

MAS 117 - Mathematics II – Section 1 - Summer 2006

Quiz 1

Every correct answer is 25 points. A wrong mark gives -5 points (discount)

**SOLUTION.**

1. The parametric equations of the line whose vector equation is  $x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = 2\mathbf{j} - \mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$  are

1.  $x = t, y = 1 - 2t, z = -1 + 2t$
2.  $x = t, y = -t, z = 1 + t$
3.  $x = t, y = 1 - t, z = 1 + t$
4.  $x = 1 + t, y = 2 - t, z = 2 + t$
5. none of the above

**The correct answer is 5.**

This is a line passing through the point  $P(0, 2, -1)$  and parallel to the vector  $\mathbf{v}(1, -2, 2)$

-o-o-o-

2. The point of intersection of the line  $x = 2 + 2t, y = 1 - t, z = 4 + 2t$  and the plane  $2x - 3y + 2z + 2 = 0$  is

1.  $(0, 2, 2)$
2.  $(-1, 1, 0)$
3.  $(1, -1, 2)$
4.  $(0, 0, 1)$
5. none of the above

**The correct answer is 1.**

Substituting  $x, y,$  and  $z$  in the equation of the plane by its expressions in  $t$  and solving for  $t$ , we have

$$2(2 + 2t) - 3(1 - t) + 2(4 + 2t) + 2 = 0$$

$$11t + 11 = 0$$

$$t = -1$$

substituting  $t$  by  $-1$  in the parametric equations of the line we get  $(0, 2, 2)$

Problem 3. The vectors  $\mathbf{u} = (2, 0, 2)$  and  $\mathbf{v} = (1, 0, 0)$  are

1. perpendicular
2. have same direction
3. have opposite directions
- 4.** form and angle of  $\pi/4$
5. none of the above

**The correct answer is 4.**

The angle  $\theta$  between the vectors is

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

and

$$\|\mathbf{u}\| = \sqrt{2^2 + 0^2 + 2^2} = \sqrt{8}, \quad \|\mathbf{v}\| = \sqrt{1^2 + 0^2 + 0^2} = 1$$

Hence,

$$\cos \theta = \frac{2 \times 1 + 0 \times 0 + 2 \times 0}{\sqrt{8}} = \frac{\sqrt{2}}{2}$$

and the angle is  $\pi/4$

-0-0-0-0-

Problem 4. Let  $\mathbf{u} = (1, 1, 0)$  and  $\mathbf{v} = (1, 0, -1)$ . The direction cosines of  $\mathbf{u} \times \mathbf{v}$  are

- 1.**  $\cos \alpha = \frac{-1}{\sqrt{3}}, \cos \beta = \frac{1}{\sqrt{3}}, \cos \gamma = \frac{-1}{\sqrt{3}}$
2.  $\cos \alpha = \frac{1}{\sqrt{3}}, \cos \beta = \frac{1}{\sqrt{3}}, \cos \gamma = \frac{1}{\sqrt{3}}$
3.  $\cos \alpha = \frac{\sqrt{2}}{\sqrt{3}}, \cos \beta = \frac{\sqrt{2}}{\sqrt{3}}, \cos \gamma = -1$
4.  $\cos \alpha = \sqrt{3}, \cos \beta = \frac{1}{2}, \cos \gamma = \frac{-1}{2}$
5. none of the above

**The correct answer is 1.**

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = -\mathbf{i} + \mathbf{j} - \mathbf{k} \quad \text{and} \quad \|\mathbf{u} \times \mathbf{v}\| = \sqrt{(-1)^2 + 1^2 + (-1)^2} = \sqrt{3}$$

Direct application of the cosine directions formula yields answer 1.