SIIT MAS117 - SUMMER 2007 - QUIZ 1 - Section 2
SOLUTION

problem 1	a	b	c	(d)	e
problem 2	(a)	b	c	d	e
problem 3	(a)	b	c	d	e
problem 4	(a)	b	c	d	e

1. Evaluate $\lim\limits_{x\to((2,2)} \dfrac{x^2 - xy - x + y}{x^2 - xy}$

(a) 4/3 (b) 0 (c) Does not exist (d) 1/2 (e) 4

The answer is (d)

$$\lim_{x\to((2,2)} \frac{x^2 - xy - x + y}{x^2 - xy} = \lim_{x\to((2,2)} \frac{x(x-y)-(x-y)}{x(x-y)} = \lim_{x\to((2,2)} \frac{(x-1)(x-y)}{x(x-y)} =$$

$$\lim_{x\to((2,2)} \frac{x-1}{x} = \frac{1}{2}$$

2. The limit $\lim\limits_{x\to((0,0)} \dfrac{x-y}{x+y}$ along the path $y = 2x$ is

(a) $-\frac{1}{3}$ (b) $\frac{0}{0}$ (c) 2 (d) does not exist (e) ∞

The answer is (a)

$$\lim_{x\to((0,0)} \frac{x-y}{x+y} = \lim_{x\to((0,0)} \frac{x-2x}{x+2x} = \lim_{x\to((0,0)} \frac{-x}{3x} = = -\frac{1}{3}$$

3. The partial derivatives of $z = x^2y - \cos(x^3 + y)$ at $(x_0, y_0) = (-1, 1)$ are

(a) $\frac{\partial z}{\partial x}(-1,1) = -2, \ \frac{\partial z}{\partial y}(-1,1) = 1$ (b) $\frac{\partial z}{\partial x}(-1,1) = 3 \ \frac{\partial z}{\partial y}(-1,1) = 0$;

(c) $\frac{\partial z}{\partial x}(-1,1) = \frac{\sqrt{3}}{2}, \ \frac{\partial z}{\partial y}(-1,1) = \frac{1}{\sqrt{2}}$; (d) $\frac{\partial z}{\partial x}(-1,1) = 0, \ \frac{\partial z}{\partial y}(-1,1) = 2$

(e) $\frac{\partial z}{\partial x}(-1,1) = \frac{\pi}{3}, \ \frac{\partial z}{\partial y}(-1,1) = \frac{\pi}{2}$

The answer is (a)

$$\frac{\partial z}{\partial x} = 2xy + 3x^2 \sin(x^3 + y)$$ therefore

$$\frac{\partial z}{\partial x}(-1,1) = 2(-1)(1) + 3(1^2)\sin(1^3 - 1) = -2 + 3\sin 0 = -2$$

$$\frac{\partial z}{\partial y} = x^2 + \sin(x^3 + y)$$ therefore

$$\frac{\partial z}{\partial y}(1,-1) = (1^2) + \sin(1^3 - 1) = 1 + \sin 0 = 1$$

4. The directional derivative of $f(x, y) = x^2y - 2x$ in the direction of $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$ at P: (1, 1) is

(a) 3 (b) 0 (c) 1 (d) ∞ (d) -1

(a) 3 (b) 0 (c) 1 (d) ∞ (d) -1

The answer is (a)

$f_x = 2xy - 2$. $f_x(1, 1) = 0$

$f_y = x^2$. $f_y(1, -1) = 1$

Therefore, $D_{\mathbf{u}} = f_x((1, -1)(u_1) + f_y(1, -1)(u_2) = (0)(2) + (1)(3) = 3$