SIIT MAS 117 SUMMER 2006 QUIZ 4 SECTION 2

Circle your answer.

Prob. 1	Prob. 2	Prob. 3	Prob. 4
1	1	1	**1**
2	2	2	2
3	**3**	3	3
4	4	**4**	4
5	5	5	5

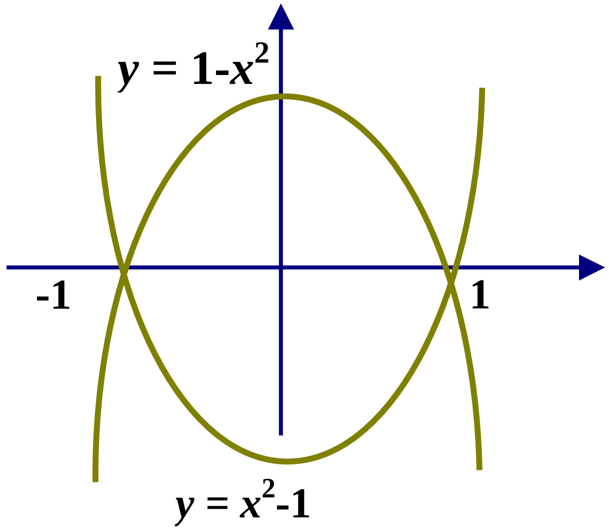
SOLUTION

Problem 1. Find the area enclosed by the curves $x^2 + y - 1 = 0$ and $x^2 - y - 1 = 0$

1. 17/4   2. 16/3   3. -2   4. 8/3   5. None of the above

The correct answer is 2.

Intersecting the curves:



$$\begin{cases} y = 1 - x^2 \\ y = x^2 - 1 \end{cases} \Rightarrow 1 - x^2 = x^2 - 1 \Rightarrow x^2 - 1 = 0 \Rightarrow (x-1)(x+1)$$

$x = -1$, $x = 1$ Hence

$$A = \int_{-1}^{1}\int_{x^2-1}^{1-x^2} 1\,dy\,dx; \quad \int_{x^2-1}^{1-x^2} 1\,dy = y\Big|_{x^2-1}^{1-x^2} = 2 - 2x^2$$

$$= \int_{-1}^{1}\left(2 - 2x^2\right)dx = \left(2x - \frac{2x^3}{3}\right)\Bigg|_{-1}^{1} = \left(2 - \frac{2}{3}\right) - \left(-2 + \frac{2}{3}\right) = \frac{8}{3}$$

Problem 2. Find the area enclosed by the rays $\theta = 0$ and $\theta = \pi/6$, and the curves $r = 1 + \sin\theta$ and $r = 1 - \sin\theta$

1. $\dfrac{\sqrt{2}-\pi}{2}$ 2. $\dfrac{3\pi}{4}$ 3. $2-\sqrt{3}$ 4. $\dfrac{\sqrt{2}}{2}-\dfrac{\pi}{3}$ 5. None of the above

The correct answer is 3.

Setting the integral $\int_0^{\pi/6}\int_{1-\sin\theta}^{1+\sin\theta} r\,dr\,d\theta$ then

$$\int_{1-siin\theta}^{1+\sin\theta} r\,dr = \frac{r^2}{2}\Big|_{1-\sin\theta}^{1+\sin\theta} = \frac{(1+\sin\theta)^2-(1-\sin\theta)^2}{2} = 2\sin\theta$$

$$\int_0^{\pi/6} 2\sin\theta\,d\theta = -2\cos\theta\Big|_0^{\pi/6} = 2-\sqrt{3}$$

Problem 3. Evaluate $\int_C x^2 y\, ds$ where C is the segment line between (0,0) and (1,4)

1. 7/3 2. -3 3. $\sqrt{20}$ 4. $\sqrt{17}$ 5. None of the above

The correct answer is 4.

$x(t) = t$, $y(t) = 4t$. $\sqrt{x'^2(t) + y'^2(t)} = \sqrt{1^2 + 4^2} = \sqrt{17}$

$$\int_C x^2 y\, ds = \int_0^1 t^2 (4t)\sqrt{17}\, dt = \sqrt{17}\int_0^1 4t^3\, dt = \sqrt{17}\, t^4 \Big|_0^1 = \sqrt{17}$$

Problem 4. Evaluate $\oint y^2\,dx + x^2 dy$, where C is the square of vertices (0,0) (1,0) (0,1), (1,1) (hint, use Green's theorem)

1. 0   2. -1   3. 1   4. 3/2   5. None of the above

The correct answer is 1.

$$\oint y^2\,dx + x^2 dy = \iint_C \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA = \int_0^1 \int_0^1 (2y - 2x)\,dy\,dx$$

$$\int_0^1 (2x - 2y)\,dy = 2xy - y^2\Big|_0^1 = 2x - 1$$

$$\int_0^1 (2x - 1)\,dx = x^2 - x\Big|_0^1 = 1 - 1 = 0$$