Chapter 2 Homework from Kreyszig

1 Addition and Scalar Multiplication of Matrices and Vectors (Kreyszig page 277)

For questions 1 to 8 in this section use

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 4 \\ -1 & 2 & 2 \\ 6 & 5 & -4 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & -5 & -3 \\ -5 & 2 & 4 \\ -3 & 4 & 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0 & 2 \\ 2 & 4 \\ 1 & 3 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 6 & 1 \\ -4 & 7 \\ -8 & 3 \end{bmatrix},$$
$$\mathbf{u} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -4.5 \\ 0.8 \\ 1.2 \end{bmatrix}.$$

Find the following expressions or give reasons why they are undefined.

1. $\mathbf{C} + \mathbf{D}$, $\mathbf{D} + \mathbf{C}$, $6(\mathbf{D} - \mathbf{C})$, $6\mathbf{C} - 6\mathbf{D}$
2. $4\mathbf{C}$, $2\mathbf{D}$, $4\mathbf{C} + 2\mathbf{D}$, $8\mathbf{C} - 0\mathbf{D}$
3. $\mathbf{A} + \mathbf{C} - \mathbf{D}$, $\mathbf{C} - \mathbf{D}$, $\mathbf{D} - \mathbf{C}$, $\mathbf{B} + 2\mathbf{C} + 4\mathbf{D}$
4. $2(\mathbf{A} + \mathbf{B})$, $2\mathbf{A} + 2\mathbf{B}$, $5\mathbf{A} - \frac{1}{2}\mathbf{B}$, $\mathbf{A} + \mathbf{B} + \mathbf{C}$
5. $3\mathbf{C} - 8\mathbf{D}$, $4(3\mathbf{A})$, $(4 \times 3)\mathbf{A}$, $\mathbf{B} - \frac{1}{10}\mathbf{A}$
6. $5\mathbf{A} - 3\mathbf{C}$, $\mathbf{A} - \mathbf{B} + \mathbf{D}$, $4(\mathbf{B} - 6\mathbf{A})$, $4\mathbf{B} - 24\mathbf{A}$
7. $33\mathbf{u}$, $4\mathbf{v} + 9\mathbf{u}$, $4(\mathbf{v} + 2.25\mathbf{u})$, $\mathbf{u} - \mathbf{v}$
8. $\mathbf{A} + \mathbf{u}$, $12\mathbf{u} + 10\mathbf{v}$, $0(\mathbf{B} - \mathbf{v})$, $0\mathbf{B} + \mathbf{u}$

2 Multiplication, Addition, and Transposition of Matrices and Vectors (Kreyszig page 286)

For questions 1 to 11 in this section use

$$\mathbf{A} = \begin{bmatrix} 6 & -2 & -2 \\ 10 & -3 & 1 \\ -10 & 5 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 9 & 4 & -4 \\ 4 & 7 & 0 \\ -4 & 0 & 11 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 3 & 1 \\ 0 & -2 \\ 4 & 0 \end{bmatrix}, \quad \mathbf{a} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 & 0 & 8 \end{bmatrix}.$$

1. $\mathbf{Aa}$, $\mathbf{Ab}$, $\mathbf{Ab}^\mathsf{T}$, $\mathbf{AB}$
2. $\mathbf{Ab}^\mathsf{T} + \mathbf{Bb}^\mathsf{T}$, $(\mathbf{A} + \mathbf{B})\mathbf{b}^\mathsf{T}$, $\mathbf{bA}$, $\mathbf{B} - \mathbf{B}^\mathsf{T}$
3. $\mathbf{AB}$, $\mathbf{BA}$, $\mathbf{AA}^\mathsf{T}$, $\mathbf{A}^\mathsf{T}\mathbf{A}$
4. $\mathbf{A}^2$, $\mathbf{B}^2$, $(\mathbf{A}^\mathsf{T})^2$, $(\mathbf{A}^2)^\mathsf{T}$
5. $\mathbf{a}^\mathsf{T}\mathbf{A}$, $\mathbf{bA}$, $5\mathbf{B}(3\mathbf{a} + 2\mathbf{b}^\mathsf{T})$, $15\mathbf{Ba} + 10\mathbf{Bb}^\mathsf{T}$

6. $\mathbf{A}^\mathsf{T}\mathbf{b}$, $\mathbf{b}^\mathsf{T}\mathbf{B}$, $(3\mathbf{A}-2\mathbf{B})^\mathsf{T}\mathbf{a}$, $\mathbf{a}^\mathsf{T}(3\mathbf{A}-2\mathbf{B})$
7. $\mathbf{ab}$, $\mathbf{ba}$, $(\mathbf{ab})\mathbf{A}$, $\mathbf{a}(\mathbf{bA})$
8. $\mathbf{ab}-\mathbf{ba}$, $-(4\mathbf{b})(7\mathbf{a})$, $-28\mathbf{ba}$, $5\mathbf{abB}$
9. $(\mathbf{A}+\mathbf{B})^2$, $\mathbf{A}^2+\mathbf{AB}+\mathbf{BA}+\mathbf{B}^2$, $\mathbf{A}^2+2\mathbf{AB}+\mathbf{B}^2$
10. $(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B})$, $\mathbf{A}^2-\mathbf{AB}+\mathbf{BA}-\mathbf{B}^2$, $\mathbf{A}^2-\mathbf{B}^2$
11. $\mathbf{A}^2\mathbf{B}$, $\mathbf{A}^3$, $(\mathbf{AB})^2$, $\mathbf{A}^2\mathbf{B}^2$

3 Rank, Row Space, Column Space (Kreyszig page 301)

Find the rank and a basis for the row space and for the column space. *Hint.* Row-reduce the matrix and its transpose. (You may omit obvious factors from the vectors of these bases.)

1. $\begin{bmatrix} 1 & -2 \\ 0 & 0 \\ -3 & 6 \end{bmatrix}$
2. $\begin{bmatrix} 8 & 2 & 5 \\ 16 & 6 & 29 \\ 4 & 0 & -7 \end{bmatrix}$
3. $\begin{bmatrix} 0 & -2 & 1 & 3 \\ 1 & 4 & 0 & 7 \\ 5 & 5 & 5 & 5 \end{bmatrix}$
4. $\begin{bmatrix} a & b & c \\ b & a & c \end{bmatrix}$
5. $\begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & -5 \\ -4 & 5 & 0 \end{bmatrix}$
6. $\begin{bmatrix} 1 & 1 & a \\ 1 & a & 1 \\ a & 1 & 1 \end{bmatrix}$
7. $\begin{bmatrix} 8 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 2 \\ 0 & 4 & 0 \end{bmatrix}$
8. $\begin{bmatrix} 1 & -2 & 3 & -4 \\ 2 & -3 & 4 & -1 \\ 3 & -4 & 1 & -2 \\ 4 & -1 & 2 & -3 \end{bmatrix}$
9. $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 5 & 8 & -37 \\ 3 & 8 & 7 & 0 \\ 0 & -37 & 0 & 37 \end{bmatrix}$
10. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$
11. $\begin{bmatrix} 2 & 4 & 8 & 16 \\ 16 & 8 & 4 & 2 \\ 4 & 8 & 16 & 2 \\ 2 & 16 & 8 & 4 \end{bmatrix}$
12. $\begin{bmatrix} 0 & 0 & -7 & 1 \\ 0 & 0 & 5 & 0 \\ -7 & 5 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix}$

4 Evaluation of Determinants (Kreyszig page 314)

Evaluate the following determinants

5. $\begin{vmatrix} 13 & 8 \\ -2 & 7 \end{vmatrix}$
6. $\begin{vmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{vmatrix}$
7. $\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$
8. $\begin{vmatrix} 14 & 2 & 5 \\ 2 & 0 & 8 \\ 5 & 8 & -2 \end{vmatrix}$
9. $\begin{vmatrix} 70.4 & 0.3 & 0.8 \\ 0 & 0.5 & 2.6 \\ 0 & 0 & -1.9 \end{vmatrix}$
10. $\begin{vmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{vmatrix}$

5 Cramer's Rule (Kreyszig page 314)

Solve by Cramer's rule and check by Gauss-Jordan elimination.

18.
$$\begin{aligned} 2x - 5y &= 23 \\ 4x + 6y &= -2 \end{aligned}$$

19.
$$\begin{aligned} 3y + 4z &= 14.8 \\ 4x + 2y - z &= -6.3 \\ x - y + 5z &= 13.5 \end{aligned}$$

20.
$$\begin{aligned} w + 2x - 3z &= 30 \\ 4x - 5y + 2z &= 13 \\ 2w + 8x - 4y + z &= 42 \\ 3w + y - 5z &= 35 \end{aligned}$$

6 Inverse of a Matrix (Kreyszig pages 322–323)

Find the inverse by Gauss-Jordan or state that it does not exist. Check by using $\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$.

1. $\begin{bmatrix} 1.20 & 4.64 \\ 0.50 & 3.60 \end{bmatrix}$

2. $\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$

3. $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

4. $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix}$

5. $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

6. $\begin{bmatrix} 29 & -11 & 10 \\ -160 & 61 & -55 \\ 55 & -21 & 19 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & 2 \\ 2 & 4 & 11 \end{bmatrix}$

9. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10. $\begin{bmatrix} 0 & 8 & 0 \\ 0 & 0 & 4 \\ 2 & 0 & 0 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & 2 \\ 2 & 4 & 10 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 2 & -9 \\ -2 & -4 & 19 \\ 0 & -1 & 2 \end{bmatrix}$