MAS 117, Section 2 - Mathematics II – Summer 2006

Quiz 1

SOLUTION.

1. Let ||**u**|| = 1, ||**v**|| = 2, and the angle between them is $\pi/6$. Find **u**•**v**

1. **u**•**v** = $\frac{\sqrt{3}}{2}$
2. **u**•**v** = $\sqrt{3}$
3. **u**•**v** = $\sin(\pi/3)$
4. **u**•**v** = $\frac{1}{2}$
5. none of the above

The correct answer is 2.

Because $\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$, it follows that $u \cdot v = \|u\|\|v\|\cos\theta$. And since $\cos(\pi/6) = \sqrt{3}/2$, substituting in this equation we get $u \cdot v = (1)(2)(\sqrt{3}/2) = \sqrt{3}$

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2. The equation of the plane passing through the point P(1, -1, 0) and normal to the vector **n** (1, 2, 3) is
   1. $x - 2y + 3z + 2 = 0$
   2. $2x + y + 3z + 1 = 0$
   3. $-x + 2y - 3z + 3 = 0$
   4. $x + 2y + 3z + 1 = 0$
   5. none of the above

The correct answer is 4.

Applying the formula of the equation of the plane passing through a point and normal to a given vector we obtain

$$1(x - 1) + 2(y + 1) + 3(z - 0) = 0$$

Simplifying we get answer number 4.

3. The slope of the tangent line to the curve $r = 2\sin\theta$ at the point where $\theta = \pi/6$ is

1. $\frac{\sqrt{3}}{2}$
2. $\sqrt{3}$
3. $\frac{\sqrt{2}}{2}$
4. $\frac{1}{2}$
5. None of the above

The correct answer is 2.

The formula for finding slope of a curve in polar coordinates is

$$\frac{dy}{dx} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta} = \frac{2\sin\theta\cos\theta + 2\cos\theta\sin\theta}{-2\sin^2\theta + 2\cos^2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

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4. The terminal point of $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ with initial point (-1, 2, -1) is
   1. (1, -1, 1)
   2. (0, 4, -4)
   3. (2, 0, 0)
   4. (1, 4, -2)
   5. None of the above

The correct answer is 2.

Let P_1 and P_2 be the initial and terminal points. Then

$$\vec{P_1P_2} = P_2 - P_1 = u \;\Rightarrow\; P_2 = P_1 + u = (-1,2,-1) + (1,2,-3) = (-1+1,\ 2+2,\ -1-3) = (0,4,-4)$$