SIIT - MAS 117 - MATHEMATICS II MIDTERM EXAM SUMMER 2006

No calculator, closed book. For questions 1-8, multiple-choice questions, circle the correct answer. Each correct answer in this part is 8 points. Each answer incorrect discounts 1.6 points. More than 1 circle on a question invalidates the answer (0 point). **Good luck!!**

SOLUTION

Problem 1. The slope of the tangent line to the curve $r = \sin(2\theta)$ at the point $\theta = \pi/6$ is

1. $\sqrt{3}/2+3$ 2. $\sqrt{3}+2$ 3. $\sqrt{3}/2$ 4. 1/2 5. None of the above

The correct answer is 2.

$$\frac{dr}{d\theta}=\frac{dy}{dx}=\frac{r\cos\theta+r'\sin\theta}{-r\sin\theta+r'\cos\theta}=\frac{\sin 2\theta\cos\theta+2\cos 2\theta\sin\theta}{-\sin 2\theta\sin\theta+2\cos 2\theta\cos\theta}=\frac{(\sqrt{3}/2)(\sqrt{3}/2)+2(1/2)(\sqrt{3}/2)}{-(\sqrt{3}/2)(1/2)+2(1/2)(\sqrt{3}/2)}=\sqrt{3}+2$$

Problem 2. If $\mathbf{u} = \mathbf{i} - \mathbf{j}$, $\mathbf{v} = \mathbf{j} + \mathbf{k}$, $\mathbf{w} = \mathbf{u}\times\mathbf{v}$, and $\mathbf{s} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$, the vectors **w** and **s** are

1. Parallel 2. Perpendicular 3. Form an angle of $\pi/4$ 4. Form an angle of $\pi/6$ **5. None of the above**

The correct answer is 2. $\mathbf{w} = \mathbf{u}\times\mathbf{v} = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = -\mathbf{i} - \mathbf{j} + \mathbf{k}$. Thus the angle θ between **w** and **s** is

$$\cos\theta = \frac{w\cdot s}{\|w\|\|s\|}=\frac{(-1,-1,1)\cdot(1,1,-2)}{\sqrt{3}\sqrt{5}}=\frac{-4}{\sqrt{15}}$$

Problem 3. Let $\mathbf{u} = (1, 0, -1)$ and $\mathbf{v} = (1, 2, 0)$. Then the equation of the plane normal to $\mathbf{u}\times\mathbf{v}$ passing through the point P (1, 3, -1) is

1. $3(x-1) + 2(y-3) - 2(z+1) = 0$ 2. $3x + 2y - 2z - 2 = 0$ 3. $2x + y + 2z - 11 = 0$ **4. $2x + y + 2z + 1 = 0$** 5. None of the above

The correct answer is 4.
$\mathbf{u}\times\mathbf{v} = (-2, 1, 2)$ Thus, the equation of the plane is $-2(x-1) + (y-3) + 2(z+1) = 0$. Simplifying we get

Problem 4. The parametric equations of the line of intersection of the planes
$2x + y - z - 1 = 0$ and $-x - y + 2z + 3 = 0$ is

1. $x = -2 - t,\ y = 5 + 3t,\ z = 2t$
2. $x = -2 - t,\ y = 3 - t,\ z = t$
3. $x = 2 + 3t,\ y = 5 + 3t,\ z = t$
4. **$x = -2 - t,\ y = 5 + 3t,\ z = t$**
5. None of the above

The correct answer is 4. Solving the system

$2x + y - z - 1 = 0$
$-x - y + 2z + 3 = 0$ we get $x = -2 - z$. Making $z = t$ we get $x = -2 - t$. Then solving for y in one of the equations we get $y = 5 + 3t$

Problem 5. The parametric equations of the line passing through the point of intersection of the line $x = 2 - 2t,\ y = -4 - t,\ z = t$ and the plane $x - y + 2z + 1 = 0$ that is parallel to the vector u = (1, 1, 1) are

1. **$x = 16 + t,\ y = 3 + t,\ z = -7 + t$**
2. $x = 12 + t,\ y = -11 + t,\ z = -1 + t$
3. $x = 1 + t,\ y = -3 + t,\ z = -1 + t$
4. $x = 5 + t,\ y = -3 + t,\ z = -12 + t$
5. None of the above

The correct answer is 1.
To intercept the line with the plane, substitute in the equation of the plane, the values of *x, y,* and *z by* their corresponding values in *t*.

$2 - 2t - (-4 - t) + 2t + 1 = 0$, that is, $t = -7$. Substituting this value of t on the equations of the line, we get the point of intersection of the line and the plane, as
(16, 3, -7). Thus the answer is 1

Problem 6. The equation of the plane passing through the points $P_1(1, 1, 1)$, $P_2(0, 1, 0)$, $P_3(0, -1, -1)$ is

1. $y - 1 = 0$
2. $x + y - 1 = 0$
3. **$2x - y + 2z - 3 = 0$**
4. $2y - z - 4 = 0$
5. None of the above

The correct answer is 3.

Let the vectors $P_2 - P_1 = (0, 1, 0) - (1, 1, 1) = (-1, 0, -1)$ and
$P_3 - P_1 = (0, -1, -1) - (1, 1, 1) = (-1, -2, -2)$. The plane is normal to

$$(P_2 - P_1) \times (P_2 - P_1) = \begin{vmatrix} i & j & k \\ -1 & 0 & -1 \\ -1 & -2 & -2 \end{vmatrix} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$ and passes through P_1, thus

$2(x - 1) - (y - 1) + 2(z - 1) = 0$ and the answer is 3.

Problem 7. Let **u** be the vector parallel to the line $x = 3 + t, y = -4 - t, z = 1$. The direction angles of **u** are

1. $\alpha = \pi/3,\ \beta = \pi/6,\ \gamma = \pi/2$
2. $\alpha = \pi/2,\ \beta = \pi/6,\ \gamma = \pi/3$
3. $\alpha = \pi/2,\ \beta = 2\pi/3,\ \gamma = \pi/3$
4. $\alpha = \pi/4,\ \beta = 3\pi/4,\ \gamma = \pi/2$
5. None of the above

The correct answer is 4.

u = (1, -1, 0) and ||**u**|| = $\sqrt{2}$. Thus, $\cos\alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, $\cos\beta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$, $\cos\gamma = 0$

and the answer is 4.

Problem 8. The initial point of the vector parallel to the line $x = 2 - t,\ y = -1 + 2t,$ $z = 1 + t$, whose terminal point P_2 is (1, -1, 1), is

1. **(2, -3, 0)** 2. (1, -1, 1) 3. (2, -1, 1) 4. (-1, 2, 1) 5.None of the above

The correct answer is 1.

The vector parallel to the given line has coordinates (-1, 2, 1). Thus
u $= P_2 - P_1$ and thus, $P_1 = P_2 -$ **u** = (1, -1, 1) – (-1, 2, 1) = (2, -3, 0)

MAS117 Mathematics II
Midterm Examination
Part II: Chapter 14 Partial Derivatives
Dr Anotai Suksangpanomrung

Instructions

1. Examination contains 4 questions and all are compulsory
2. Calculator is not allow in the examination room
3. Textbooks and lecture notes are not allow in the examination room
4. The student must shows details calculation in order to obtain a full mark in each question

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1. (Limit of function of two variables)

Calculating the limit of $\lim\limits_{(x,y)\to(1,1)} \dfrac{x^2 - y^2}{x - y}$

2. (Partial derivative)

Given $f(x,y) = x^2 y + \cos y + y \sin x$; Find $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial^2 f}{\partial x \partial y}$.

3. (Directional derivative and Gradient)

Finding the derivative of the function $f(x,y) = 2xy - 3y^2$ at point $P_0(5, 5)$ in the direction of $\vec{v} = 4\vec{i} + 3\vec{j}$

4. (Extreme values and Saddle Points)

Locate all relative maxima, relative minima, and saddle points of $f(x,y) = x^2 + xy + y^2 - 3x$

Hints: The second derivative test for local extreme values

- **i.** f has a **local maximum** at (a, b) if $f_{xx} < 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b).
- **ii.** f has a **local minimum** at (a, b) if $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b).
- **iii.** f has a **saddle point** at (a, b) if $f_{xx}f_{yy} - f_{xy}^2 < 0$ at (a, b).
- **iv.** **The test is inconclusive** at (a, b) if $f_{xx}f_{yy} - f_{xy}^2 = 0$ at (a, b). In this case, we must find some other way to determine the behavior of f at (a, b).

Name______________________________ID____________________No________