SIIT – MAS210-8 Midterm Exam

1. Solve the following systems of equations. If it is incompatible, write incompatible. If it has one single solution, find it. If it has infinitely many solutions, find the solution set.

(a)
$$\begin{aligned} x + y + z + w &= 1 \\ y + z - w &= 2 \\ x + y - 2w &= 1 \end{aligned}$$

(b)
$$\begin{aligned} x - y &= 1 \\ x + y &= 3 \\ x - 3y &= 0 \end{aligned}$$

2. Find the rank of the coefficient matrices of the systems in Problem 1.

(a) rank(A) = (b) rank A =

3. (a) Find the inverse of $A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ if it exists. If it does not exist, write *singular*

(b) Solve the homogeneous system A**x** = **0**. (that means, "find all solutions")

4. Are the vectors $\mathbf{v}_1 = (3, 11, 7)$, $\mathbf{v}_2 = (5, -13, 7)$, $\mathbf{v}_3 = (8, 6, 19)$, $\mathbf{v}_4 = (9, 6, 17)$ linearly independent? (yes/no, explain)

5. Do the vectors $\mathbf{v}_1 = (3, 0, 2, 1)$, $\mathbf{v}_2 = (15, -12, -7, 0)$, $\mathbf{v}_3 = (7, 5, 31, -11)$ form a basis for R^4? (yes/no, explain)

6. Given the vectors $\mathbf{v}_1 = (1, -1, 0)$, $\mathbf{v}_2 = (-1, -1, 0)$, $\mathbf{v}_3 = (1, 1, 1)$
   (a) Are they linearly independent? (yes/no, explain)
   (b) Do they span R^3? (yes/no, explain)
   (c) Do they form a basis on R^3? (yes/no, explain)

7. Let V the subset of R^3 formed by triplets of the form $(x, y, x-y)$, whose third component is difference of the first minus the second.
   (a) Is V a subspace of R^3? (yes/no explain)
   (b) Prove that the vectors $\mathbf{u}_1 = (1, 1, 0)$, $\mathbf{u}_2 = (1, 2, -1)$, $\mathbf{u}_3 = (0, 1, -1)$ span V
   (c) Prove that the vectors $\mathbf{v}_1 = (1, 0, 1)$, $\mathbf{v}_2 = (0, 1, -1)$ form a basis for V
   (d) What is the dimension of V?

8. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 3 & 1 \end{bmatrix}$ and S(A) the vector space of solutions of the homogeneous system A**x** = **0**.
   (a) Find one vector of S(A)
   (b) What is the dimension of S(A)?
   (c) What is the rank of A?
   (d) Calculate det (A)
   (e) Find A^{-1}. If it does not exist write *singular*.

9. Let V be the subset of R^3 of triplets such that at least one component is zero. Is V a subspace of R^3? (yes/no explain)