

# List of Problems 4

**Problem 1.** Find maximum and minimum of  $f(x,y)$  subject to the condition  $g(x,y) = 0$

a.  $f(x,y) = x^2 - xy$ ,  $g(x,y) = x^2 - 2xy = 1$     b.  $f(x,y) = 4x^2 + y^2$ ,  $g(x,y) = 2x^2 + y^2 = 1$

**Problem 2.** Evaluate the following integrals

a.  $\int_{-1}^2 \int_0^1 3x^2 y^2 dy dx$     b.  $\int_0^{\pi/2} \int_0^1 y \sin x dy dx$     c.  $\int_0^1 \int_{x^2}^{1/x} 4xy^3 dy dx$

d.  $\int_0^1 \int_{-1}^1 \int_0^1 2x^3 y^2 z dz dy dx$     e.  $\int_0^1 \int_0^x \int_0^{1/xy} 2x^3 y^2 z dz dy dx$     f.

$\int_0^{\pi/2} \int_0^{\sin y} 3x^2 \cos y dx dy$

g.  $\int_0^1 \int_0^{1-x} 2xy dy dx$     h.  $\int_1^2 \int_1^{\sqrt{x}} \int_0^{1/xy} 2x^2 y^2 dz dy dx$

**Problem 3.** Evaluate  $\iint_R xy dA$  where  $R$  is the region

- The triangle of vertices  $(0,2)$ ,  $(0,0)$ ,  $(1,0)$
- The rectangle  $[0,1] \times [1,2]$
- The triangle of vertices  $(1,0)$ ,  $(2,3)$ ,  $(3,0)$
- The region enclosed by the curves  $y = 2x - y = 0$  and  $y = x^2$ .

**Problem 4** Calculate the area enclosed by the curves  $x^2 - y = 0$  and  $2x - y - 1 = 0$

**Problem 5.** Find the volume of the solid under the surface  $f(x,y) = x^2 + y$ , bounded by the three coordinate planes and the plane  $x + 2y = 1$ .

**Problem 6.** Find the volume of the solid that lays in the first octant and is bounded by the three coordinate planes and under the plane  $x + 3y + 5z - 15 = 0$

**Problem 7.** Evaluate the following line integrals

a.  $\int_C (x + 2y)dx + (x - y)dy$ ,  $C : x = \cos t$ ,  $y = 4 \sin t$   $0 \leq t \leq \pi/4$

b.  $\int_C (y - x)dx + x^2 y dy$ ,  $C : y^2 = x^3$ , from  $(1,-1)$  to  $(1,1)$

c.  $\int_C (y - x)dx + xy dy$ ,  $C$ : the line segment from  $(3,4)$  to  $(2,1)$

d.  $\int_C \frac{x}{1+y^2} ds$   $C: x=1+2t, y=t, 0 \leq t \leq 1$

e.  $\int_C (x+y) ds$   $C: x=2t, y=t, 0 \leq t \leq 1$

f.  $\int_C (x+2y) dx$   $C: x=1+t, y=5t, 0 \leq t \leq 1$

g.  $\int_C (x+2y) dy$   $C: x=1+t, y=5t, 0 \leq t \leq 1$

h.  $\int_C x^2 dx + 2y dy$   $C: x=3t, y=2-3t, 0 \leq t \leq 1$

**Problem 8.** Use Green's Theorem to evaluate  $\oint_C x^2 dx - xy dy$ , where  $C$  is

a. The triangle of vertices  $(0,0)$ ,  $(2,0)$ ,  $(0,1)$

b. The triangle of vertices  $(0,0)$ ,  $(1,2)$ ,  $(2,0)$

c. The rectangle bounded by  $x=0$ ,  $y=0$ ,  $x=1$ ,  $y=2$