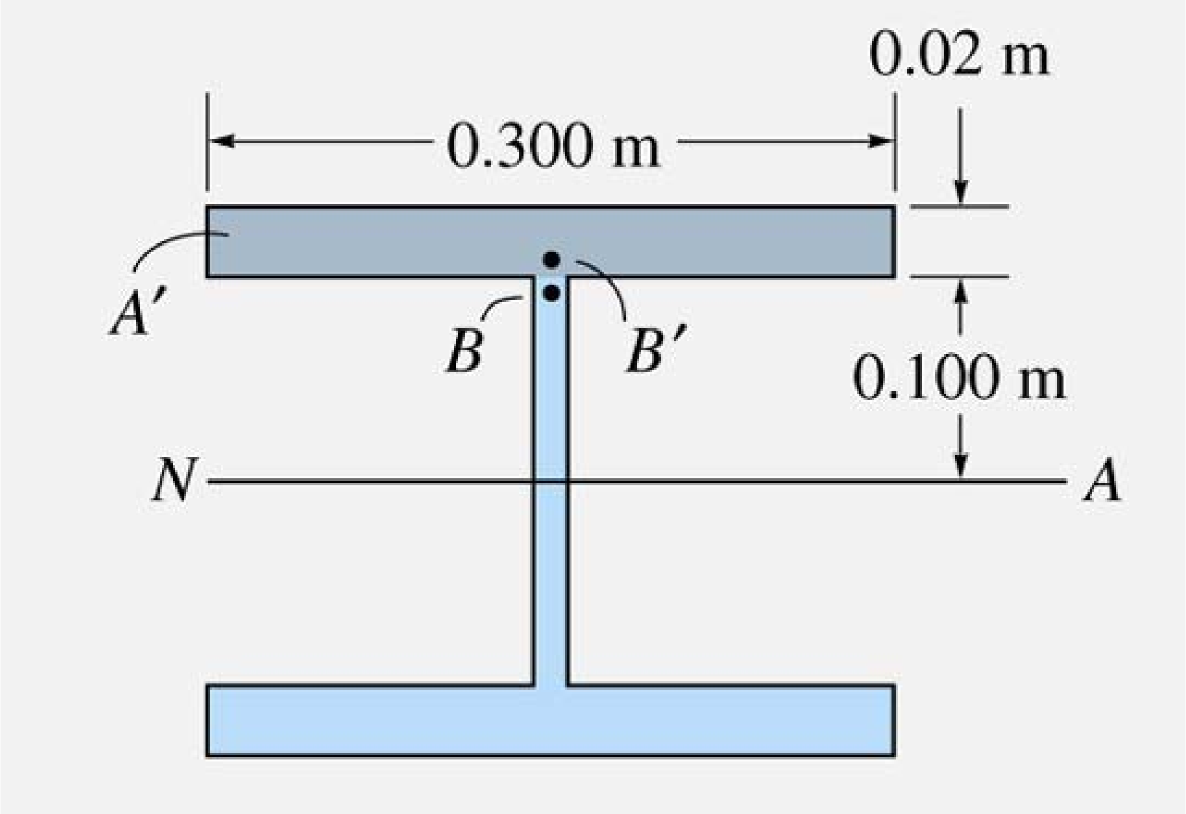
Due to symmetry only the shear stresses at points B’, B A and C have to be computed

B’ belongs to the flange, B belongs to the web but both are considered to be at the flange-web junction. $Q_{B'} = Q_B$





Profile view

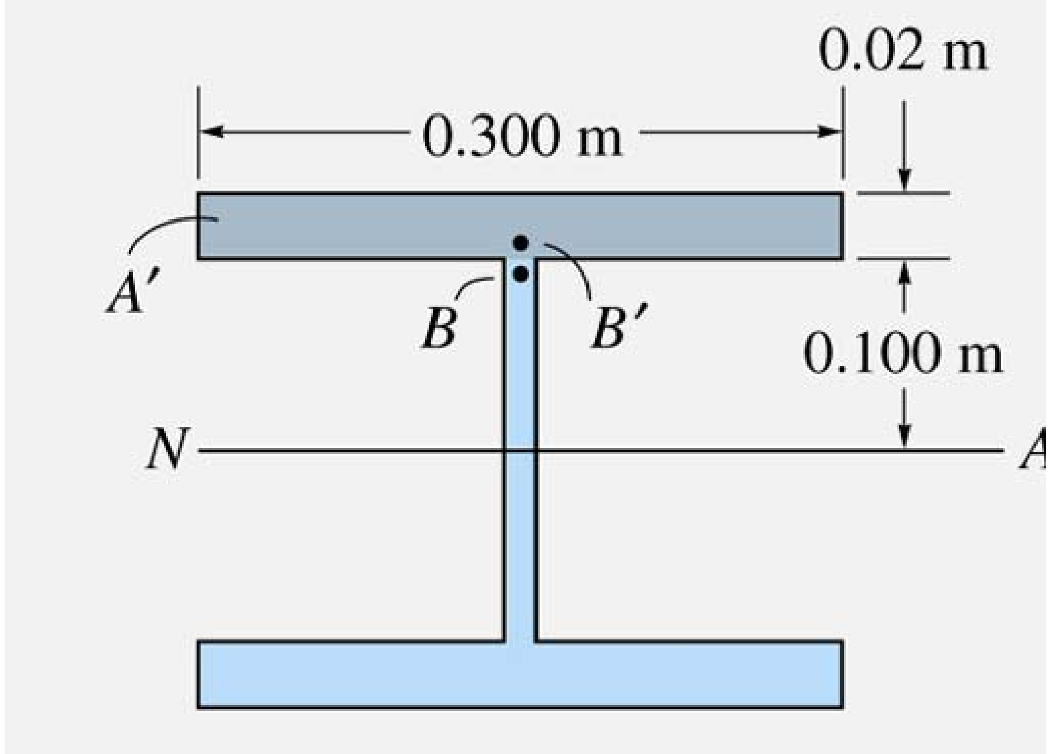


For point B', $t_{B'} = 0.300$ m, and A' is the dark shaded area shown in Fig. 7–11*c*. Thus,

$$Q_{B'} = \bar{y}'A' = [0.110 \text{ m}](0.300 \text{ m})(0.02 \text{ m}) = 0.660(10^{-3}) \text{ m}^3$$

so that

$$\tau_{B'} = \frac{VQ_{B'}}{It_{B'}} = \frac{80 \text{ kN}(0.660(10^{-3}) \text{ m}^3)}{155.6(10^{-6}) \text{ m}^4(0.300 \text{ m})} = 1.13 \text{ MPa}$$



For point B, $t_B = 0.015$ m and $Q_B = Q_{B'}$, Fig. 7–11c. Hence

$$\tau_B = \frac{VQ_B}{It_B} = \frac{80\text{ kN}(0.660(10^{-3})\text{ m}^3)}{155.6(10^{-6})\text{ m}^4(0.015\text{ m})} = 22.6\text{ MPa}$$

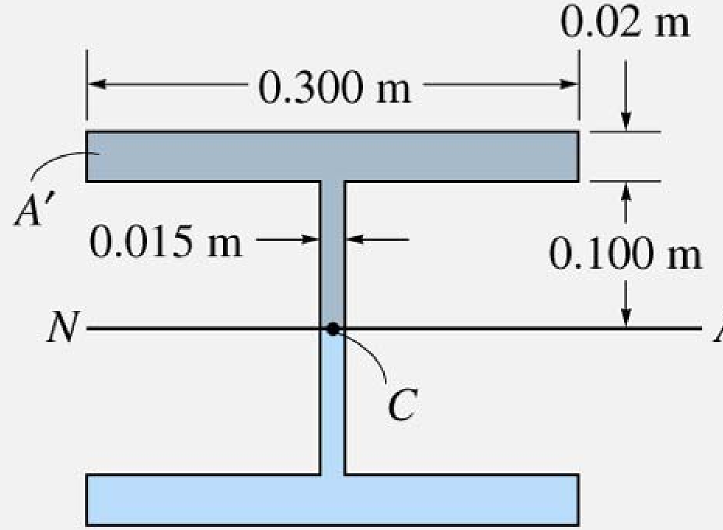
For point C, $t_C = 0.015$ m and A' is the dark shaded area shown in Fig. 7–11*d*. Considering this area to be composed of two rectangles, we have

$$Q_C = \Sigma\, \bar{y}'A' = [0.110 \text{ m}](0.300 \text{ m})(0.02 \text{ m}) + [0.05 \text{ m}](0.015 \text{ m})(0.100 \text{ m})$$

$$= 0.735(10^{-3}) \text{ m}^3$$

Thus,

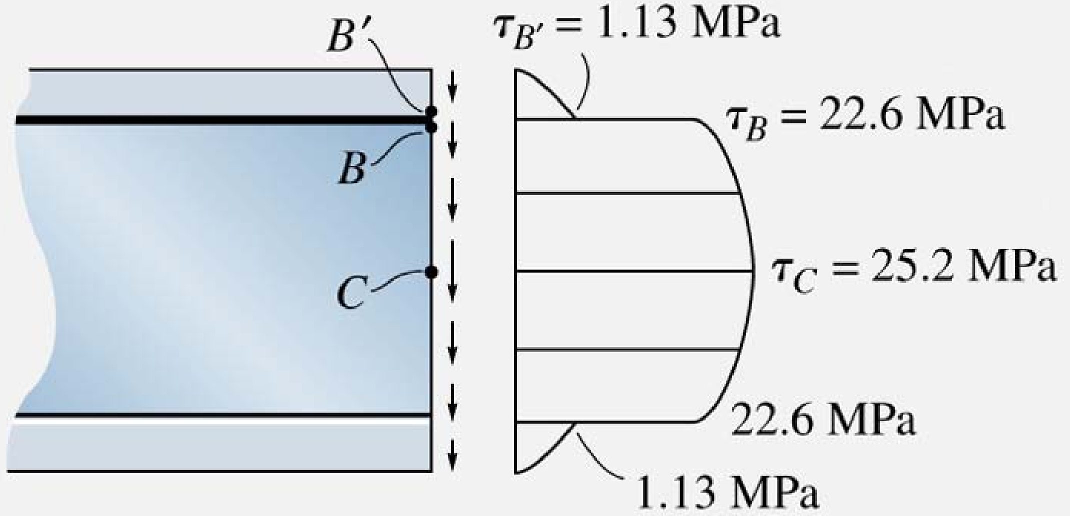
$$\tau_C = \tau_{max} = \frac{VQ_C}{It_C} = \frac{80 \text{ kN}[0.735(10^{-3}) \text{ m}^3]}{155.6(10^{-6}) \text{ m}^4(0.015 \text{ m})} = 25.2 \text{ MPa}$$



$$\tau_{B'} = \frac{VQ_{B'}}{It_{B'}} = \frac{80\text{ kN}(0.660(10^{-3})\text{ m}^3)}{155.6(10^{-6})\text{ m}^4(0.300\text{ m})} = 1.13\text{ MPa}$$

$$\tau_B = \frac{VQ_B}{It_B} = \frac{80\text{ kN}(0.660(10^{-3})\text{ m}^3)}{155.6(10^{-6})\text{ m}^4(0.015\text{ m})} = 22.6\text{ MPa}$$

$$\tau_C = \tau_{max} = \frac{VQ_C}{It_C} = \frac{80\text{ kN}[0.735(10^{-3})\text{ m}^3]}{155.6(10^{-6})\text{ m}^4(0.015\text{ m})} = 25.2\text{ MPa}$$



(b)

Part (b). The shear force in the web will be determined by first formulating the shear stress at the *arbitrary* location *y* within the web, Fig. 7–11*e*. Using units of meters, we have

$$I = 155.6(10^{-6})\ \text{m}^4$$

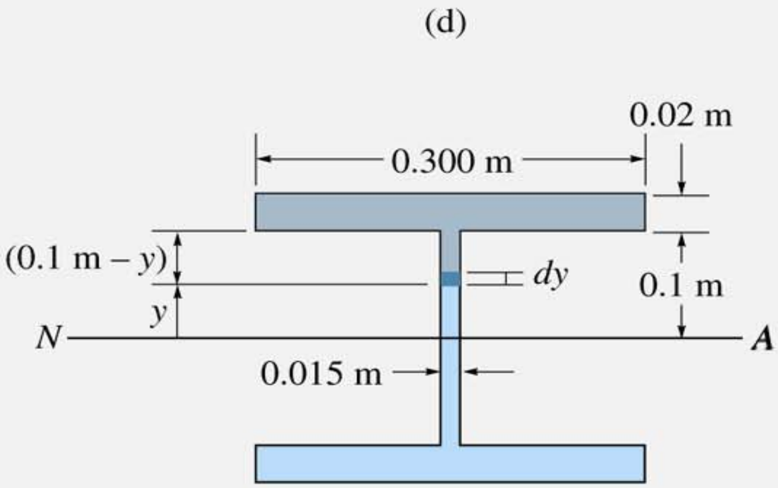
$$t = 0.015\ \text{m}$$

$$A' = (0.300\ \text{m})(0.02\ \text{m}) + (0.015\ \text{m})(0.1\ \text{m} - y)$$

$$Q = \Sigma \bar{y}'A' = (0.11\ \text{m})(0.300\ \text{m})(0.02\ \text{m})$$

$$+[y + \tfrac{1}{2}(0.1\ \text{m} - y)](0.015\ \text{m})(0.1\ \text{m} - y)$$

$$= (0.735 - 7.50\,y^2)(10^{-3})\ \text{m}^3$$



$$\tau = \frac{VQ}{It} = \frac{80\ \text{kN}(0.735 - 7.50\,y^2)(10^{-3})\ \text{m}^3}{(155.6(10^{-6})\ \text{m}^4)(0.015\ \text{m})}$$

$$= (25.192 - 257.07\,y^2)\ \text{MPa}$$

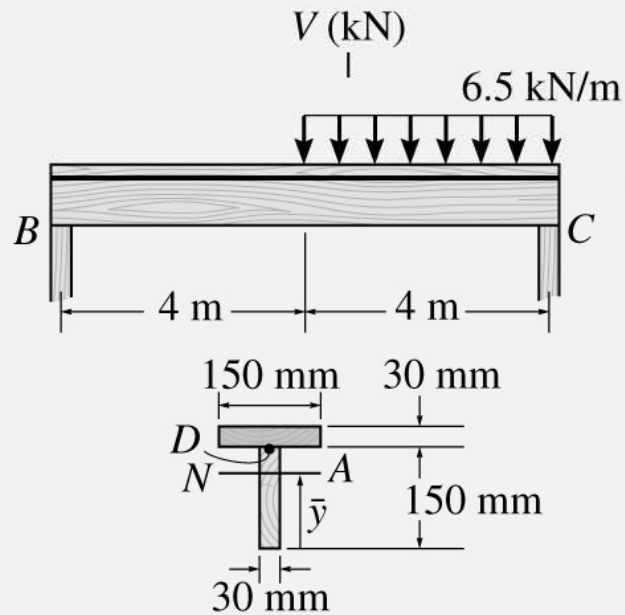
This stress acts on the area strip $dA = 0.015\,dy$ shown in Fig. 7–11*e*, and therefore the shear force resisted by the web is

$$V_w = \int_{A_w} \tau\,dA = \int_{-0.1\ \text{m}}^{0.1\ \text{m}} (25.192 - 257.07\,y^2)(10^6)(0.015\ \text{m})\,dy$$

$$V_w = 73.0\ \text{kN}$$ ***Ans.***

EXAMPLE 7–3

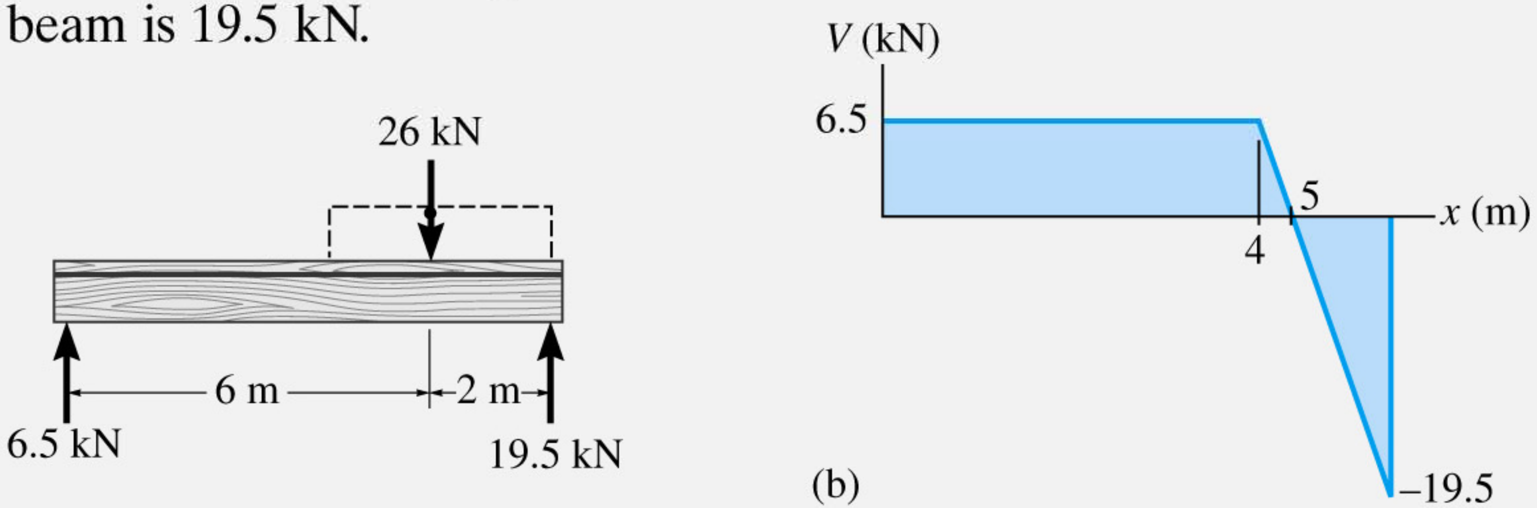
The beam shown in Fig. 7–12*a* is made from two boards. Determine the maximum shear stress in the glue necessary to hold the boards together along the seam where they are joined. The supports at *B* and *C* exert only vertical reactions on the beam.



(a)

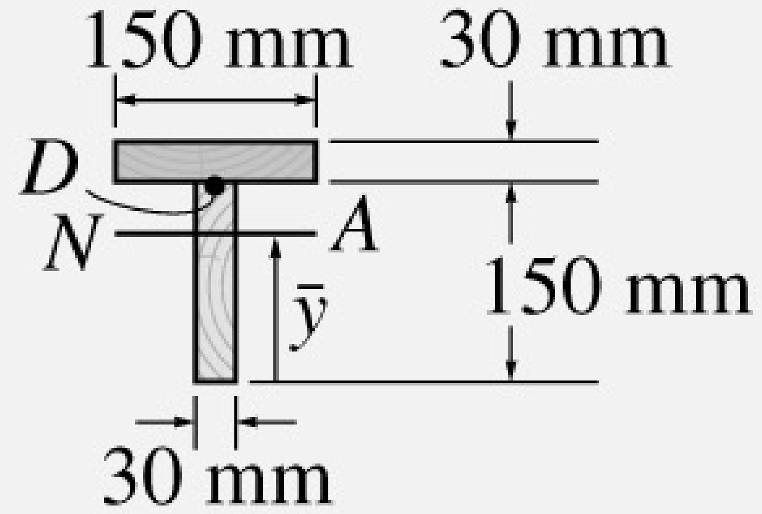
SOLUTION

Internal Shear. The support reactions and the shear diagram for the beam are shown in Fig. 7–12*b*. It is seen that the maximum shear in the beam is 19.5 kN.



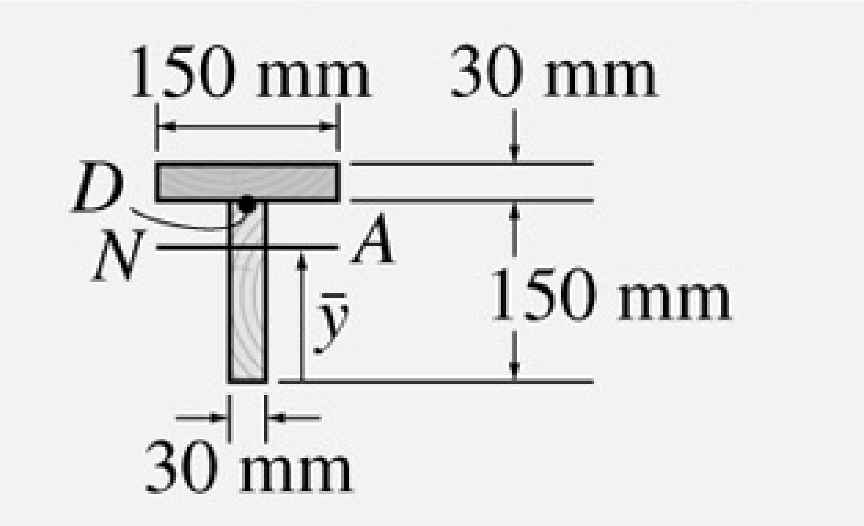
(b)

The neutral axis and the centroid will be determined from the references axis placed at the bottom of the cross section



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{[0.075\text{ m}](0.150\text{ m})(0.030\text{ m}) + [0.165\text{ m}](0.030\text{ m})(0.150\text{ m})}{(0.150\text{ m})(0.030\text{ m}) + (0.030\text{ m})(0.150\text{ m})} = 0.120\text{ m}$$

$$I = \left[\frac{1}{12}(0.030\text{ m})(0.150\text{ m})^3 + (0.150\text{ m})(0.030\text{ m})(0.120\text{ m} - 0.075\text{ m})^2\right] + \left[\frac{1}{12}(0.150\text{ m})(0.030\text{ m})^3 + (0.030\text{ m})(0.150\text{ m})(0.165\text{ m} - 0.120\text{ m})^2\right] = 27.0(10^{-6})\text{ m}^4$$



The glue will be applied over the thickness 0.03 m (point D), thus A' is defined to be the area above point D (area of the top board).

$$Q = \bar{y}'A' = [0.180\text{ m} - 0.015\text{ m} - 0.120\text{ m}](0.03\text{ m})(0.150\text{ m})$$

$$= 0.2025(10^{-3})\text{ m}^3$$

Shear Stress. Using the above data and applying the shear formula yields

$$\tau_{max} = \frac{VQ}{It} = \frac{19.5\text{ kN}(0.2025(10^{-3})\text{ m}^3)}{27.0(10^{-6})\text{ m}^4(0.030\text{ m})} = 4.88\text{ MPa} \qquad \textit{Ans.}$$