

Quiz 1 SOLUTION

Prob. 1 Prob. 2 Prob. 3 Prob. 4

1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5

Problem 1. The general solution of $2y' + 6 - 8y = 0$ is

- (1) $y = 3/4 + Ce^{4x}$ (2) $y = 4/9 + Ce^{3x}$ (3) $y = (y' + 6)/2$ (4) $y = x + C \ln |x + 1|$ (5) $y = 9x + Ce^{6x}$

Write $y' = 4y - 3$. It is *linear with constant coefficients* with $p = 4$ and $q = -3$. The general solution is $y = -q/p + Ce^{px}$, that is, $y = 3/4 + Ce^{4x}$,

so the answer is (1).

Problem 2. The general solution of $(5x - 2y)y' + 2x = -5y$ is

- (1) $x + 2x^2y + y^2 = C$ (2) $(3/2)x^2 + xy - y^2 = C$ (3) $x^2 + 5xy - y^2 = C$
 (4) $x^2y + 2y + y^2 = C$ (5) $2x + 2x + y = C$

This equation is *exact*. Indeed, write $(2x + 5y) dx + (5x - 2y) dy = 0$. Then

$$\frac{\partial M}{\partial y} = 5 \quad \text{and} \quad \frac{\partial N}{\partial x} = 5$$

Integrating M , $F(x, y) = \int (2x + 5y) dx = x^2 + 5xy + C_1(y)$.

Differentiating in y , $F_y(x, y) = N(x, y)$, that is

$$5x + C'(y) = 5x - 2y \Rightarrow C'(y) = -2y \Rightarrow C(y) = -y^2 + C.$$

The general solution, therefore, is

$$x^2 + 5xy - y^2 = C$$

and the answer is (3).

Problem 3. The general solution of $3y^2y' = 2x$ is

(1) $y = \ln(x^2 + C)$ (2) $y = Ce^{2x^2}$ (3) $y = Cx^3$

(4) $y + xy + Cx^2 = 0$ (5) $y = \sqrt[3]{x^2 + C}$

The equation can be written $3y^2 dy = 2x dx$, and therefore, it is *separable*. Integrating

$$\int 3y^2 dy = \int 2x dx \Rightarrow y^3 = x^2 + C \Rightarrow y = \sqrt[3]{x^2 + C}$$

The answer is (5).

Problem 4. The general solution of $2y' + 12xy = -12x$ is

(1) $y = x^2 + cx$ (2) $y = \ln x^2 + Cx$ (3) $y + x^2y + x^2 = 0$

(4) $y = \frac{1}{3} + Ce^{-3x^2}$ (5) $y = -1 + Ce^{-3x^2}$

This equation is first-order linear, write $y' + 6xy = -6x$ so $a(x) = 6x$ and $b(x) = -6x$.

The solution is

$$y = \frac{\int \mu(x)b(x)dx + c}{\mu(x)} \quad \text{with} \quad \mu(x) = \exp \int a(x)dx$$

Hence, $\mu(x) = \exp \int 6x dx = e^{3x^2}$ and $y =$

$$y = \frac{\int -6xe^{3x^2} dx + C}{e^{3x^2}} = e^{-3x^2} (-e^{3x^2} + C) = -1 + Ce^{-3x^2}$$

The answer is, (5).