# SIIT MAS 117 SUMMER 2006 QUIZ 4 SECTION 1

**Circle your answer.**

| Prob. 1 | Prob. 2 | Prob. 3 | Prob. 4 |
|---|---|---|---|
| **1** | 1 | 1 | 1 |
| 2 | 2 | 2 | **2** |
| 3 | 3 | **3** | 3 |
| 4 | **4** | 4 | 4 |
| 5 | 5 | 5 | 5 |

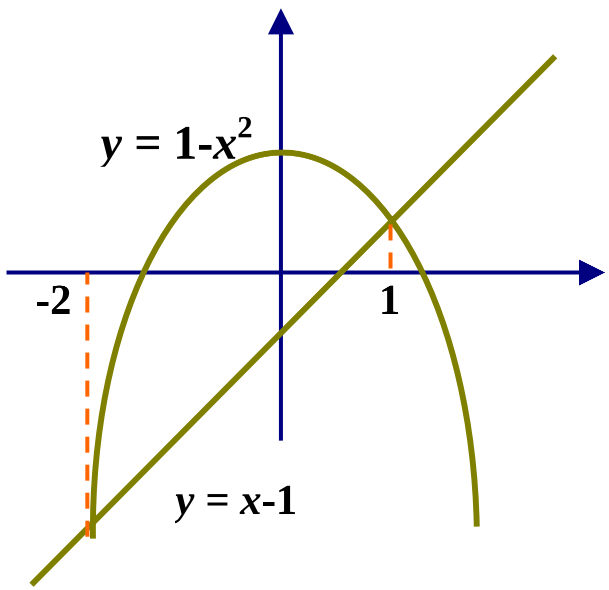
## SOLUTION

**Problem 1.** Find the area enclosed by the curves $x^2 + y - 1 = 0$ and $x - y - 1 = 0$

1. 9/2  2. 5  3. 5  4. 8/3  5. None of the above

**The correct answer is 1.**

Intersecting the curves:




$$\begin{cases} y = 1 - x^2 \\ y = x - 1 \end{cases} \Rightarrow 1 - x^2 = x - 1 \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1)$$

$x = -2$, $x = 1$ Hence

$$A = \int_{-2}^{1}\int_{x-1}^{1-x^2} 1\,dy\,dx; \quad \int_{x-1}^{1-x^2} 1\,dy = y\Big|_{x-1}^{1-x^2} = -x^2 - x + 2$$

$$= \int_{-2}^{1}\left(-x^2 - x + 2\right)dx = \left(-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right)\Bigg|_{-2}^{1} = \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(-\frac{-8}{3} - \frac{4}{2} - 4\right) = \frac{9}{2}$$

**Problem 2**. Find the area enclosed by the rays $\theta = 0$ and $\theta = \pi/4$, and the curves $r = \cos\theta$ and $r = 1 + \cos\theta$

1. $\dfrac{\sqrt{2}}{2}$  2. $\dfrac{\pi}{4}$  3. $\dfrac{\sqrt{3}}{2} - \dfrac{\pi}{3}$  4. $\dfrac{\sqrt{2}}{2} + \dfrac{\pi}{8}$  5. None of the above

**The correct answer is 4**.

Setting the integral $\int_0^{\pi/4}\int_{\cos\theta}^{1+\cos\theta} r\,dr\,d\theta$ then

$$\int_{\cos\theta}^{1+\cos\theta} r\,dr = \frac{r^2}{2}\Bigg|_{\cos\theta}^{1+\cos\theta} = \frac{(1+\cos\theta)^2 - \cos^2\theta}{2} = \cos\theta + \frac{1}{2}$$

$$\int_0^{\pi/4} (\cos\theta + 1/2)\,d\theta = \sin\theta + \theta/2\Big|_0^{\pi/4} = \frac{\sqrt{2}}{2} + \frac{\pi}{8}$$

**Problem 3**. Evaluate $\int_C xy\, ds$ where $C$ is the segment line between (0,0) and (1,3)

1. 7/2  2. 5  3. $\sqrt{10}$  4. $\sqrt{18}$  5. None of the above

**The correct answer is 3**.

$x(t) = t,\ \ y(t) = 3t.\ \ \sqrt{x'^2(t) + y'^2(t)} = \sqrt{1^2 + 3^2} = \sqrt{10}$

$$\int_C xy\, ds = \int_0^1 y(3t)\sqrt{10}\, dt = \sqrt{10}\int_0^1 3t^2\, dt = \sqrt{10}\, t^3\Big|_0^1 = \sqrt{10}$$

**Problem 4**. Integrate by converting to polar coordinates $\iint_R 2e^{x^2+y^2}\,dA$ ,
where $R$ is the region enclosed by the circle $x^2+y^2 = 1$

1. $\pi - 2e$ **2.** $2\pi(e - 1)$ 3. $1+ 2\pi$ 4. $\pi+e$ 5. None of the above

**The correct answer is 2**.

$$\int_0^{2\pi}\int_0^1 2r\,e^{r^2}\,dr$$

$$\int_0^1 2re^{r^2}dr = e^{r^2}\Big|_0^1 = e-1 \quad \int_0^{2\pi}(e-1)\,d\theta = 2\pi(e-1)$$