SIIT MAS210 First Semester 2006

Quiz 1 Solution

Prob. 1	Prob. 2	Prob. 3	Prob. 4
1	1	1	1
2	**2**	**2**	2
3	3	3	3
4	4	4	**4**
5	5	5	5

Problem 1 Given the reduced-echelon matrix below, the solution set is

$$\left(\begin{array}{cccccc|c} 1 & 0 & 3 & 0 & 0 & 0 & 3 \\ 0 & 1 & -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

(1) $x_1 = 3,\ x_2 = 1,\ x_3 = 1,\ x_4 = 0;\ x_5 = 2$
(2) $x_1 = 3 - 3t,\ x_2 = 1 + t - 3s,\ x_3 = t,\ x_4 = 1 + s,\ x_5 = s,\ x_6 = 2$
(3) $x_1 = 3t,\ x_2 = t,\ x_3 = s,\ x_4 = 0 + s,\ x_5 = t + s,\ x_6 = 2$
(4) The system is incompatible
(5) The matrix is not in reduced-echelon format

The leading term of the fourth row is not 1.

Problem 2. Which one is false? Mark the statement that is false.

(1) A homogeneous system has always solution
(2) A system with infinitely many solutions is incompatible
(3) If the coefficient matrix of a square system is nonsingular, then the system admits solution unique
(4) A linear system cannot have exactly two solutions
(5) If a square matrix is nonsingular, then it has an inverse

Problem 3. Solve the following system

$$\begin{aligned} x + y + z &= 3 \\ x - y - z &= -2 \\ x + y - z &= 1 \end{aligned}$$

(1) $x = t, \quad y = s, \quad z = z$

(2) $x = 1, \; y = 1,$

(3) incompatible

(4) $x = 1, \; z = -3$

(5) $y = 3, \; z = -2,$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 \end{array}\right) \Rightarrow \begin{array}{c} r_1 \\ r_1 - r_2 \\ r_1 - r_3 \end{array}\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 2 & 2 \end{array}\right) \Rightarrow \begin{array}{c} r_1 \\ r/2 \\ r_1 - r_3 \end{array}\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow \begin{array}{c} r_1 - r_2 \\ r_2 - r_1 \\ r_3 \end{array}\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

Solution: $x = 1, \quad y = 1, \quad z = 1$

Problem 4. **The reduced-echelon matrix of**

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\begin{array}{c} r_1 \\ r_1 - r_2 \\ r_3 \end{array}\begin{pmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix} \Rightarrow \begin{array}{c} r_1 \\ r_2 \\ -r_2 + 3r_3 \end{array}\begin{pmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{pmatrix} \Rightarrow$$

$$\begin{array}{c} 3r_1 - r_2 \\ r_2 \\ r_3/8 \end{array}\begin{pmatrix} 3 & 0 & 5 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{array}{c} r_1 - 5r_3 \\ r_2 - r_3 \\ r_3 \end{array}\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{array}{c} r_1/3 \\ r_2/3 \\ r_3 \end{array}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(1) $\begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$; (2) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$; (3) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$; **(4)** $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; (5) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$