

SIIT MAS117 - SUMMER 2007 - QUIZ 1 - Section 2

problem 1	1	2	3	4	5
problem 2	1	2	3	4	5
problem 3	1	2	3	4	5
problem 4	1	2	3	4	5

1. The slope of the tangent line to the circle $r = 2\sin\theta$ at $\theta = \pi/3$ is

- 1) -1; 2) $\frac{\sqrt{3}+1}{2}$; 3) $\frac{1-\sqrt{2}}{2}$; 4) 0; 5) $-\sqrt{3}$

The answer is 5)

Applying the formula for the derivative of a polar function,

$$\begin{aligned} \frac{dr}{d\theta} &= \frac{r \cos\theta + \sin\theta r'}{-r \sin\theta + \cos\theta r'} = \frac{2\sin\theta \cos\theta + 2\sin\theta \cos\theta}{-2\sin^2\theta + 2\cos^2\theta} = \frac{4\sin\theta \cos\theta}{-2\sin^2\theta + 2\cos^2\theta} \\ &= \frac{2\sin\theta \cos\theta}{-\sin^2\theta + \cos^2\theta} = \frac{2 \frac{\sqrt{3}}{2} \frac{1}{2}}{-\frac{3}{4} + \frac{1}{4}} = \frac{\sqrt{3}}{-\frac{1}{2}} = -\sqrt{3} \end{aligned}$$

Evaluated at $\theta = \pi/3$, we obtain $-\sqrt{3}$

2. The angle between the vectors $\mathbf{u} = 2\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - 2\mathbf{k}$ is

- 1) $\pi/6$, 2) $\pi/3$; 3) 0; 4) $\pi/2$; 5) $\pi/4$

The answer is 2)

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{(2 \times 2 + 2 \times 0 + 0 \times (-2))}{\sqrt{8} \sqrt{8}} = \frac{4}{8} = \frac{1}{2}$$

3) The graph of the polar function $r = 2\sin\theta$ is

- 1) A four-petal rose curve; 2) A circle with center at the origin and radius 1;
3) A circle with center at (0, 1) and radius 1; 4) A spiral; 5) None of the above

The answer is 3)

4) The direction angles of the vector $\mathbf{u} = \mathbf{i} + \mathbf{k}$ are:

1) $\alpha = \pi/3, \beta = \pi/4, \gamma = 4\pi/3$; 2) $\alpha = \pi/4, \beta = \pi/2, \gamma = \pi/4$; 3) $\alpha = \pi, \beta = \pi/2, \gamma = -\pi/2$;

4) $\alpha = \pi/3, \beta = \pi/6, \gamma = 3\pi/2$; 5) $\alpha = \pi/6, \beta = \pi/6, \gamma = 0$;

The answer is 2)

Applying the direction angles formula

$$\cos \alpha = \frac{u_1}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4} \quad \cos \beta = \frac{u_2}{\|\mathbf{u}\|} = \frac{0}{\sqrt{2}} \Rightarrow \beta = \frac{\pi}{2} \quad \cos \gamma = \frac{u_3}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4}$$