MAS210 Dr. Ruben

HOMEWORK IN PREPARATION FOR THE FINAL EXAM

1. Given $z_1 = 2 + 3i$ and $z_2 = 1 + i$ and $z_3 = -3 + 2i$, calculate
(a) $z_3(z_1 + 2z_2)$ (b) $z_1\,\bar{z}_2$ (c) $1/z_1$ (d) z_1/z_2 (e) $\mathrm{Re}\left(z_1/\bar{z}_1\right)$ (f) $(z_1)^{16}$

2. Evaluate $\sqrt[4]{z}$ and $\sqrt[5]{z}$ for
(a) $z = 2 - 2i$ (b) $z = 3\sqrt{3} + 3i$ (c) $z = 4\sqrt{4} + 4i$

3. Given $z = \sqrt{3} + i$, find
(a) the polar form of z
(b) write z^3 in polar form and in regular form
(c) $\sqrt[3]{z}$

4. Let $z = -\sqrt{2} + i\sqrt{2}$. Calculate z^5.

5. Differentiate
(a) $f(z) = \exp(z^2+1)$, (b) $f(z) = \ln(z^3 - 2z)$

6. Use Cauchy-Riemann equations to figure out whether the given functions are analytic
(a) $f(z) = z^2 - 2z + 1$ (b) $f(z) = \bar{z}/z$ (c)) $f(z) = \mathrm{Im}(z)$ (d)) $f(z) = \mathrm{Re}(z) + 2\mathrm{Im}(z)$
(e) $f(z) = \mathrm{Re}(z) + \mathrm{Im}(z)$

7. Calculate exactly $\ln z$ where
(a) $z = (3 - 3i)$ (b) $\sqrt{3} - i$ (c) $z = 2e^{i\pi/3}$ (d) $z = 2\left(\cos 5\pi/7 + i\sin 5\pi/7\right)$

8. Find the exact values of $\sin z$ and $\cos z$, for
(a) $z = 0$ (b) $z = \pi$ (c) $z = \pi/2$ (d) $z = \pi i$ (e) $z = \pi i/4$ (f) $z = \pi i/3$ (g) $z = \pi i/2$

In problems 9 - 19 solve the given integral.

9. $\int_C e^{-3z}\,dz$ C: $z(t) = 2 - t^2 + i(t + 1)$ $1 \le t \le 2$

10. $\int_C \sin z\,dz$ C: $z(t) = 1 + t + it$ $0 \le t \le \pi$

11. $\int_C (\mathrm{Re}(z+3))\,dz$ C: $z(t) = t^2 + i(t+1)$ $0 \le t \le 1$

12. $\int_C \mathrm{Re}(z^2)\,dz$ C: $z(t) = 2t + i$ $0 \le t \le 1$

13. $\oint_C \left(\tan z - 2z^3 e^{z^2}\right)dz$ $z(t) = 1 - 3i + 2e^{it}$ $0 \le t \le 2\pi$

14. $\oint_c \frac{1}{z-i}\,dz \qquad C\text{: } z(t) = i + e^{it} \quad 0 \le t \le 2\pi$

15. $\oint_c \frac{1}{(z-i)^2}\,dz \qquad C\text{: } z(t) = i + e^{it} \quad 0 \le t \le 2\pi$

16. $\int_C \frac{1}{z-i}\,dz \qquad C\text{: } z(t) = i + e^{it} \quad 0 \le t \le \pi$

17. $\int_C \frac{1}{z-i}$ where C is the segment line between 0 and $1 + i$

18. $\oint_c \frac{1}{2i-z}\,dz \qquad C\text{: : } z(t) = e^{it} \quad 0 \le t \le 2\pi$

19. $\oint_C \frac{2}{z-2+i}\,dz$

(a) C is the rectangle of vertices 0, 3, $3+3i$, $3i$ oriented counterclockwise
(b) C is the rectangle of vertices 0, 3, $3-3i$, $-3i$ oriented counterclockwise

20. $\oint_C \frac{e^z}{(z-i\pi/2)}\,dz$

(a) C: $z(t) = i\pi/2 + 3e^{it} \quad 0 \le t \le 2\pi$
(b) C: $z(t) = -i\pi/2 + e^{it} \quad 0 \le t \le 2\pi$

21. $\oint_C \frac{e^z}{z^2-2z+2}\,dz \quad C: z(t) = 1+i+e^{it} \quad 0 \le t \le 2\pi$

22. $\oint_C \frac{\sin z}{(z-i)^2}\,dz$

23. $\oint_C \frac{\sin z}{(z-i)^3}\,dz$ (a) $z(t) = i + 2e^{it} \; 0 \le t \le 2\pi$ (b) $z(t) = i + 2e^{it} \; 0 \le t \le \pi$

24. $\oint_C \frac{1-3z}{(z-i)^4}\,dz \quad C: z(t) = 1 + e^{it} \quad 0 \le t \le 2\pi$