SIIT GTS211 Section 1 First Semester 2006 Dr. Ruben

Quiz 1

SOLUTION

Problem 1. The general solution of $3y' + 6 - 9y = 0$ is

Write $y' = 3y - 2$. It is ***linear with constant coefficients*** with $p = 3$ and $q = -2$. The general solution is

$y = -q/p + C e^{px}$, that is, $y' = 2/3 + Ce^{3x}$, so the answer is (1).

Problem 2. The general solution of of $(2x - 4y)y' = -6x - 2y$ is

This equation is ***exact***. Indeed, write $(6x + 2y)\, dx + (2x - 4y)\, dy = 0$. Then

$$\frac{\partial M}{\partial y} = 2 = \frac{\partial N}{\partial x}$$

Integrating M, $F(x,y) = \int (6x + 2y)dx = 3x^2 + 2xy + C(y)$.

Differentiating in y, $F_y(x,y) = N(x,y)$, that is

$$2x + C'(y) = 2x - 4y \Rightarrow C'(y) = -4y \Rightarrow C(y) = -2y^2 + C.$$

The general solution, therefore, is

$$\mathbf{3x^2 + 2xy - 2y^2 = C}$$

and the answer is (2).

Problem 3. The general solution of $\frac{y'}{x} = 2y$ is

The equation can be written $\frac{dy}{y} = 2x\,dx$ and therefore, it is ***separable***. Integrating

$$\int \frac{dy}{y} = \int 2x\,dx \Rightarrow \ln|y| = x^2 + C$$

In explicit form, $y = e^{x^2 + C}$ or, $y = Ce^{x^2}$, where the constants in each equation are arbitrary but not the same. Thus **the answer is (2)**.

Problem 4. The general solution of $2y' + 12xy = 4x$ is

This equation is first-order linear, write $y' + 6xy = 2x$, $a(x) = 6x$ and $b(x) = 2x$. The solution is

$$y = \frac{\int \mu(x) b(x)\,dx + c}{\mu(x)} \quad \text{with } \mu(x) = \exp \int a(x)\,dx$$

Hence, $\mu(x) = \exp\int 6x\,dx = e^{3x^2}$ and $\dfrac{\int 2xe^{3x^2}\,dx + C}{e^{3x^2}} = e^{-3x^2}\left(\dfrac{1}{3}e^{3x^2} + C\right) = \dfrac{1}{3} + Ce^{-3x^2}$

The answer is, (4).