



Sirindhorn International Institute of Technology

Thammasat University

*GTS 211 Differential Equations and
Numerical Methods*

Class Notes

Part I:
DIFFERENTIAL EQUATIONS
and
LAPLACE TRANSFORMS

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PRELIMINARIES

These notes are a summary of the material presented in class for the course GTS211. Their purpose is to provide supplemental material to the book, including examples and homework. By no means they substitute the book or attendance to class.

The course GTS211 consists of three entirely unrelated topics: Part I deals with differential equations; Part II undertakes the study of Fourier Analysis; Part III covers concepts of numerical analysis. This paper only includes only the first part of the course.

A differential equation is an equation involving a function $y(x)$ and some of its derivatives. Solving differential equations requires a systematic methodology. There are several types of differential equations; for each type there is a unique procedure to obtain the solution. The task to the student to solve a problem of differential equations entails to first recognize the type of the equation and second apply the appropriate method of solution. This means that the student who practices the solution of each different type of equation is in good condition to perform well in the exams.

Chapter 1 presents introductory concepts. Chapter 2 deals with first order differential equations, which involve only the first derivative of the function. Chapter 3 tackles differential equations of second order, involving a second derivative. Chapter 4 takes on Laplace Transform.

At the end of each chapter, a list of homework is provided. Homework are not compulsory. This is just a suggested list of problems for practicing. The problems given in the exams are always similar to the problems suggested in the homework sections.

I wish you a successful semester.

Ruben N. Mera

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