

## MAS210 FINAL EXAM 1<sup>ST</sup> SEMESTER 2006 –Dr. Ruben

1. Exam consists of 11 problems, and all are compulsory. Exam total points is 100.
2. Only calculator Casio FX-991MS is allowed. Textbook and notes are not allowed.
3. Problems 1 – 5 are multiple-choice, mark the correct answer, no work needs to be shown. Correct answers in problems 1-5 value 8 points; wrong answers discount 2 points.
4. Problems 6-11 value 10 points each and require the full procedure to be shown. All necessary steps to solve these problems must be clearly written. No points discounted on problems 6-11.
- 5 Please, write your final answer in the provided box

Answers to problems 1-5					
Circle the correct answer					
Problem 1	a	b	c	d	e
Problem 2	a	b	c	d	e
Problem 3	a	b	c	d	e
Problem 4	a	b	c	d	e
Problem 5	a	b	c	d	e

**Problem 1.** If  $f(x)$  is periodic of period  $\pi$ , then the period  $p$  of  $g(x) = f(\pi x/2)$  is

- (a)  $p = 1/2$                       (b)  $p = \pi$                       (c)  $p = 2$   
 (d)  $p = 2/\pi$                       (e)  $g(x)$  is not periodic

**Problem 2.** Let  $f(x)$  be an odd function,  $g(x)$  be even, and  $h(x) = f(x)g(x)$ . Then the Fourier series of  $h(x)$  can only be one of the following

- (a)  $\frac{2}{\pi} \left[ \frac{\cos \pi x}{1} - \frac{\cos 2\pi x}{2} + \frac{\cos 3\pi x}{3} - \dots \right]$                       (b)  $\frac{\sin \pi x}{1} - \frac{\sin 2\pi x}{2} + \frac{\sin 3\pi x}{3} - \dots$   
 (c)  $\frac{\cos \pi}{1^2} - \frac{\sin \pi}{1^2} - \frac{\cos 2\pi}{2^2} + \frac{\sin 2\pi}{2^2} + \frac{\cos 3\pi}{3^2} - \frac{\sin 3\pi}{3^2} - \dots$                       (d)  $1 + \cos x - \sin 2x + \sin 3x \dots$   
 (e) None of the above

**Problem 3.** Using floating point representation with 4 significant digits, and using the *round-off* method, the numbers  $x_1 = 0.00345600$ ,  $x_2 = 173.0001$ ,  $x_3 = 151$  become

- (a)  $x_1 = 0.31E04$ ,  $x_2 = 17.3E01$ ,  $x_3 = .15E02$   
 (b)  $x_1 = 3456E-3$ ,  $x_2 = .1735600$   $x_3 = 151E01$   
 (c)  $x_1 = .3456E2$ ,  $x_2 = .1730E05$ ,  $x_3 = 1510E-3$   
 (d)  $x_1 = 3456$ ,  $x_2 = 173.0$   $x_3 = 151.0$   
 (e)  $x_1 = .3456E-2$   $x_2 = .1730E03$   $x_3 = .1510E03$

**Problem 4.** Five numbers in floating point system with 5 significant digits,  $x_1 = .13571E02$ ,  $x_2 = .90900E-3$ ,  $x_3 = .11111E04$ ,  $x_4 = .30199E02$ ,  $x_5 = .00171E-3$ . Let  $S = x_1 + x_2 + x_3 + x_4 + x_5$ . The error bound  $\beta$  for the sum  $S$  is equal to

- (a)  $\beta = 10^{-3} + 10^{-8} + (.5) \times 10^{-1}$                       (b)  $\beta = 3 \times (.5) \times 10^{-3}$                       (c)  $\beta = 0.005$   
 (d)  $\beta = 10^{-2} + 10^{-1} - 10^{-5}$                       (e)  $\beta = 0.5 \times 10^{-1}$

**Problem 5.** Using mathematical software that provides answers with 5 significant digits, the unique root of the polynomial  $p(x) = 7x^3 - 4x^2 - 4x - 11$  is found to be  $.15714E01$ . If the software rounds-off, what is the error bound  $\beta$ ? (read carefully the text, the answer is very simple).

- (a)  $\beta = 0.000005$                       (b)  $\beta = 1/2$                       (c)  $\beta = .5714$                       (d)  $\beta = 0.00005$                       (e)  $\beta = .0005$

**Problem 6.** Find the Fourier series of  $f(x) = \begin{cases} -x & \text{if } -1 \leq x < 0 \\ x & \text{if } 0 \leq x < 1 \end{cases}$ ,  $f(x+2) = f(x)$

answer

**Problem 7.** Let  $f(x) = \sin x$

(a) Write Taylor's expansion of  $f(x)$  up to the fourth derivative around the point  $a = 0$  (MacLauring expansion).

(b) Use part (a) to approximate the value of  $\sin(1/2)$ . Provide your answer with 5 significant digits

answer

(a)

  
  
  
  

(b)

**Problem 8.** Let  $P(x) = x^3 + 2x - 1$ .

(a) Find an interval where  $P(x)$  has one root (hint: calculate  $P(-1)$ ,  $P(0.2)$ ,  $P(0.9)$  and  $P(1.2)$ ).

(b) Use the result in (a) to choose a suitable initial point  $x_0$  and find a valid algorithm to approximate the root of  $P(x) = 0$ ; perform 4 iterations and write the answer with 5 significant digits

answer:

(a)

  
  
  
  

(b)

**Problem 9.** Use Chauchy-Raphson method to approximate a root of the equation.  $f(x) = e^x - 2 = 0$  (hint: evaluate  $f(-1)$ ,  $f(0)$  and  $f(1)$  to chose a good initial point  $x_0$  ) Provide your answer with 5 significant digits

answer:

**Problem 10.** Use the Cauchy-Raphson method to evaluate  $\ln 1.3$ . Use as starting point  $x_0 = 0.2$  and perform 3 iterations.

answer:

**Problem 11.** Find the polynomial  $p(x)$  that interpolates the function  $f(x) = \sin x$  at the points  $x_0 = 0$ ,  $x_1 = \pi/6$ ,  $x_2 = \pi/3$ ,  $x_3 = \pi/2$ . (no need to simplify)

answer: