

SIIT MASJIT - SUMMER 2007 - QUIZ 2 - Section I
SOLUTION

problem 1	a	b	c	d	e
problem 2	a	b	c	d	e
problem 3	a	b	c	d	e
problem 4	a	b	c	d	e

1. Evaluate $\lim_{x \rightarrow ((2,1))} \frac{x^2 - 4y^2}{xy - 2y^2}$

- (a) 4/3 (b) 0 (c) Does not exist (d) 1/2 (e) 4

The answer is (e)

$$\lim_{x \rightarrow ((2,1))} \frac{x^2 - 4y^2}{xy - 2y^2} = \lim_{x \rightarrow ((2,1))} \frac{(x-2y)(x+2y)}{y(x-2y)} = \lim_{x \rightarrow ((2,1))} \frac{x+2y}{y} = 4$$

2. The limit $\lim_{x \rightarrow ((0,0))} \frac{x-y}{2x+y}$ along the path $y = -x$ is

- (a) 0 (b) $\frac{0}{0}$ (c) 2 (d) does not exist (e) ∞

The answer is (c)

$$\lim_{x \rightarrow ((0,0))} \frac{x-y}{2x+y} = \lim_{x \rightarrow ((0,0))} \frac{x+x}{2x-x} = \lim_{x \rightarrow ((0,0))} \frac{2x}{x} = 2$$

3. The partial derivatives of $z = x^2y + \sin(x^3 + y)$ at $(x_0, y_0) = (1, -1)$ are

(a) $\frac{\partial z}{\partial x}(1, -1) = -2, \frac{\partial z}{\partial y}(1, -1) = 1$ (b) $\frac{\partial z}{\partial x}(1, -1) = 3, \frac{\partial z}{\partial y}(1, -1) = 0;$

(c) $\frac{\partial z}{\partial x}(1, -1) = \frac{\sqrt{3}}{2}, \frac{\partial z}{\partial y}(1, -1) = \frac{1}{\sqrt{2}};$ (d) $\frac{\partial z}{\partial x}(1, -1) = 1, \frac{\partial z}{\partial y}(1, -1) = 2$

(e) $\frac{\partial z}{\partial x}(1, -1) = \frac{\pi}{3}, \frac{\partial z}{\partial y}(1, -1) = \frac{\pi}{2}$

The answer is (d)

$$\frac{\partial z}{\partial x} = 2xy + 3x^2 \cos(x^3 + y) \text{ therefore}$$

$$\frac{\partial z}{\partial x}(1, -1) = 2(1)(-1) + 3(1^2) \cos(1^3 - 1) = -2 + 3 \cos 0 = 1$$

$$\frac{\partial z}{\partial y} = x^2 + \cos(x^3 + y) \text{ therefore}$$

$$\frac{\partial z}{\partial y}(1, -1) = (1^2) + \cos(1^3 - 1) = 1 + \cos 0 = 2$$

4. The directional derivative of $f(x, y) = x^2y - 2x$ in the direction of $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$ at $P: (1, -1)$ is

(a) -11 (b) 0 (c) 1 (d) ∞ (e) -1

The answer is (a)

$$f_x = 2xy - 2. \quad f_x(1, -1) = -4.$$

$$f_y = x^2. \quad f_y(1, -1) = 1$$

$$\text{Therefore, } D_{\mathbf{u}} = f_x((1, -1)(u_1) + f_y(1, -1)(u_2) = (-4)(2) + (1)(-3) = -11$$