SIIT GTS211 MIDTERM EXAM 1[ST] SEMESTER 2006 DR. RUBEN

Solution

Problem 1. Find a particular solution of $y'' - y' - 2y = e^{2x}$

Homogeneous equation, characteristic equation $k^2 - k - 2 = (k-2)(k+1)$, $y_1 = e^{-x}$, $y_2 = e^{2x}$.
Undetermined coefficients, particular solution is

$$y = Axe^{2x},\quad y' = Ae^{2x}(2x+1),\quad y'' = A\,e^{2x}(4x+4).$$

Plugging in the equation we get $3Ae^{2x}$, therefore, $A = 1/3$.

Problem 2. Find the general solution of $(1+x)\,y' = \dfrac{1}{\cos y}$

Separable. $\cos y\,dy = \dfrac{1}{1+x}dx \;\Rightarrow\; \int \cos y\,dy = \int \dfrac{1}{1+x}dx \;\Rightarrow\; \sin y = \ln|1+x| + C$

Problem 3. Find the general solution of $y' - 2y = 5$

Linear constant coefficients. Writing in standard form, $y' = 2y + 5$. The general solution is

Applying the formula $y = -q/p + Ce^{px}$, with $p = 2$ and $q = 5$. Therefore, $y = -5/2 + Ce^{2c}$.

Problem 4. Using a table of Laplace transforms find the inverse Laplace transform of

$\Im(s) = \dfrac{2s-2}{s^2-2s-8}$

$$\Im(s) = 2\frac{s}{(s-4(s-2)} - 2\frac{1}{(s-4)(s-2)} = 2\frac{1}{6}\left(4e^{4t} - (-2)e^{-2t}\right) - 2\frac{1}{6}\left(e^{4t} - e^{-2t}\right) = e^{4t} + e^{2t}$$

Problem 5. Find the general solution of $y' = 2t\cos t^2$

$$y = \int 2t\cos t^2\,dt = \sin t^2 + C$$

Problem 6. Find in explicit form the general solution of

$$2x\sin y\,dx + x^2\cos y\,dy = 0$$

Exact equation. $\int 2x\sin y\,dx = x^2\sin y + C(y)$, Differentiating, $x^2\cos y + C'(y) = x^2\cos y$,

thus $C'(y) = 0$, $C(y) = C$. The solution **is $x^2\cos y = C$**

Problem 7. Solve the initial value problem

$$y'' - 6y' + 9 = 0, \quad y(0) = 1, \quad y'(0) = -2$$

$k^2 - 6k + 9 = (k-3)^2$. General solution is $y = e^{3x}(C_1x + C_2)$.

$y(0) = C_2 = 1$. $y' = 3e^{3x}(C_1x + C_2) + e^{3x}C_1 = e^{3x}(C_1x + C_2 + C_1)$

$y'(0) = 1 + C_1 = -2$, $C_1 = -3$. The answer is $\boxed{y = e^{3x}(-3x + 1)}$

Problem 8. . Knowing that $y_1 = x^{-2}$ is a solution of

$$y'' + \frac{y'}{x} - \frac{4y}{x} = 0,$$

use the method of reduction of order to find a second, linearly independent solution y_2