# SIIT GTS211 Section 1 First Semester 2007 Dr. Ruben

## Quiz 1 SOLUTION

| Prob. 1 | Prob. 2 | Prob. 3 | Prob. 4 |
|---|---|---|---|
| (1) | 1 | 1 | 1 |
| 2 | (2) | 2 | 2 |
| 3 | 3 | (3) | 3 |
| 4 | 4 | 4 | (4) |
| 5 | 5 | 5 | 5 |

**Problem 1**.The general solution of $2y' + 6 - 8y = 0$ is

(1) $y = 3/4 + Ce^{4x}$ (2) $y = -2 + Ce^{-x}$ (3) $y = (y' + 6)/2$ (4) $y = x + C \ln |x + 1|$ (5) $y = 9x + Ce^{6x}$

Write $y' = -3 + 4y$. It is ***linear with constant coefficients*** with $p = 4$ and $q = -3$. The general solution is $\boldsymbol{y = -q/p + C e^{px}}$, that is, $\boldsymbol{y = 3/4 + Ce^{4x}}$, so the answer is (1).

**Problem 2**. The general solution of $(x - 2y)y' = -3x - y$ is

(1) $x + 2x^2y + y^2 = C$ (2) $(3/2)x^2 + xy - y^2 = C$ (3) $2x^2 + 2y^2 + y^2 = C$

(4) $x^2y + 2y + y^2 = C$ (5) $2x + 2x + y = C$

This equation is ***exact***. Indeed, write $(3x + y)\, dx + (x - 2y)\, dy = 0$. Then

$$\frac{\partial M}{\partial y} = 1 \quad \text{and} \quad \frac{\partial N}{\partial x} = 1$$

Integrating $M$, $F(x, y) = \int (3x + y)dx = \frac{3}{2}x^2 + xy + C_1(y)$.

Differentiating in $y$, $F_y(x,y) = N(x,y)$, that is

$$x + C'(y) = x - 2y \Rightarrow C'(y) = -2y \Rightarrow C(y) = -y^2 + C.$$

The general solution, therefore, is

$$\boldsymbol{(3/2)x^2 + xy - y^2 = C}$$

**and the answer is (2)**.

**Problem 3.** The general solution of $y' = \dfrac{3y}{x}$ is

(1) $y = \ln(x^2 + C)$ (2) $y = Ce^{2x^2}$ (3) $y = Cx^3$

(4) $y + xy + Cx^2 = 0$ (5) $y = 3/2 + Cxy$

The equation can be written $\dfrac{dy}{y} = \dfrac{3}{x}dx$ and therefore, it is ***separable***. Integrating

$$\int \frac{dy}{y} = \int \frac{3}{x}dx \Rightarrow \ln|y| = 3\ln|x| + C$$

In explicit form, $y = e^{3\ln|x|+K} = e^k e^{2\ln x|} = Cx^3$ Thus **the answer is (3)**.

**Problem 4.** The general solution of $y' + 6xy = 2x$ is

(1) $y = x^2 + cx$ (2) $y = \ln x^2 + Cx$ (3) $y + x^2y + x^2 = 0$

(4) $y = \dfrac{1}{3} + Ce^{-3x^2}$ (5) $y = x + Cxy$

This equation is first-order linear, with $a(x) = 6x$ and $b(x) = 2x$. The solution is

$$y = \frac{\int \mu(x)b(x)\,dx + c}{\mu(x)} \quad \text{with} \quad \mu(x) = \exp\int a(x)\,dx$$

Hence, $\mu(x) = \exp\int 6x\,dx = e^{3x^2}$ and $y = \dfrac{\int 2xe^{3x^2}dx + C}{e^{3x^2}} = e^{-3x^2}\left(\dfrac{1}{3}e^{3x^2} + C\right) = \dfrac{1}{3} + Ce^{-3x^2}$

**The answer is, (4).**