

List of Problems 2

Problem 1. Calculate the following limits

a. $\lim_{(x,y) \rightarrow (1,2)} \frac{x^3 - y^3}{x + y}$

b. $\lim_{(x,y) \rightarrow (1,1)} \frac{x^4 - y^4}{x^2 - y^2}$

c. $\lim_{(x,y) \rightarrow (0,0)} \frac{x - y}{\sqrt{x} - \sqrt{y}}$

d. $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$

Problem 2. Show that the following limits do not exist

a. $\lim_{(x,y) \rightarrow (0,0)} \frac{x - y}{x^2 + y^2}$

b. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{(x - y)^2}$

Problem 3. Given the following functions, find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$

a. $f(x,y) = x^2y$

b. $f(x,y) = \sin(xy)$

c. $f(x,y) = x^2 \cos(xy) - x/y$

Problem 4. Find the directional derivative of the following functions $f(x,y)$, at the given points P , in the direction of the indicated vectors \mathbf{u} .

a. $f(x,y) = 3x^2y - 2x^4$, $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j}$, $P(-1, 2)$

b. $f(x,y) = 2x^2 - 2x^4y$, $\mathbf{u} = 2\mathbf{i} + \mathbf{j}$, $P(1, 0)$

Problem 5. Find all relative extrema (maxima and minima) and saddle points of

a. $f(x,y) = x^2 - xy + y^3$.

b. $f(x,y) = x^2y - 8x - y^2$.

Problem 6. Find the gradient of the functions in problem 5 at the point $(-1, 1)$.

Problem 7. Given the following functions, find the direction angles of the directional derivative of maximum slope

a. $f(x,y) = 2x^2y - 2x^4 - y$,

b. $f(x,y) = 2x^2 - 2x^3y + xy^2$,

Problem 8. Find the absolute maximum and absolute minimum of $f(x,y)$ subject to the given condition $g(x,y) = 0$

a. $f(x,y) = x^2 - y^2$, $g(x,y) = x^2 + y^2 = 25$

b. $f(x,y) = 4x^2 + y^2$, $g(x,y) = 2x^2 + y^2 = 1$