

Quiz 2 SOLUTION

Prob. 1 Prob. 2 Prob. 3 Prob. 4

1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5

Problem 1. The general solution of $y'' = 6y' - 9y$ is

- (1) $y = c_1 e^{4x} + c_2 e^{-x}$ (2) $y = c_1 e^{2x} + c_2 x$ (3) $y = e^x (c_1 \cos 2x + c_2 \sin 2x)$
 (4) $y = x + C(x + 1)$ (5) $y = e^{3x} (c_1 x + c_2)$

Regrouping, $y'' - 6y' + 9y = 0$. The characteristic equation is

$k^2 - 6k - 9 = (k - 3)^2$, with double-root $k = 3$. The general solution is

$$y = e^{3x} (c_1 x + c_2)$$

The answer is 5.

Problem 2. The general solution of $y'' - 2y' + 4y = 0$ is

- (1) $y = c_1 e^{2x} + c_2 e^{5x}$ (2) $y = c_1 x^2 + c_2 x$ (3) $e^{2x} (c_1 + c_2 x)$
 (4) $e^x (c_1 \cos \sqrt{3}x + c_2 \sin \sqrt{3}x)$ (5) $2x + x^2 + y = C$

The characteristic equation is

$$k^2 - 2k + 4 = 0 \quad k = \frac{2 \pm \sqrt{4 - 16}}{2} = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm \sqrt{3}i}{2} = 1 \pm \sqrt{3}i. \text{ The general solution is}$$

$$y = e^x (\cos 3x + \sin 3x).$$

The answer is 4.

Problem 3. Find a particular solution of $y'' + 2y' + y = 2e^{2x}$

- (1) $Y = \ln(x^2 + C)$ (2) $Y = (2/9)e^{2x}$ (3) $y = Cx^3$
(4) $Y = 3 \sin x + 2 \cos x$ (5) $Y = 2x - 4$

By the method of the undetermined coefficients, we try $Y = Ae^{2x}$.

$Y' = 2Ae^{2x}$, $Y'' = 4Ae^{2x}$. Plugging in the equation

$$Ae^{2x}(4 + 4 + 1) = 2e^{2x} \Rightarrow 9A = 2. \quad A = 2/9$$

Therefore, the particular solution is

$$Y = (2/9)e^{2x}$$

The answer is 2

Problem 4. Solve the initial value problem $y'' - 2y' + y = 0$, $y(0) = 0$, $y'(0) = 1$

- (1) $y = e^x(1 - x)$ (2) $y = e^x(c_1x + c_2)$ (3) $y = e^{2x}(3x + 1)$
(4) $y = e^{2x}(\cos x - 2 \sin x)$ (5) $y = xe^x$

Characteristic equation is $k^2 - 2k + 1 = (k - 1)^2$. Double root $k = 1$.

General solution is $y = e^x(c_1x + c_2)$; $y' = e^x(c_1x + c_2) + c_1e^x = e^x(c_1x + c_1 + c_2)$

$$y(0) = c_2 = 0$$

$$y'(0) = c_1 + c_2 = 1 \Rightarrow c_1 = 1$$

The initial value problem is $y = xe^x$

The answer is (5)