SIIT, MAS 210, MATHEMATICS III – FINAL EXAM-
Section 8 **First Semester 2005** Ruben Mera

1. Given the complex numbers $z_1 = 1 - i$, $z_2 = 2 + i$, $z_3 = 1 - 3i$, $z_4 = 2[\cos(\pi/4) + i\sin(\pi/4)]$, find the result of the following operations

(a) $z_1 \bar{z}_3$ (b) $\dfrac{z_1}{z_2}$ (c) $|z_3 - z_1|$ (d) z_4^3 (e) $\sqrt[3]{z_4}$

2. Calculate the derivative of $f(z) = \dfrac{\sin z}{z}$

3.. Use the Cauchy-Riemann equations to show whether the given functions are analytic or not

(a) $f(z) = e^{2z}$ (b) $f(z) = \mathrm{Im}\,(z)$

4. Given the function $f(z) = \dfrac{\cos z}{z^2 + z + 1}$,

(a) find the points where it is not analytic

(b) for the curve $C: z(t) = -1 + \frac{1}{2}e^{it}$, $0 \le t \le 2\pi$, find a simply connected domain D containing the curve and such that *f* is analytic in *D*. If it is not possible, explain why not. (you can show you answer in a graph if you wish or analytically)

5. Calculate exactly $\ln\left(\dfrac{\sqrt{2}}{2} + i\dfrac{\sqrt{2}}{2}\right)$. Give your answer in the form $x + iy$

6. Calculate the following integrals

(a) $\int_C e^{2z}\,dz \quad C: z(t) = t^3 + i\sqrt{t} \quad 0 \le t \le 1$ (b) $\oint_C \sin z\, e^{5z}\,dz \quad C: z(t) = 2 + 5i + 3e^{it} \quad 0 \le t \le 2\pi$

(c) $\oint_C \dfrac{dz}{z - i} \quad C: z(t) = i + e^{it} \quad 0 \le t \le 2\pi$ (d) $\oint_C \dfrac{dz}{(z-i)^2} \quad C: z(t) = i + e^{it} \quad 0 \le t \le 2\pi$

(e) $\oint_C \dfrac{\sin z}{z^2 + 4}\,dz \quad C: z(t) = 3e^{it} \quad 0 \le t \le 2\pi$ (f) $\oint_C \dfrac{\sin z}{z^2 + 4}\,dz \quad C: z(t) = 2i + e^{it} \quad 0 \le t \le 2\pi$

(g) $\oint_C \dfrac{e^{-z^2}}{z + 1}\,dz$ where C is the triangle of vertices *i*, -*i*, -2. (h) $\int_C \mathrm{Im}\, z\; dz \quad C: z(t) = t + it^2 \quad 0 \le t \le 1$