

Quiz 1 SOLUTION

Prob. 1 Prob. 2 Prob. 3 Prob. 4

①	1	1	①
2	2	2	2
3	3	3	3
4	④	4	4
5	5	⑤	5

Problem 1. The general solution of $y'' = 3y' + 4y$ is

- (1) $y = c_1 e^{4x} + c_2 e^{-x}$ (2) $y = c_1 e^{2x} + c_2 x$ (3) $y = e^x (c_1 \cos 2x + c_2 \sin 2x)$
 (4) $y = x + C(x + 1)$ (5) $y = -2x + C e^{5x}$

Regrouping, $y'' - 3y' - 4y = 0$. The characteristic equation is

$k^2 - 3k - 4 = (k - 4)(k + 1)$, with roots $k = 4, -1$. The general solution is

$$y = c_1 e^{4x} + c_2 e^{-x}.$$

The answer is 1.

Problem 2. The general solution of $y'' - 2y' + 2y = 0$ is

- (1) $y = c_1 e^{2x} + c_2 e^{5x}$ (2) $y = c_1 x^2 + c_2 x$ (3) $e^{2x}(c_1 + c_2 x)$
 (4) $e^x (c_1 \cos x + c_2 \sin x)$ (5) $2x + x^2 + y = C$

The characteristic equation is

$k^2 - 2k + 2 = 0$ $k = \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$. The general solution is

$$y = e^x (\cos x + \sin x).$$

The answer is 4.

Problem 3. Find a particular solution of $y'' + 2y' + y = 2x$

- (1) $Y = \ln(x^2 + C)$ (2) $Y = 3e^{2x}$ (3) $y = Cx^3$
(4) $Y = 3 \sin x + 2 \cos x$ (5) $Y = 2x - 4$

By the method of the undetermined coefficients, we try $Y = Ax + B$.

$Y' = A$, $Y'' = 0$. Plugging in the equation

$2A + Ax + B = 2x \Rightarrow Ax + 2A + B = 2x$. We have the system

$A = 2$; $2A + B = 0 \Rightarrow B = -4$. Therefore, the particular solution is

$$Y = 2x - 4$$

The answer is 5

Problem 4. Solve the initial value problem $y'' - 2y' + y = 0$, $y(0) = 1$, $y'(0) = 0$

- (1) $y = e^x(1 - x)$ (2) $y = e^x(c_1x + c_2)$ (3) $y = e^{2x}(3x + 1)$
(4) $y = e^{2x}(\cos x - 2 \sin x)$ (5) $y = 1 + 3x$

Characteristic equation is $k^2 - 2k + 1 = (k - 1)^2$. Double root $k = 1$.

General solution is $y = e^x(c_1x + c_2)$; $y' = e^x(c_1x + c_2) + c_1e^x = e^x(c_1x + c_1 + c_2)$

$$y(0) = c_2 = 1$$

$$y'(0) = c_1 + c_2 = 0 \Rightarrow c_1 = -1$$

The initial value problem is $y = e^x(-x + 1)$.

The answer is (1)