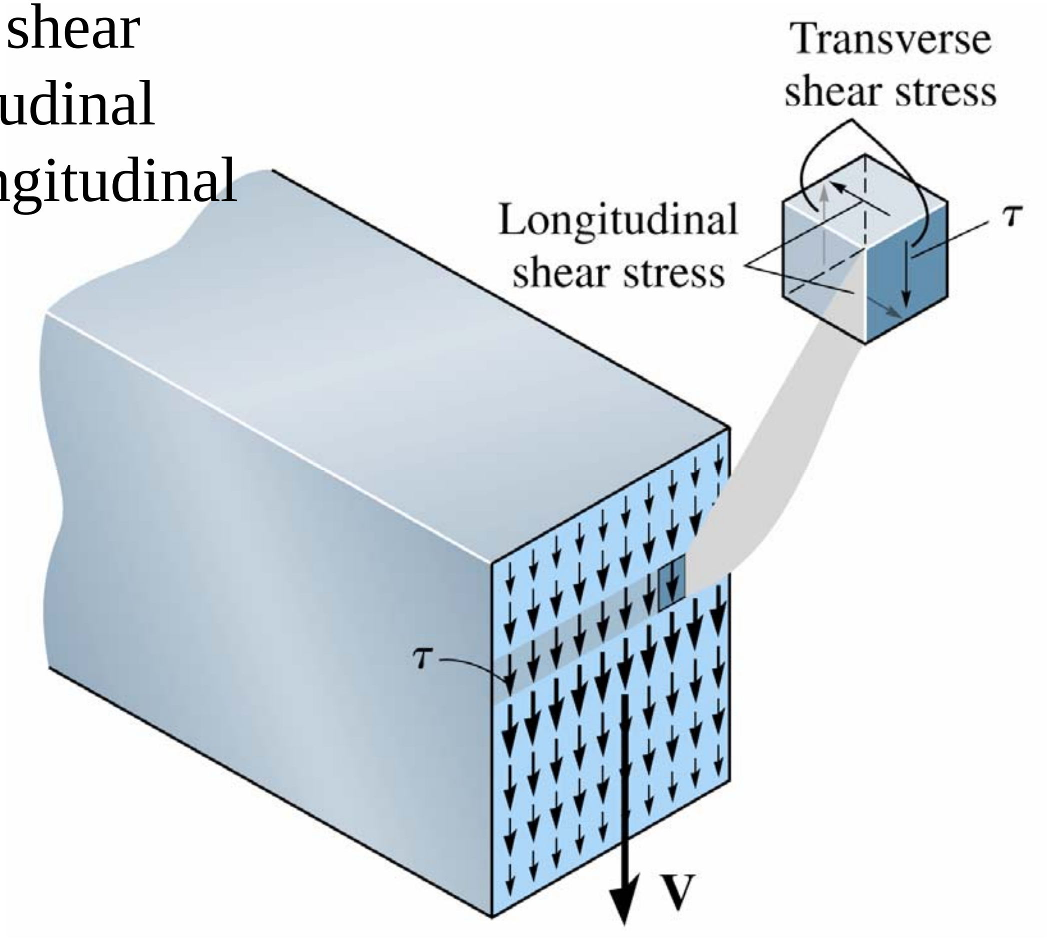
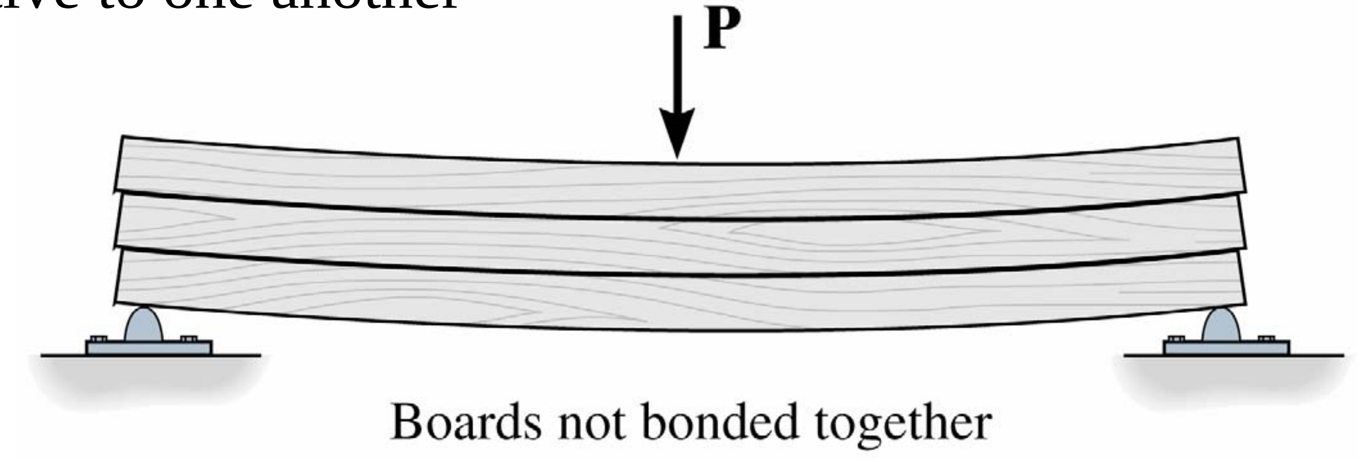
Transverse shear

V is the result of transverse shear stress that acts over the beam cross section. Notice that due to the complementary property of shear there is an associated longitudinal shear stress acting along longitudinal planes of the beam.

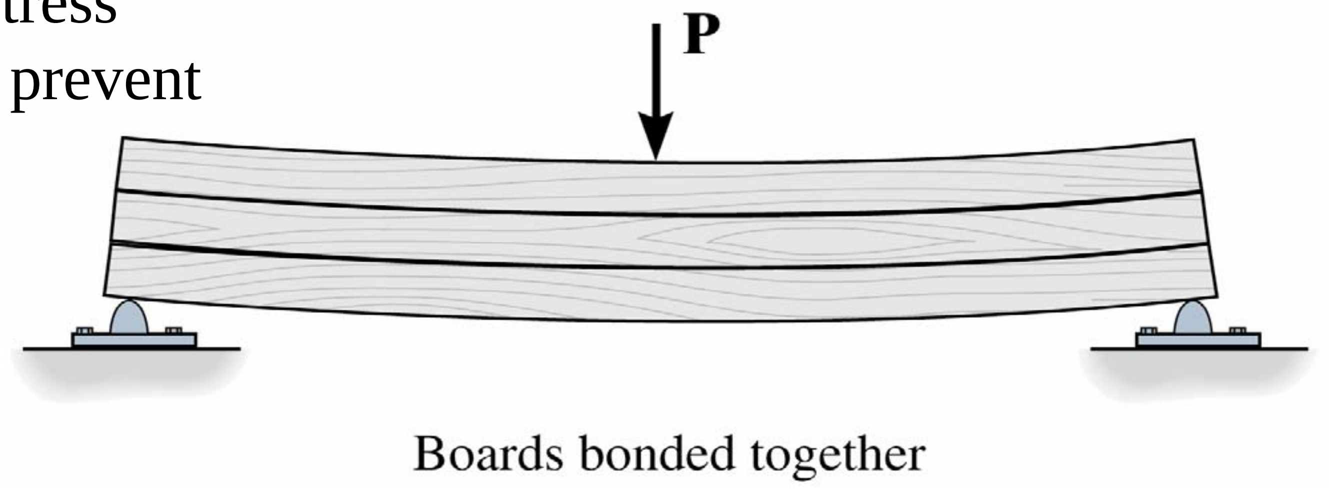


Longitudinal shear stress

Boards slide relative to one another



Longitudinal shear stress between boards will prevent their relative sliding

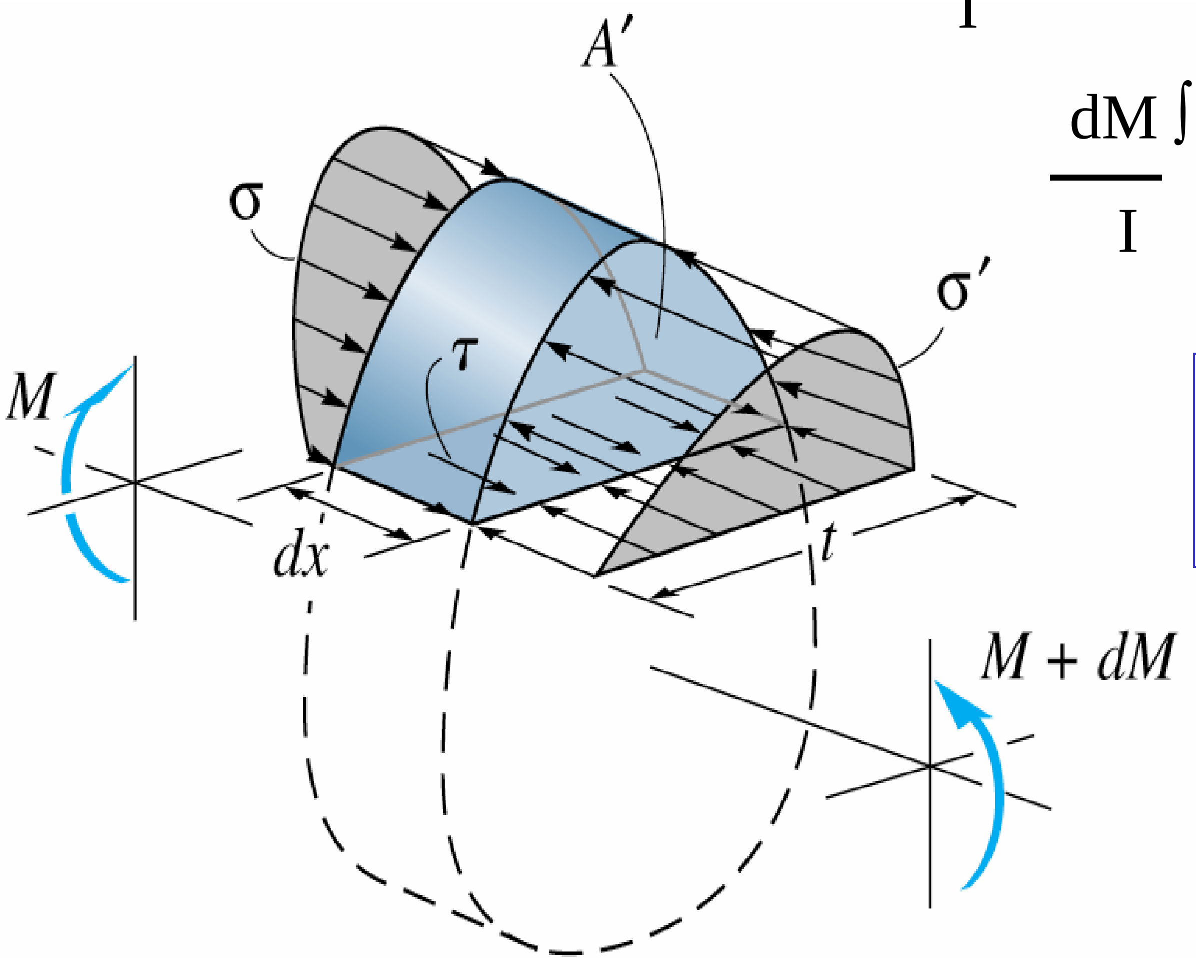


Shear Formula

$\leftarrow \Sigma F_x = 0$ $\int \sigma' \, dA - \int \sigma \, dA - \tau\,(t\,dx) = 0$

$$\int \frac{(M + dM)\,y}{I}\,dA - \int \frac{My}{I}\,dA - \tau\,(t\,dx) = 0$$

$$\frac{dM}{I} \int y\,dA = \tau\,(t\,dx)$$



Solving for τ

$$\tau = \frac{dM}{(I\,t)dx} \int y\,dA$$

$Q = \int y\,dA$

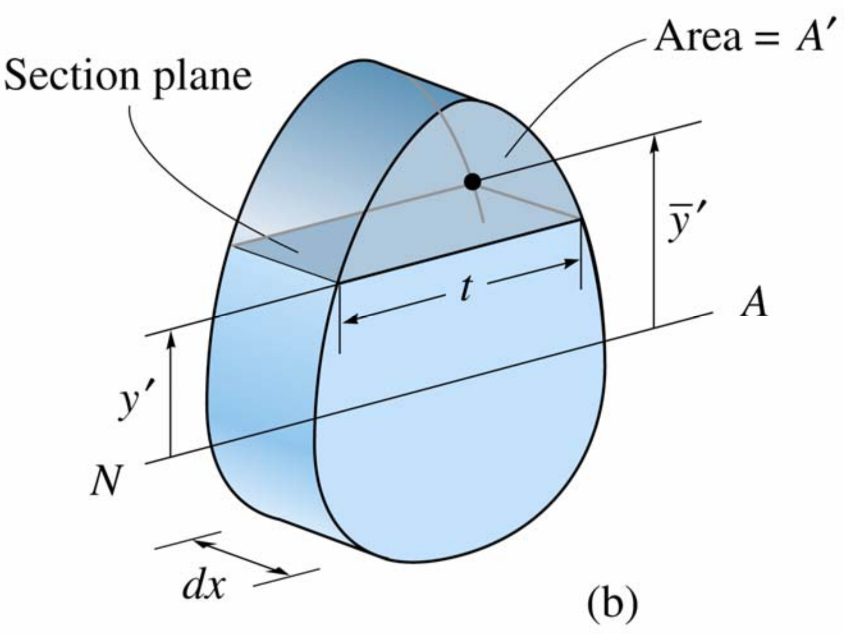
First moment of area

$V = dM / dx$

$$\tau = \frac{V\,Q}{I\,t}$$

$Q = \int y \, dA$

First moment of area



Location of the centroid ($\bar{y}'$) of area A' is determined by:

$\bar{y}' = \int y \, dA / A'$

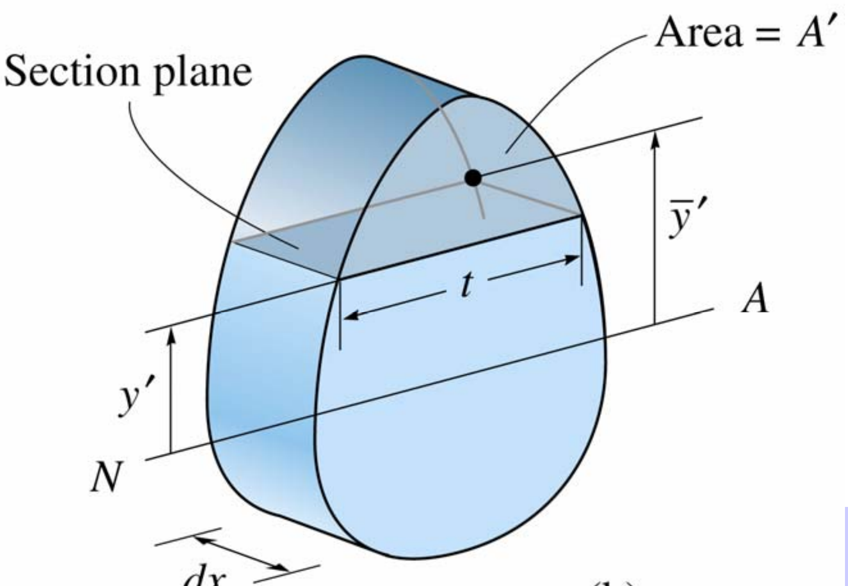
Thus

$Q = \bar{y}' A'$

$\bar{y}'$: is the distance between the **centroid point** of the section and the **principle neutral axis**.

A' is the **top** or **bottom** portion of the members cross section area, defined from the section where t is measured

$$\tau = \frac{VQ}{It}$$



(b)

Where:

τ: shear stress in the member at a point located a distance y’ from the neutral axis.

V: internal resultant shear force determined by the method of section.

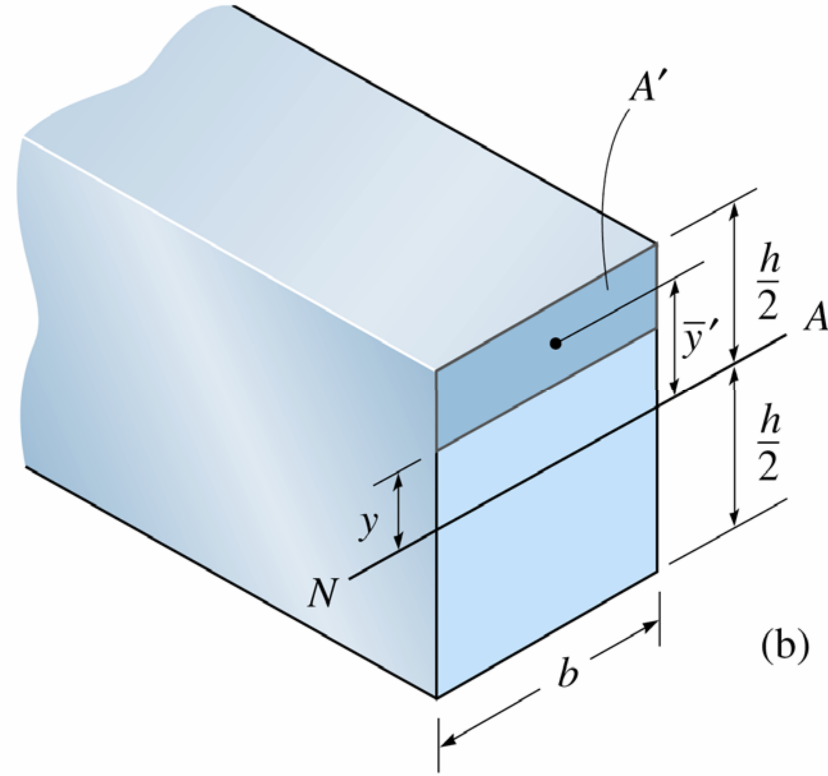
I: moment of inertia of the entire cross section area about the neutral axis

t: width of the member’s cross section area, measured at the point where τ is to be determined.

Q: $\int y\, dA = \bar{y}' A'$, where A’ is the top or bottom portion of the members cross section area, defined from the section where t is measured, $\bar{y}'$ is the distance from the neutral axis to the centroid of A’.

Shear stress in beams

Rectangular cross section



$Q = \bar{y}' A' = [\, y + [(h/2)-y)]^* \ ½ \,] * [(h/2) - y]\, b$

$Q = ½ \left[\dfrac{h^2}{4} - y^2\right] b$

Applying the shear formula:

$$\tau = VQ / It = \frac{V\, ½\, [h^2/4 - y^2]\, b}{(1/12\, b\, h^3)\, b}$$

$$\tau = \frac{6V}{bh^3}(h^2/4 - y^2)$$

$$\tau = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

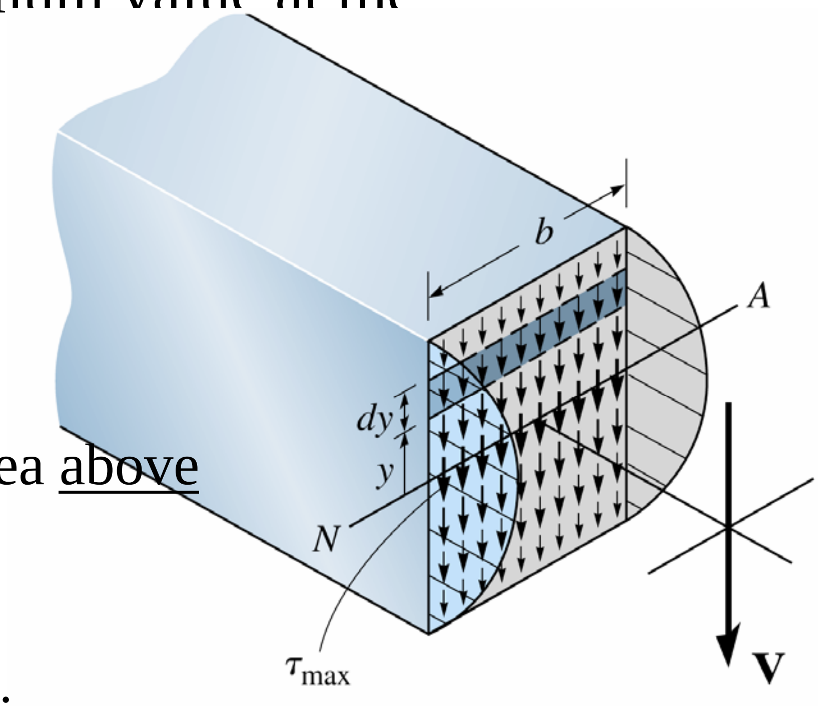
Shear stress distribution is parabolic, being zero at the top and bottom ($y=+h/2$, $-h/2$) to a maximum value at the neutral axis ($y=0$).

$$\tau = \frac{6V}{bh^3}\left(\frac{h^2}{4}\right)$$

$$\tau_{max} = 1.5\ V / A$$

N.B. Qmax is achieved by considering the area above or below the neutral axis.

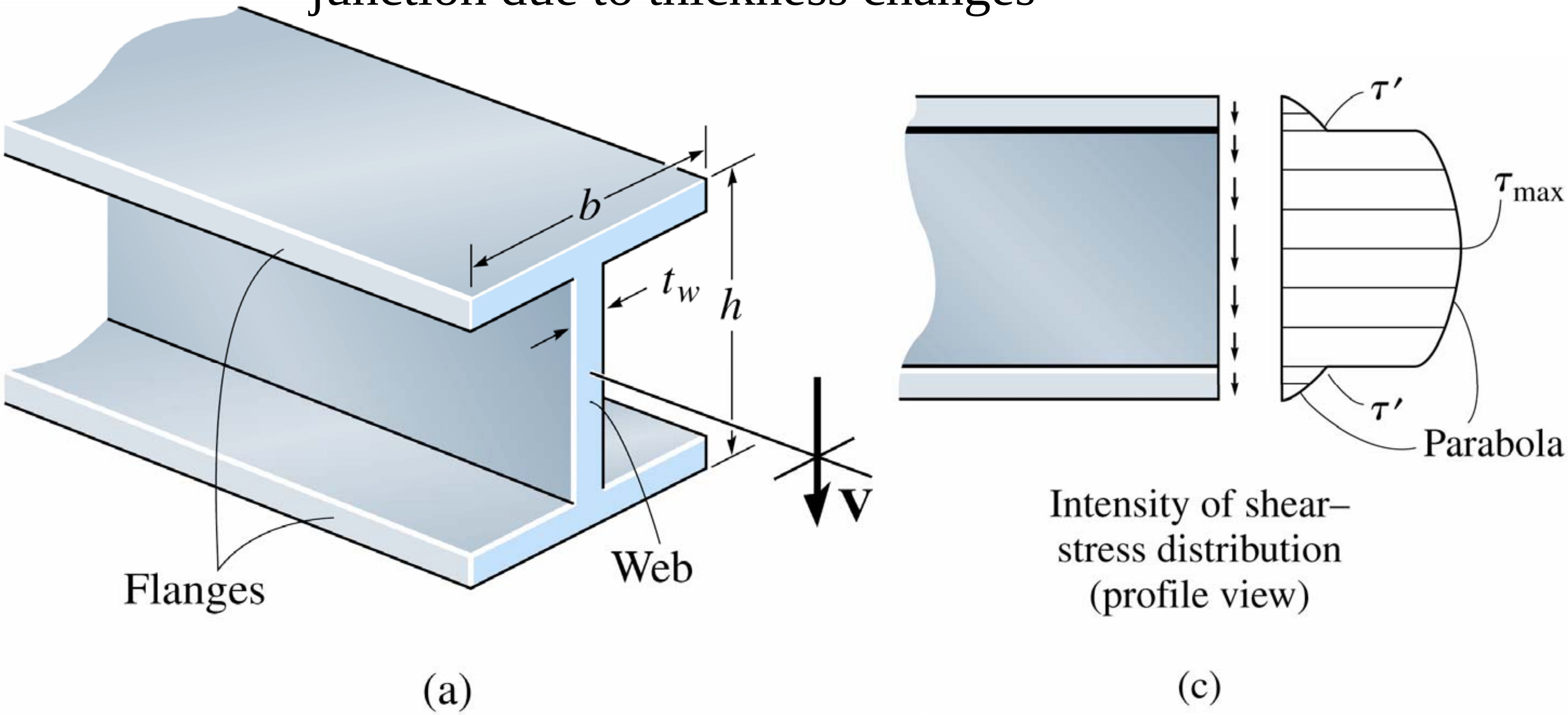
τ max occurs at **Qmax**, which mean that it is maximum at the neutral axis, provided that **t** is minimum at this neutral axis



Shear–stress distribution

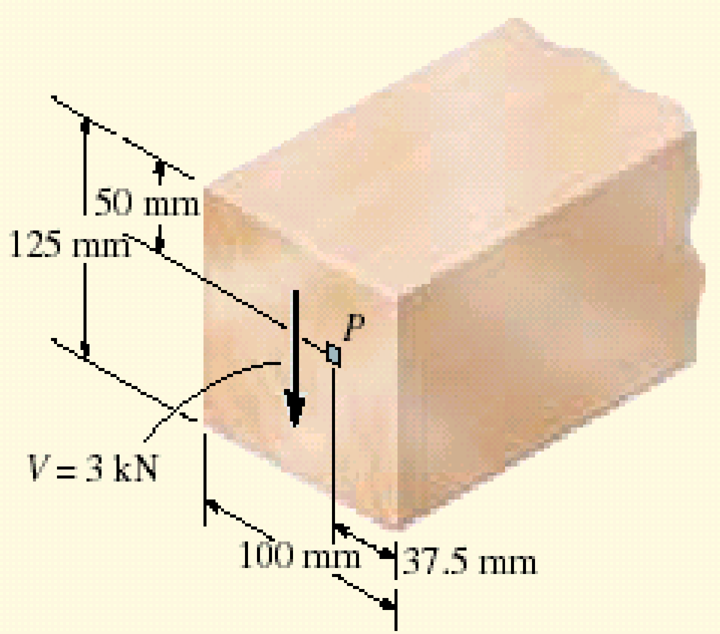
Shear stress distribution in wide flange beam

- Note that the web will carry more of the shear force compared to the flange.
- Notice the jump in the shear stress in the web-flange junction due to thickness changes



EXAMPLE 7.1

The beam shown in Fig. 7–10*a* is made of wood and is subjected to a resultant internal vertical shear force of $V = 3$ kN. (a) Determine the shear stress in the beam at point *P*, and (b) compute the maximum shear stress in the beam.



Solution

Part (a).

Section Properties. The moment of inertia of the cross-sectional area computed about the neutral axis is

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(100\text{ mm})(125\text{ mm})^2 = 16.28 \times 10^6\text{ mm}^4$$

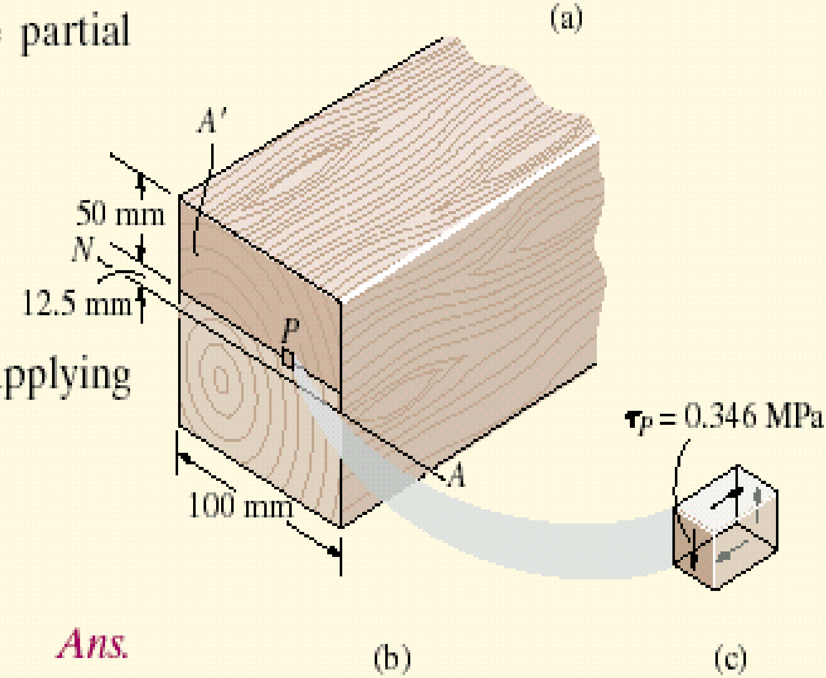
A horizontal section line is drawn through point P and the partial area A' is shown shaded in Fig. 7–10b. Hence

$$Q = \bar{y}'A' = \left[12.5\text{ mm} + \frac{1}{2}(50\text{ mm})\right](50\text{ mm})(100\text{ mm})$$
$$= 18.75\text{ mm} \times 10^4\text{ mm}^3$$

Shear Stress. The shear force at the section is $V = 3$ kN. Applying the shear formula, we have

$$\tau_P = \frac{VQ}{It} = \frac{(3\text{ kN})(18.75 \times 10^4\text{ mm}^3)}{(16.28 \times 10^6\text{ mm}^4)(100\text{ mm})}$$

$$= 3.46 \times 10^{-4}\text{ kN/mm}^2 = 0.346\text{ MPa} \qquad \textit{Ans.}$$

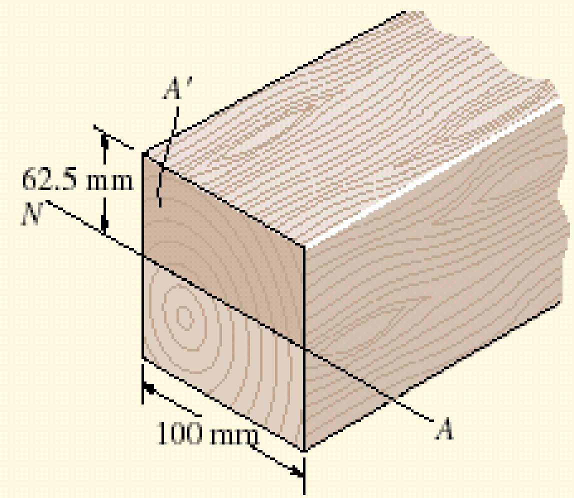


Part (b).

Section Properties. Maximum shear stress occurs at the neutral axis, since t is constant throughout the cross section and Q is largest for this case. For the dark shaded area A' in Fig. 7–10d, we have

$$Q = \overline{y}'A' = \left[\frac{62.5\text{ mm}}{2}\right](100\text{ mm})(62.5\text{ mm}) = 19.53 \times 10^4\text{ mm}^3$$

Shear Stress. Applying the shear formula yields



$$\tau_{max} = \frac{VQ}{It} = \frac{(3\text{ kN})(19.53 \times 10^4\text{ mm}^3)}{(16.28 \times 10^6\text{ mm}^4)(100\text{ mm})}$$

$$= 3.60 \times 10^{-4}\text{ kN/mm}^2 = 0.360\text{ MPa}$$

Note that this is equivalent to

$$\tau_{max} = 1.5\frac{V}{A} = 1.5\,\frac{3\text{ kN}}{(100\text{ mm})(125\text{ mm})} = 3.6 \times 10^{-4}\text{ kN/mm}^2 = 0.36\text{ MPa}$$

Ans.