SIIT GTS211 Section 1 First Semester 2006 Dr. Ruben

Quiz 2 Solution

Each correct answer = 25 points, each incorrect answer is –6 points.

The letter A, used in the answers, is actually a number. It is given to prevent the non-solving a problem by substituting the answers in the given equation.

1. Find the general solution of $y'' - 8y' + 15y = 0$

1. $y = c_1e^{2x} + c_2\,x\,e^{Ax}$ 2. $y = c_1e^{3x} + c_2e^{Ax}$ 3. $y = e^{2x}(c_1 \sin x + c_2 \cos x)$
4. $y = c_1e^{-x} + c_2\,e^{Ax}$ 5. $y = e^{-x}(c_1 + c_2 \ln |x|)$

The characteristic polynomial is $k^2 - 8k + 15 = (k - 5)(k - 3)$, the general solution is

$y = c_1e^{3x} + c_2e^{5x}$

The answer is 2.

2. Solve the initial value problem $y'' - 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

1. $y = e^{-2x}(c_1 + c_2x)$ 2. $y = e^{2x}(1 + \mathrm{A}x)$ 3. $y = e^{x}(\sin x - \cos x)$
4. $y = e^{x} + \mathrm{A}e^{-x}$ 5. $y = xe^{x} + Ax$

The characteristic polynomial is $k^2 - 4k + 4 = (k - 2)^2$. The general solution is
$y = e^{2x}(c_1 + c_2x)$. Thus $y' = 2e^{2x}(c_1 + c_2x) + e^{2x}c_2 = e^{2x}(2c_1 + c_2 + 2c_2x)$. With the constraints we get
$c_1 = 1$, $c_2 = -2$. Hence the solution is $y = e^{2x}(1 - 2x)$
The answer is 2.

3. Find a particular solution of $y'' - y' + y = 6e^{2x}$

1. $y = 3e^{x}$ 2. $y = 2e^{x} + 2e^{-2x}$ 3. $y = \cos x - 2 \sin x$
4. $y = 2e^{2x}$ 5. $y = 3e^{2x}$

We try by the method of undetermined coefficients $y = Ae^{2x}$.
$y' = 2Ae^{2x}$, $y'' = 4Ae^{2x}$. Plugging in the equation, since it this is not a solution of the homogenous equation,

$Ae^{2x}(4 - 2 + 1) = 6e^{2x}$. Thus $A = 2$.

The answer is 4.

4. Find the general solution of of *y*” – 2*y*’ + 5*y* = 0

1\. $y = \cos x - \sin x$ 2. $y = e^x(\cos Ax + \sin Ax)$ 3. $y = 2e^x + 2e^{-2x}$
4\. $y = xe^x + Ax$ 5. $y = Ax^2 + Bx$

The characteristic polynomial is $k^2 - 2k + 5 = 0$, which has complex roots
$\frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$. The general solution is $y = e^x(c_1 \cos 2x + c_2 \sin 2x)$.
The answer is 2.