MAS210 First Semester 2007 Dr. Ruben

Quiz 1 SOLUTION

Prob. 1	Prob. 2	Prob. 3	Prob. 4
1	**1**	1	1
2	2	2	2
3	3	**3**	3
4	4	4	**4**
5	5	5	5

Problem 1 Given the reduced-echelon matrix below, the solution set is

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 4 & 0 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 3 & 0 & -2 \\ 0 & 0 & 0 & 1 & 0 & 3 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(1) $x_1 = 1 - 4t - 2s$, $x_2 = -2 - t - 3s$, $x_3 = t$, $x_4 = 4 - 3s$; $x_5 = -7s$; $x_6 = s$; $x_7 = 3$
(2) $x_1 = 3 - 3t$, $x_2 = 1 + t - 3s$, $x_3 = t$, $x_4 = 1 + s$, $x_5 = s$, $x_6 = 2$; $x_7 = 3$
(3) $x_1 = 1$, $x_2 = -2$ $x_3 = 4$, $x_4 = 0$, $x_5 = 3 + s$, $x_6 = 0$; $x_7 = 0$
(4) The system is incompatible
(5) The matrix is not in reduced-echelon format

Problem 2. Which one is false? Mark the statement that is false.

(1) A homogeneous system has always infinitely many solutions
(2) The row-echelon matrix of a square system with unique solution is the identity
(3) If the coefficient matrix of a square system is nonsingular, then the system admits solution unique
(4) A linear system cannot have exactly two solutions
(5) If a square matrix is nonsingular, then it has an inverse

The answer is (1), because it is the only one false statement (all the others are true)

Problem 3. Solve the following system

$$x + y + z = 1$$
$$x - y - z = 1$$
$$x + y - z = 1$$

(1) $x = t, \ \ y = s, \ \ z = z;$ (2) $x = 1, \ y = 1,$ (3) incompatible
(4) $x = 1, \ y = 0; \ z = 0$ (5) y = 3, z = -2,

$$\begin{pmatrix} 1 & 1 & 1 & | & 1 \\ 1 & -1 & -1 & | & 1 \\ 1 & 1 & -1 & | & 1 \end{pmatrix} \Rightarrow \begin{matrix} r_1 \\ r_1 - r_2 \\ r_1 - r_3 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 2 & 2 & | & 0 \\ 0 & 0 & 2 & | & 0 \end{pmatrix} \Rightarrow \begin{matrix} r_1 \\ r_2/2 \\ r_3/2 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 1 & | & 0 \end{pmatrix} \Rightarrow \begin{matrix} r_1 - r_2 \\ r_2 - r_3 \\ r_3/2 \end{matrix} \begin{pmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \end{pmatrix}$$

The answer is 4

Problem 4. The reduced row-echelon matrix of $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is

(1) $\begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$; (2) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$; (3) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$; (4) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; (5) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$

$$\begin{matrix} r_1 \\ r_1 - r_2 \\ r_3 \end{matrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 0 \\ 0 & 1 & 3 \end{pmatrix} \Rightarrow \begin{matrix} r_1 \\ r_2 \\ -r_2 + 3r_3 \end{matrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 9 \end{pmatrix} \Rightarrow$$

$$\begin{matrix} 3r_1 - r_2 \\ r_2 \\ r_3/9 \end{matrix} \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{matrix} r_1 - 3r_3 \\ r_2 - r_3 \\ r_3 \end{matrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{matrix} r_1/3 \\ r_2/3 \\ r_3 \end{matrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$