SIIT MAS 117 SUMMER 2006 QUIZ 2 SECTION 2

SOLUTION

Prob. 1	Prob. 2	Prob. 3	Prob. 4
1	**1**	**1**	1
2	2	2	2
3	3	3	**3**
4	4	4	4
5	5	5	5

Problem 1. Calculate $\lim\limits_{(x,y)\to(1,-1)} \dfrac{x^3 - xy^2}{x+y}$

1. 1/2 [✓ 2.] 2 3. 0 4. ∞ 5. None of the above

Multiplying by $x - y$ both numerator and denominator and factoring the numerator,

$$\lim_{(x,y)\to(1,-1)} \frac{x(x^2-y^2)(x-y)}{(x+y)(x-y)} = \lim_{(x,y)\to(1,-1)} \frac{x(x^2-y^2)(x-y)}{x^2-y^2} = \lim_{(x,y)\to(1,-1)} x(x-y) = 2$$

Problem 2. Let $f(x, y) = y\sin xy$. Evaluate $\frac{\partial^2 f}{\partial x \partial y}(0,1)$

1. [✓ 2]   2. -1 3. $\pi/3$ 4. $\pi/4$ 5. None of the above

$f_x = y^2 \cos xy$. Hence, $f_{xy} = 2y \cos xy - xy^2 \sin xy = 2$

Problem 3. Given the function $f(\text{x, y}) = x^2 + xy + y^2 - 3x$. It has

[✓ 1.] Relative minimum at (2, -1)
2. Relative maximum at (1, 1)
3. Saddle point at (2, -2)
4. No relative maximum, no relative minimum and no saddle points.
5. None of the above

$f_x = 2x + y - 3 = 0$ The solution of the system is the critical point (2, -1)
$f_y = x + 2y = 0$

$f_{xx} = 2, \quad f_{xy} = 1, \quad f_{yy} = 2$

Therefore, $D = f_{xx} f_{yy} - f^2_{xy} = (2)(2) - 1^2 > 0$ and $f_{xx} > 0$. Therefore it is relative minimum at (2, -1)

Problem 4. Calculate the directional derivative of $f(x,y) = x^2 + xy^3 - 17$ at the point $P(1, -1)$ in the direction of $\mathbf{u} = 3\mathbf{i} - 3\mathbf{j}$.

1. 0   2. 5/3   [✓ 3. $-\sqrt{2}$]   4. –11/2   5. None of the above

$f_x = 2x + y^3$ $f_x(1, -1) = 1$

$f_y = 3\,xy^2$ $f_y(1, -1) = 3$ $\|\mathbf{u}\| = 3\sqrt{2}$

Then $D_{\mathbf{u}} = (1)\frac{1}{\sqrt{2}} + (3)\left(-\frac{1}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} = -\sqrt{2}$