A Theory of Teams

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Contents

Double Moral Hazard in Teams

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Abstract

This paper studies the optimal incentive schemes in a small team managed ¯rm where the owner of the ¯rm has the option to quit from production and be a pure residual claimant. The optimal incentive scheme suggests that: 1, the dependence between earnings and ¯rm's output should be higher for the ¯rm owners when their actions a®ect the ¯rm's output, even if they are risk-averse; 2, the incentive parameter for the managers should be higher if the owners of the ¯rm do not participate in decision making and management.

Do not impose others what yourself do not desire.

— — — Confucius

1 Introduction

The problem of double (or two-sided) moral hazard was not studied until lately. Cooper and Thomas (1985) is probably the first one who studied the double moral hazard problem through examining the optimal product warranty when the failure rate of a product is affected by both the firm's and the consumer's privately observed actions taken on the product. Mann and Wissink (1988), Dybvig and Lutz (1993) also discussed the same issue. In this literature, the optimal warranty is based on the notion of Nash equilibrium of the noncooperative game. When arriving at a conclusion about the level of the warranty jointly, both parties anticipate their reciprocal pattern of unobserved behavior, they consequently maximize their joint surplus under the restriction of the Nash equilibrium. The conclusion of the double moral hazard warranty model is that, first, parties who feel unobserved when carrying out their product investments normally agree to a partial warranty; second, this voluntary agreement solves the double moral hazard problem in a suboptimal manner. Mann and Wissink (1988) discussed the case of a voluntary money-back warranty. In their model, the buyer is allowed to return the product within a period specified beforehand. The authors conclude that under extreme conditions the double moral hazard problem is solved by the ¯rst-best levels of care-taking.

For the form of the optimal compensation schemes, Romano (1994) and Bhattacharyya and Lafontaine (1995) show that a simple linear contract in which both principal and agent share the output with constant proportions after a certain amount of transfer is made between them when both the principal and the agent are risk neutral is optimal. When the principal is risk neutral and the agent is risk averse, Kim and Wang (1998) argue that the optimal contract is generally not linear if ¯rst order approach is valid. Agrawal (1999) models the double moral hazard scenario between a landlord and a tenant with di®erent levels of farming e± ciency. The optimal contract maximizes the output net of the risk-taking and agency costs with risk averse landlord and tenant mutually monitoring each other. The primary ¯nding in Agrawal (1999) is that the farming e± ciency di®erence is the principal determinant of the contract.

In an example of Bhattacharyya and Lafontaine (1995) on franchising, it is showed that with some speci¯c assumptions[2], the optimal share proportion is independent of market size or competition, franchisees' disutility parameters and the size of the franchise chain. Lutz (1995) studies the dynamic double moral hazard problem in

[2]Cobb-Douglas production technology, constant marginal cost of private e®ort for the franchisor and CARA disutility functions for heterogeneous franchisees.

franchising in which she found that franchising may be the preferred organizational form when the local manager's e®ort has a relatively small e®ect on the unit's current pro¯t, but a large e®ect on the unit's future pro¯t.

Many have considered the remedies for double moral hazard. Feess and Nell (1998) consider a double moral hazard problem in a one-manager-one-auditor game. They show that an e±cient liability rule without punitive penalties can be solved through contingent auditing fees and fair insurance contracts where deductibles above the damages are not required. Demski and Sappington (1991) show that the double moral hazard problem can be completely and costlessly resolved if both parties have the option for the risk-averse agent to purchase the ¯rm at a prenegotiated price. Hence the agent will not be exploited under the buyout scheme, nor will he bear any unnecessary uncertainty. However, they noticed that in order to make the buyout scheme feasible, the agent will have to have su±cient wealth to buy out the ¯rm, thus, the model is restricted to the literature on the vertical relations between manufacturer and retailer. Tsoulouhas (1999) proposed that a piece rate tournament can eliminate double moral hazard with multiple agents. However this holds only if two conditions hold: 1. the principal su±ciently saves in transaction costs by employing a tournament; 2. the number of agents is su±ciently large.

Recently, Creightney (2000) tries to generalize some properties including optimal risk sharing in a comprehensive double moral hazard model in a repeated setting.

It is found that the optimal sensitivity is lower than that of a single moral hazard problem. Also, the marginal utility ratio is changing across periods, suggesting an history independent dynamic contract.

However the current literature on double moral hazard is problematic in the sense that double moral hazard is implicitly assumed as the cause of some inefficiencies. If, the conjecture is false or misunderstood, then the remedies offered will be naturally rejected whatsoever. Second, most models are static, therefore any interactions between the two sides of the relationship due to the repeated nature of the relationship is ignored, making the model less vigorous and convincing. Third, output of an agent in the real world does not only depend on the agent's effort but also his ability, especially entrepreneurial for a manager. Therefore the tournament propose in Tsoulouhas (1999) is not acceptable to us in this sense. Fourth, most models use very general functional forms on production technology, players' utility, and information structure etc. This makes the model somewhat convincing but less manipulable and therefore less capable to capture some insights of a problem.

With the above points in mind, our model differs in at least four aspects: 1, the inefficiency of double moral hazard, if it ever exists, is not presumed before hand; 2, the timing in our model allows learning to nest; 3, the decision to participate the production or not by the principal is endogenous depending on his own ability, the information he has and risk bearing; 4, we seek the close form solutions with aggregate production technology, CARA utility functions and commonly adopted monitoring technologies. Our major findings are: the optimal incentive sensitivity is higher for the principal if her effort affects the output positively and stochastically, implying that it is optimal for the principal to have more residual claims; optimal incentive sensitivity is lower for the agents in a double moral hazard scenario.

The structure of the rest of this paper follows: section 2 introduces the model, section 3 analyzes the optimal scheme for the second period, section 4 derives the optimal incentive scheme in the first period, section5 concludes the model.

2 The Model

The firm is constituted by two agents and one principal. The production is a function of both agents' and the principal's labour inputs plus a random shock,

$$\begin{aligned} y &= f(a; b; e) + \varepsilon \\ &= a + b + e + \eta_a + \eta_b + \varsigma + \varepsilon; \end{aligned} \tag{2.1}$$

where $a; b; e$ are the effort exerted by agent 1, 2 and principal, respectively, $\eta_a; \eta_b; \varsigma$ are the abilities to agent 1, 2 and the principal, respectively, and all distributed as $N(m_0; \sigma^2)$ a prior with zero correlation and ε is a random transient shock, distributed as $N(0; \sigma_\varepsilon^2)$. There are two periods in the model, therefore learning about team mates' ability is involved. Only aggregate output in each period is observable and verifiable to all parties. We use backwards induction to analyze the two-sided moral hazard.

Consider first in the second period, output is

$$y_2 = a_2 + b_2 + e_2 + \eta_a + \eta_b + \varsigma + \varepsilon_2;$$

where the subscripts denote the period[3]. Following Holmstrom and Milgrom (1987), we assume a linear incentive contract in the following form,

$$S_2 = \alpha_2 + \beta_2 y_2: \tag{2.2}$$

Therefore agent 1's[4] and principal's utility maximization problems are

$$\max_{a_2} E[U(\alpha_2 + \beta_2 y_2) - h(a_2)]; \tag{2.3}$$

and

$$\max_{e_2} E[V((1 - 2\beta_2) y_2 - 2\alpha_2) - g(e_2)]; \tag{2.4}$$

where $U(\cdot)$; $V(\cdot)$ are the utility functions of the agents and principal respectively. $h(\cdot)$; $g(\cdot)$ are disutilities of effort.

Formally, the principal's utility maximization problem is

$$\max_{e_2} E[V((1 - 2\beta_2)(a_2 + b_2 + e_2 + \eta_a + \eta_b + \varsigma + \varepsilon_2) - 2\alpha_2) - g(e_2)] \tag{2.5}$$

subject to

$$\max_{a_2} E[U(\alpha_2 + \beta_2 y_2) - h(a_2)]; \tag{2.6}$$

[3] The intrinsic abilities do not change over time.

[4] Because of information symmetry, we take agent 1 as representative agent.

$$\max_{b_2} E[U(\alpha_2 + \beta_2 y_2) - h(b_2)]; \tag{2.7}$$

$$E[U(\alpha_2 + \beta_2 y_2) - h(a_2)] \geq \underline{U}; \tag{2.8}$$

$$E[U(\alpha_2 + \beta_2 y_2) - h(b_2)] \geq \underline{U}; \tag{2.9}$$

where $\underline{U}$ is the agent's reservation utility, $(2{:}6)$; $(2{:}7)$ are agents 1 and 2's incentive compatibility constraints, and $(2{:}8)$; $(2{:}9)$ are the agent's participation constraints.

We assume that both the principal and the agents have CARA utility functions as we do not impose ad hoc assumption on the asymmetry on the two parties risk aversion. In our point of view, the major di®erence between a principal and an agent is the authority within the ¯rm. Firms are di®erent sorts of agencies, although the relationship between the owner and the managers of a ¯rm can be depicted by principal-agent models, it is di®erent from between a patient and a doctor, or a defendant and her attorney. The ¯rm owner can supervise or monitor her sta® to get a less vague picture on her fellow employees' abilities and e®ort on work while the reverse is much harder. This is not because the ¯rm owner's activities are less relevant to the ¯rm's output, but because of three reasons: ¯rst, the boss is a worker who works under the shadow, so it is harder to measure his talent or work that has been done; second, it is too costly to the agents to measure the owner's e®ort while the owner could be using information generated from each department of the ¯rm to measure and evaluate employees, these could be seen as some by-products from the production process and hence costless; finally, the owner has the authority, or voting rights to implement his action, using resources within the firm, this implies that even if monitoring is costly, she can still do it. Therefore, we impose a costless monitoring technology to the principal where by the end of the first period she can find out the true level of effort exerted by the agents with probability π; and with $1 - \pi$ she does not retrieve any information. The principal's conditional variance of estimated sum of the agents' abilities, σ_P^2, is

$$\begin{aligned}\sigma_P^2 \equiv Var(\eta_a + \eta_b | y_1) &= \pi \left(\frac{2\sigma_\varepsilon^2 \sigma_\eta^2}{2\sigma_\eta^2 + \sigma_\varepsilon^2} + \sigma_\varepsilon^2 \right) + (1 - \pi) \left(2\sigma_\eta^2 + \sigma_\varepsilon^2 \right) \\ &= \frac{2\pi\sigma_\varepsilon^2\sigma_\eta^2}{2\sigma_\eta^2 + \sigma_\varepsilon^2} + 2(1 - \pi)\sigma_\eta^2 + \sigma_\varepsilon^2.\end{aligned}$$

3 Second Period Double Moral Hazard

We can now rewrite the principal's utility maximization problem in period 2 into

$$\max_{e_2, \beta_2} - \exp\left\{ -R \left[(1 - 2\beta_2) E_P[y_2] - 2\alpha_2^* - g(e_2) - \frac{1}{2} R (1 - 2\beta_2)^2 \sigma_P^2 \right] \right\}; \tag{3.1}$$

where R is the principal's coefficient of absolute risk aversion, and α_2^* can be derived from agent's participation constraint (2.8),

$$\alpha_2^* = \underline{u} - \beta_2^* E_A[y_2] + h(\hat{e}_2) + \frac{1}{2} r \beta_2^{*2} \sigma_A^2; \tag{3.2}$$

where $\underline{U} = -\exp(-r\underline{u})$; superscript * denotes equilibrium level, $\hat{a}_2$ is agent 1's second period equilibrium effort level, which is expected by the principal, and the agent's posterior belief on randomness, $\varepsilon_2 + \eta_b + \zeta$; is

$$\sigma_A^2 = 2\sigma_\eta^2 + \sigma_\varepsilon^2;$$

which is equal to the agent's a prior belief since neither of them has monitoring technology. Therefore (3:1) can be rewritten as

$$\max_{e_2;\,\beta_2} -\exp\left\{-R\left[E_P[y_2] - 2\underline{u} - (m_1 - 2m_0) - 2h(\hat{a}_2) - g(e_2) - r\beta_2^2\sigma_A^2 - \frac{1}{2}R(1-2\beta_2)^2\sigma_P^2\right]\right\} \qquad (3.3)$$

where the principal's posterior belief on $\eta_a + \eta_b$; m_1; is

$$m_1 = 2\pi\frac{\sigma_\varepsilon^2 + 2\sigma_\eta^2(y_1 - \tilde{a}_1 - \tilde{b}_1 - \tilde{e}_1 - \varsigma)}{2\sigma_\eta^2 + \sigma_\varepsilon^2} + 2(1-\pi)m_0; \qquad (3.4)$$

where decoration ~denotes the associated variable is the actual value that the principal perceives.

The first order condition to (3:3) with respect to β_2 implies

$$e_2' \equiv \frac{\partial e_2}{\partial \beta_2} = -\frac{2}{\frac{\partial g'(e_2)}{\partial \beta_2}} = -\frac{2}{g''(e_2)}; \qquad (3.5)$$

and first order condition for the agent's second period utility maximization yields,

$$a_2' = \frac{1}{h''(a_2)}: \qquad (3.6)$$

Therefore the first order condition to (3.3) with respect to β_2 yields

$$\beta_2^* = \frac{\frac{1}{g''(e_2)} + R\sigma_P^2}{\frac{1}{h''(a_2)} + \frac{2}{g''(e_2)} + (r\sigma_A^2 + 2R\sigma_P^2)}. \tag{3.7}$$

where r is the agents' coefficient of absolute risk aversion. One can easily verify that if there is no uncertainly or risk aversion, the optimal incentive parameter, β_2^*, for this one principal two agent team, is $\frac{1}{3}$, because of the symmetry of the relationship between the principal and the agents.

Proposition 1 In a double moral hazard scenario, the optimal incentive for the principal is (weakly) higher than that of an agent.

Proof. Suppose the agents and the principal have the same disutility function, and at least one of them is risk averse, then refer to (14.1), $1 - 2\beta_2^* > \frac{1}{3}$. ■

Corollary 2 The optimal incentive for the principal is (weakly) higher if the principal's information on agents' abilities is more precise or she is less risk reverse.

Proof. This is to say

$$\frac{\partial \beta_2^*}{\partial \sigma_P^2} \geq 0$$

and

$$\frac{\partial \beta_2^*}{\partial R} \geq 0$$

It can be easily checked by taking partial derivatives on (14.1) with respect to σ_P^2 and R. ■

Consider now the principal has an option to quit from everyday production but just turn to be a residual claimant. Therefore the work is left to the two agents in second period. The principal will quit if and only if the utility from being a residual claimant is greater than that from directly participating in production. That is,

$$
\begin{aligned}
&(1-\beta_2^B)(a_2+b_2+m_1) - 2\alpha_2^B - \frac{1}{2}R(1-2\beta_2^B)^2\sigma_P^2 \qquad (3.8)\\
\geq\ &(1-\beta_2^*)(a_2+b_2+e_2+m_1+\varsigma) - 2\alpha_2^* - g(e_2) - \frac{1}{2}R(1-2\beta_2^*)^2\sigma_P^2;
\end{aligned}
$$

where superscript B represents the case of no principal participates in production.

To obtain conditions for inequality (3.8) to hold, we solve β_2^B and α_2^B. First order conditions of the maximization over the principal's certainty equivalent when she quits from production operation in period two yield,

$$
\beta_2^B = \frac{1+\frac{R\sigma_P^2}{h''(a_2)}}{2+\frac{r\sigma_A^2+2R\sigma_P^2}{h''(a_2)}}; \qquad (3.9)
$$

where if no uncertainty or risk aversion, $\beta_2^B = \frac{1}{2}$; implying a rental lease.

Lemma 3 Optimal incentive sensitivity for the agent in a double moral hazard scenario is lower than that of a one-sided moral hazard case in teams.

Proof. Just a few algebra to establish

$$
\beta_2^* < \beta_2^B. \qquad (3.10)
$$

■

Assuming for the moment that $h(x) = g(x) = \frac{x^2}{2}$; then (3.8) can be rewritten as

$$2\beta_2^B - \left(\beta_2^B\right)^2 - r(\beta_2^B)^2\left(\sigma_A^B\right)^2 - \frac{1}{2}R(1-2\beta_2^B)^2\sigma_P^2 \qquad (3.11)$$
$$\geq \frac{1}{2} + \varsigma - 3\beta_2^{*2} + \beta_2^* - r\beta_2^{*2}\sigma_A^2 - \frac{1}{2}R(1-2\beta_2^*)^2\sigma_P^2;$$

where $\left(\sigma_A^B\right)^2 = \sigma_\eta^2 + \sigma_\varepsilon^2 < \sigma_A^2$.

Lemma 4 If the principal's entrepreneurial ability is sufficiently low, i.e., (3.11) holds, she will quit from the production operation.

Remark 1 Note that $\left(\sigma_A^B\right)^2 < \sigma_A^2$ and $\beta_2^* < \beta_2^B$; the sufficient condition (3.11) can be simplified to a further sufficient condition,

$$\varsigma \leq \beta_2^B - \left(\beta_2^B\right)^2 + 3\beta_2^{*2} - r(\beta_2^B)^2\left(\sigma_A^B\right)^2 + r\beta_2^{*2}\sigma_A^2.$$

Lemma 5 If the principal and the agents have the same disutility function, then the sufficient and necessary condition for the principal from not shirking is

$$r \leq \left(\sigma_A^2\right)^{-1}.$$

Proof. Straightforwardly from (14.1). ■

Because uncertainty is reduced, i.e., $\left(\sigma_A^B\right)^2 < \sigma_A^2$; the trade-off between insurance effect and incentive to the agents is improved and hence the optimal incentive sensitivity is higher. Recently, Mario Lemieux, the Pittsburgh Penguins' owner who ended a 3 1/2-year retirement in December 2000, helped to take his team to the conference finals for the first time since 1996. On the other hand, Lemieux's fellow superstar, Jaromir Jagr, who is one of the best players in the history of the National Hockey League, scored only twice during the entire playoffs in 2001. Jagr will also end his 11 years draft with the Pittsburgh Penguins this summer. Our theory may explain the above story.

4 First Period Incentive Problem

With the second period optimal incentive problem solved, we advance to the first period. In the first period, the utility maximization problem to agent 1 is,

$$\max_{a_1} -\exp\{-r[\alpha_1 + \beta_1^* E_A[y_1] - h(a_1) + \delta\alpha_2^* + \delta\beta_2^* E_A[y_2] - h(\hat{a}_2) + \frac{1}{2} r\sigma_A^2]\}; \tag{4.1}$$

where variance σ_A^2 captures all the uncertainties to agent 1 at the first period perspective. The principal's problem is then

$$\max_{e_1;\,\beta_1} -\exp\{-R[(1-2\beta_1)E_P[y_1] - 2\alpha_1^* - g(e_1) \tag{4.2}$$
$$+\delta(1-2\beta_2^*)E_P[y_2] - 2\delta\alpha_2^* - \delta g(\hat{e}_2) + \frac{1}{2} R\sigma_P^2]\};$$

where variance Σ_P^2 captures all the uncertainties to the principal at the first period perspective and

$$\Sigma_P^2 = \left(1 - 2\beta_1 - \frac{\delta\pi\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2} + \delta(1 - 2\beta_2)\right)^2 \left(2\sigma_\epsilon^2\right) + \left[\left(1 - 2\beta_1 - \frac{\delta\pi\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2}\right)^2 + \delta^2(1 - 2\beta_2)^2\right]\sigma_\eta^2. \tag{4.3}$$

The first order condition for (4.1) yields

$$h'(a_1) = \beta_1 + \delta\pi\frac{\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2}.$$

To obtain β_1^*, maximize (4.2) with respect to β_1, i.e.,

$$0 = -2\left(\beta_1^* + \frac{\delta\pi\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2}\right)a_1' - \left(1 - 2\beta_1^* - \frac{2\delta\pi\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2}\right)e_1' - 2r\beta_1^*\sigma_A^2 + \frac{1}{2}R\frac{\partial\Sigma_P^2}{\partial\beta_1}, \tag{4.4}$$

where

$$\frac{\partial\Sigma_P^2}{\partial\beta_1} = 8\beta_1(2\sigma_\epsilon^2 + \sigma_\eta^2) + 4\left(\frac{\delta\pi\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2} - 1\right)(2\sigma_\epsilon^2 + \sigma_\eta^2) - 8\sigma_\epsilon^2\delta(1 - 2\beta_2). \tag{4.5}$$

Therefore some algebra manipulations from (4.4) imply that

$$\beta_1^* = \frac{\frac{1}{g''(e_1)} + 2R\delta\sigma_\epsilon^2(1 - 2\beta_2^*) - \left(\frac{1}{h''(a_1)} + \frac{2}{g''(e_1)}\right)\frac{\delta\pi\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2} + R((2 - \delta\pi)\sigma_\epsilon^2 + \sigma_\eta^2)}{\frac{1}{h''(a_1)} + \frac{2}{g''(e_1)} + (r + 2R)(2\sigma_\epsilon^2 + \sigma_\eta^2)}. \tag{4.6}$$

The first term in the numerator of (4.6) is similar to that of (14.1); the second term is the effect of principal's estimation error on agents' first period effort; the third term is to balance out the career concerns of the agents and the last term in the numerator is similar to that of (14.1), an appropriate risk aversion adjuster. Once again, if there is no uncertainty or the players are risk neutral, $\beta_1^* = \frac{1}{3}$.

Lemma 6 The principal will not shirk relative to the agents if and only if

$$\pi \geq \frac{2r(1 - 2\beta_2^*)\sigma_A^2}{r\sigma_A^2 + 2},$$

where $\sigma_A^2 = 2\sigma_\epsilon^2 + \sigma_\eta^2$.

Proof. First order conditions to (4.2) and (4.1) show that

$$h'(a_1^*) = \beta_1^* + \frac{\delta\pi\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2},$$

and

$$g'(e_1^*) = 1 - 2\beta_1^* - \frac{2\delta\pi\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2}.$$

Suppose $g(\cdot) = h(\cdot)$, then $h'(a_1^*) \geq g'(e_1^*) \Leftrightarrow a_1^* \geq e_1^*$, which is equivalent to

$$\beta_1^* \geq \frac{1}{3} - \frac{\delta\pi\sigma_\epsilon^2}{2\sigma_\epsilon^2 + \sigma_\eta^2}$$

which in turn is equivalent to

$$\pi \geq \frac{2r(1 - 2\beta_2^*)\sigma_A^2}{r\sigma_A^2 + 2}.$$

■

Lemma 6 says that a risk averse principal should have an advanced monitoring technology to convince her subordinates that she will not shirk if she ever participates in production operation.

Lemma 7 The optimal explicit incentive for the subordinates increases in a firm with double moral hazard, i.e., $\beta_2^* \geq \beta_1^*$.

Proof. Some algebras in (4.6) and (14.1) yield the result. ■

The above lemma implies that a risk averse owner of the firm will work harder in later career if he ever decides to stay in production operation. This result is against the common sense as it seems an owner quits from business as time goes. Note the analysis here assumes ability does not change over time and there is no interaction between effort and ability. Many of the facts attribute to the ability, for instance, a chair of a department, commonly a productive senior faculty, usually does not always painstakingly stay in office or lab overtime to produce good research, instead, much of her late work is due to her superior academic ability that she gained from previous years. Same thing happens in a business world[5].

[5] Recently, Lee Iacocca, the legendary former chairman of Chrysler, is now on the board of San Diego-based Online Asset Exchange, a commodities-like exchange for used corporate assets. Though selling used machine tools might not sound as glamorous as launching the original Mustang or bringing the first minivan into market or saving one of America's Big Three automakers from bankruptcy, Iacocca is pretty successful especially given that fact that about 20 dot com companies

5 Conclusion

In this paper, we study the double moral hazard problem in a ¯rm's optimal incentive scheme design. We suppose the relationship between the ¯rm owner and the salaried managers are repeated; and the owner's decision to stay in board or quit for retirement is endogenous; managerial team size is small. We do not adopt lump sum transfer or buyout as options to improve the risk sharing as the relationship considered in our model is between a ¯rm owner and professional managers where the managers' wealth is constrained.

Our model ¯nds that there is no ine± ciency in the sense of double moral hazard in a team managed ¯rm because any ine± ciencies in an organization caused by double moral hazard will be adjusted by the optimal incentive scheme and the principal's strategic decision on participation and hence any remedy to it is not necessary. Some properties that the optimal incentive scheme exhibits are consistent with commonly observed facts. These properties include: a ¯rm owner's salary is more powered than a manager's in the contract; the sensitivity of the incentive is higher for a publicly held ¯rm than a privately held ones with block shareholders.

go bankruptcy every day in the Sillicon Valley alone. Je®rey Bezos claims in a TV show that he works extremely hard to keep Amazon.com alive while Lee Iacocca is taking time in his part.

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On the Optimal Team Incentive Compensation Schemes

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Abstract

This paper studies optimal incentive schemes in a team setting and we explicitly derive a closed form solution to the compensation scheme where the underlying output processes are correlated drifted Brownian motions. Some implications of the model

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are discussed.

6 Introduction

Incentive schemes need to be robust in either single agent or multiple agent case. A simple incentive scheme may perform well across a wide range of environments as well as having low writing costs. Intricate schemes are designed for the purpose of inducing agents to exert effort in the principal's best interest in particular environments. However these fine-tuning complex incentive schemes are not very realistic for at least two reasons: first, they could hardly maintain their optimality for even a slightest change in the information structure or technological change; second, intuitively, intricate schemes are hard to be implemented, for instance, agents would take advantage of the complexity of the contracts by arbitrage-taking like behaviours, and this situation can be exaggerated by multiple agents because collusion and deviating from optimal reciprocal help level will be their options. A more rudimentary concern is monotonicity of the incentive contracts. As standard first order approach pointed out, an optimal incentive scheme has not to be monotone if MLRP does not hold. However if free disposal of the output is feasible, a practical scheme then has to be monotone despite any other argument on either information structure or stochastic production technology it may have. If sabotage is feasible to some level, the output would be undermined seriously if the scheme is not monotone. With this concern removed, we move onto the linearity of the optimal contracts.

The robustness of compensation linearity was studied in Holmstrom and Milgrom (1987) for a single agent case. Mirrlees (1974) claims that a step function scheme can induce optimal e®ort level of the agent as well as linear contract. However the e®ort level chosen by the agent is just one shot. If instead, more generally, the agent choose e®ort level everyday, over the course of a company ¯scal year, if the compensation is a step function of the aggregate output of his account, then his e®ort level chosen will vary day to day. By the end of year, he would certainly do nothing in the o± ce if the output already exceeds the critical level, the same thing if the realized aggregated output is too far below the critical level. To rescue step function in this aspect, one would possibly adjust the scheme into a day to day step function scheme, but this revisionism fails because if the agent labours under the same one-day step contract throughout the year the aggregate compensation itself is a linear function of the aggregate output. Furthermore, step functions do not converge to ¯rst-best if some assumptions on utility functions and technology are not clearly speci¯ed even under one-shot e®ort case, pointed out by Holmstrom and Milgrom (1987).

In the continuous-time principal-agent framework, Holmstrom and Milgrom(1987) and Scattler and Sung(1993, 1997) suggest an optimal sharing rule for the principal given the underlying output process is a Brownian motion. In their models, the optimal sharing rule is a linear function of aggregate output. Kim and Wang(1998) argue that the optimal incentive contract is nonlinear when the agent is risk averse and the principal and agent are set in a double moral hazard situation in which the principal also participates in the production process. Some others constantly cast doubt on the robustness of the linearity. We agree that real world schemes need to be robust. It is not enough for a scheme performs optimally in a limited environment but also a changing environment because constant changes of schemes are simply not feasible. Therefore regularities about the shape of the optimal sharing rule are essential to modeling. Linear contracts are observed across a wide range of technologies, companies and industries, on the other hand, managerial teams, instead of single manager, are on play, in most modern capitalistic and partnership firms.

Several other issues arise together with the optimality of linear incentive contracts in teams. Many have realized that the managers' role is not only exerting effort to improve the NPV of the ongoing projects, but also the choice on future projects to be carried out. Even only in the ongoing projects, an entrepreneur's function can be categorized into both mean improving and variance reducing activities, for example, creating strategic alliance with friendly rivals in the industry, spending time with potential clients, strategic actions in financial markets, everyday on-field management and supervision, or, some other activities like regular maintenance of production facilities, reducing unnecessary risks to the firm by activities such as maintaining a modest relationship with workers' union to avoid strikes, or be careful on every tip of the ongoing projects in order to minimize lawsuits ¯led by unsatis¯ed customers. While some of these activities are not too hard to observe, most are. Sung (1995) realizes this fact and claims that optimal schemes are still linear in ¯nal outcome alone even when the agent is allowed to control variance. Sung further shows that the sensitivity must be low to correct the manager's incentive to choose the right project, o®ering an alternative interpretation for Jensen-Murphy puzzle. Some chunks of our analysis were inspired by this observation, nevertheless, we focus on problems arise in teams, for instance, the optimal level of help between two team members in characterized in the optimal schemes we proposed. The incentive problem concerned in our model is two-fold, ¯rst, an optimal contract should fully exploit agents' comparative advantage on di®erent types of activities, for example, some careful and versatile people may be good at everyday human management and facility maintenance, while some very creative and knowledgable about current business lines may be good at dealing with tough clients or carrying out new projects, it is therefore better to let them work more on the tasks they specialized in; secondly, the relative performance evaluation in our model does exist but is not to ¯lter out noise, instead, to encourage cooperation between the team members by placing correct contractual incentives.

The structure of the rest of the paper follows. Section 2 introduces the framework of the model. Section 3 gives a closed form optimal linear incentive scheme under group performance evaluation. Section 4 solves the optimal incentive scheme under individual performance evaluation. Section 5 discusses team related issues like reciprocal helps and other comparative statics. Section 6 summarizes the essay.

7 The Model

We start with a two-dimensional Brownian motion where the two agents control the drift rate of the Brownian motions and the Wiener processes are correlated. Both agents and principal have negative exponential utilities with different coefficients of absolute risk aversion. Time is normalized to one.

Formally, the processes are governed by a simultaneous differential equation system of the form,

$$dX_1 = a_1(t)dt + \sigma_1 dB_1(t);$$

$$dX_2 = a_2(t)dt + \sigma_2(\rho dB_1(t) + \sqrt{1-\rho^2}dB_2(t));$$

where $dB_1; dB_2(t)$ are independent Brownian motions. Production technology is additive, i.e.,

$$dY = dX_1 + dX_2$$

$$= [a_1 + a_2]dt + dB_4(t) \times (\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2\rho)^{\frac{1}{2}};$$

where $dB_4(t) = [(\sigma_1 + \sigma_2\rho)dB_1(t) + \sqrt{1-\rho^2}dB_2(t)]=(\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2\rho)^{\frac{1}{2}}:$

The principal's problem is to find a sharing rule $S_i(Y)$ and controls $a_{ii}; a_{ij}$ to maximize his utility.

$$\max_{S(Y);\, a_{ii};\, a_{ij}} E\left[-\exp\left\{-R\left(Y(1) - \sum_{i=1}^{2} S_i(Y)\right)\right\}\right]; \tag{7.1}$$

subject to

$$dX_1 = a_1(t)dt + \sigma_1 dB_1(t); \tag{7.2}$$
$$dX_2 = a_2(t)dt + \sigma_2\left(\rho dB_1(t) + \sqrt{1-\rho^2}\, dB_2(t)\right);$$

$$E\left[-\exp\left\{-r\left(S_i(Y) - \int_0^1 c(a_i)dt\right]\right.\right. \geq -\exp\{-rW_{CE}\}; \quad \text{and} \tag{7.3}$$

$$a_i \in \arg\max_{a_i \in A} E\left[-\exp\left\{-r\left(S_i(Y) - \int_0^1 c(a_i)dt\right]\right.\right. \tag{7.4}$$

where W_{CE} is the agents' certainty equivalent at time 0 in monetary terms; $c(\cdot)$ is the disutility function for the agents; a_i is the agent i's effort; and $R; r$ are the coefficients of absolute risk aversion to the principal and agents, respectively.

8 Optimal Group Performance Evaluation

For the utility maximization problem (7.1); we have the value function as the following,

$$V\left(t; \sum x_i\right) = \max_{S;\, a} E\left[\left\{\exp\left(-R\left(-dS\left(\sum x_i\right)\right)\right)\right\} V\left(t+dt; \sum x_i\right)\right]: \tag{8.1}$$

Note that Ito's Lemma can be written

$$
\begin{aligned}
dG &= \left(\frac{\partial G}{\partial x}\mu + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}\sigma^2\right)dt + \frac{\partial G}{\partial x}\sigma dZ \\
&= \frac{\partial G(x)}{\partial x}(\mu dt + \sigma dZ) + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}\sigma^2 dt \\
&= \frac{\partial G(x)}{\partial x}dx + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}\sigma^2 dt;
\end{aligned}
$$

where $G(x)$ is a function of the drifted Brownian motion $dx = \mu dt + \sigma dZ$. Therefore for a vector drifted Brownian motion and a scalar function of the vector below,

$$
\begin{aligned}
dx(t) &= a(t)dt + \Sigma(t)dB(t) \qquad \text{and} \\
y(t) &= \psi(t; x(t));
\end{aligned}
$$

Ito's Lemma is

$$
\begin{aligned}
dy(t) = d\psi(t; x(t)) = {} & \left\{\psi_t(t; x(t)) + \psi_x(t; x(t))a(t) + \frac{1}{2}\mathrm{tr}\left\{\psi_{xx}(t; x(t))\Sigma(t)\Sigma'(t)\right\}\right\} dt \\
& + \psi_x(t; x(t))\Sigma(t)dB(t).
\end{aligned}
$$

Therefore we have the first best compensation scheme's total differentiate as

$$
\begin{aligned}
dS(t; x(t)) &= \left\{\frac{\partial S}{\partial t} + \left[\frac{\partial S}{\partial x_1}, \frac{\partial S}{\partial x_2}\right]\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \frac{1}{2}\mathrm{tr}\left\{\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}[\sigma_1; \sigma_2]\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}\right\}\right\} dt \\
&\quad + \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}\left[\sigma_1 \quad \sigma_2\right] d\begin{bmatrix} B_1(t) \\ \rho B_1(t) + \sqrt{1 - \rho^2}B_2(t)) \end{bmatrix} \\
&= \alpha dt + \beta dx
\end{aligned}
$$

where $S_i \equiv \frac{\partial S}{\partial x_1}$; $S_{ij} \equiv \frac{\partial S}{\partial x_i \partial x_j}$; $\alpha = \frac{1}{2}\mathrm{tr}\left\{ \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} [\sigma_1; \sigma_2] \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} \right\}$ and $\beta = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$.

The value function (8:1) is hence rewritten as

$$
\begin{aligned}
& V(t; x_i) && (8.2) \\
= \; & S; a \max E[\left(1 + R(\alpha dt + \beta dx) + \frac{1}{2}R^2 (\alpha dt + \beta dx)^2\right) \\
& \left(V(\cdot) + \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial x}dx + \frac{1}{2}\mathrm{tr}\left(\frac{\partial^2 V}{\partial x^2}\sigma\sigma'\right) dt\right)]:
\end{aligned}
$$

In a second best world where individual output is not observable and hence compensation could only be based on aggregate output, that is

$$
\begin{aligned}
dY(t) &= (a_1 + a_2)dt + \sigma_1 dB_1 + \rho\sigma_2 dB_1 + \sqrt{1 - \rho^2}\sigma_2 dB_2 \\
&= C dt + D \begin{bmatrix} dB_1 \\ dB_2 \end{bmatrix}
\end{aligned}
$$

where $C = a_1 + a_2$; $D = \left[\sigma_1 + \rho\sigma_2; \; \sqrt{1 - \rho^2}\sigma_2 \right]$:

The value depends on a scalar Y instead of the vector x, i.e.,

$$
\begin{aligned}
& V\left(t; \sum x_i\right) && (8.3) \\
= \; & S; a \max E[\left(1 + R(\alpha_1 dt + \beta_1 dY) + \frac{1}{2}R^2 (\alpha_1 dt + \beta_1 dY)^2\right) \\
& \left(V(t;Y) + \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial Y}dY + \frac{1}{2}\left(\frac{\partial^2 V}{\partial Y^2}DD'\right) dt\right)]:
\end{aligned}
$$

Rewriting (8.3) yields

$$
\begin{aligned}
0 \equiv\ & \frac{\partial V}{\partial t}dt + \frac{1}{2}\left(\frac{\partial^2 V}{\partial Y^2}DD'\right)dt \\
& + \max_{\alpha_1,\beta_1,a}\left[\frac{\partial V}{\partial Y}(a_1 + a_2 + R\beta_1 DD')dt + RV(\cdot)\left(\beta_1(a_1 + a_2) + \alpha_1 + \frac{R}{2}\beta_1^2 DD'\right)dt\right]
\end{aligned}
$$

with transversality condition $V(1,Y) \equiv -\exp\{-RY(1)\}$; and agents' participation constraints:

$$E[-\exp\{-r(\frac{1}{2}dS(Y) - c(a_i)dt]\geq -1,$$

for $i = 1, 2$.

The agents' participation constraints imply,

$$0 \equiv \frac{\alpha_1}{2}dt + \frac{\beta_1}{2}(a_1 + a_2)dt - c(a_i)dt - \frac{r}{8}\beta_1^2 DD'dt$$

which can be rewritten as

$$\alpha_1^* \equiv 2c(a_i^*) - 2\beta_1 a_i^* + \frac{1}{4}r\beta_1^2 DD'. \qquad (8.4)$$

Once β_1 is obtained, optimal α is calculated from the transformation of the agents' participation constraint (8.4).

With the agents' participation constraint, we now rewrite the dynamic programming problem as

$$
\begin{aligned}
0 \equiv\ & \frac{\partial V}{\partial t}dt + \frac{1}{2}\left(\frac{\partial^2 V}{\partial Y^2}DD'\right)dt \\
& + \max_{\beta_1,a}\left[\frac{\partial V}{\partial Y}(a_1 + a_2 + R\beta_1 DD')dt + RV(\cdot)\left(2c(a_i^*) + \frac{2R + r}{4}\beta_1^2 DD'\right)dt\right].
\end{aligned}
$$

The value function can be treated functionally as the principal's certainty equivalent or any linear transformation of the certainty equivalent. Again by Taylor's expansion, we have

$$0 \equiv \frac{\partial\psi}{\partial t}dt + \frac{1}{2}\left(\frac{\partial^2\psi}{\partial Y^2}DD'\right)dt - \frac{R}{2}\left(\frac{\partial\psi}{\partial Y}\right)^2 DD'dt \tag{8.5}$$
$$+\max_{\beta_1; a}\left[\frac{\partial\psi}{\partial Y}(a_1 + a_2 + R\beta_1 DD')dt - 2c(a_i^*)dt - \frac{2R+r}{4}\beta_1^2 DD'dt\right];$$

where $\psi \equiv \psi(t;Y)$; $V(t;Y) = -\exp\{-R\psi(t;Y)\}$; with terminal condition $\psi(1;Y) = Y(1)$:

Differentiating (8:5) yields the first order conditions for the principal's utility maximization problem, and these conditions are

$$\frac{\partial\psi}{\partial Y}RDD'dt - \frac{2R+r}{2}\beta_1^2 DD'dt = 0 \Rightarrow \beta_1^* \equiv \frac{R}{R+\frac{r}{2}}\frac{\partial\psi}{\partial Y}; \tag{8.6}$$

$$\frac{\partial\psi}{\partial Y} = 2c'(a_i^*): \tag{8.7}$$

Substituting the first order conditions above back to the maximand (8:5) yields

$$0 \equiv \frac{\partial\psi}{\partial t}dt + \frac{1}{2}\left(\frac{\partial^2\psi}{\partial Y^2}DD'\right)dt - \frac{R}{2}\left(\frac{\partial\psi}{\partial Y^2}\right)^2 DD'dt \tag{8.8}$$
$$+\max_{\beta_1; a}\left[\begin{array}{l} 2c'(a_i^*)\left(a_1 + a_2 + 2c'(a_i^*)\frac{R^2}{R+\frac{r}{2}}DD'\right)dt \\ - 2c(a_i^*)dt - 2\left(c'(a_i^*)\right)^2\frac{R^2}{R+\frac{r}{2}}DD'dt \end{array}\right]$$
$$= \frac{\partial\psi}{\partial t} + \frac{1}{2}\frac{\partial^2\psi}{\partial Y^2}DD' - \frac{R}{2}\left(\frac{\partial\psi}{\partial Y^2}\right)^2 DD'$$
$$+2c'(a_i^*)\left[c'(a_i^*)\frac{R^2}{R+\frac{r}{2}}DD' + a_1 + a_2\right] - 2c(a_i^*):$$

To solve the partial differential equation (8:8), we guess a solution of the form $\Psi(t;Y) = \lambda(t)Y^3 + \phi(t)Y^2 + \kappa(t)Y + \psi(t)$ with the terminal constraints $\lambda(1) = 0; \phi(1) = 0; \kappa(1) = 1;$ and $\psi(1) = 0$: Therefore we have

$$\frac{\partial \Psi}{\partial Y} = 3\lambda(t)Y^2 + 2\phi(t)Y + \kappa(t);$$

$$\frac{\partial \Psi}{\partial t} = \dot{\lambda}(t)Y^3 + \dot{\phi}(t)Y^2 + \dot{\kappa}(t)Y + \dot{\psi}(t);$$

$$\frac{\partial^2 \Psi}{\partial Y^2} = 6\lambda(t)Y + 2\phi(t);$$

$$\left(\frac{\partial \Psi}{\partial Y}\right)^2 = 9\lambda^2(t)Y^4 + 12\lambda(t)\phi(t)Y^3 + (4\phi^2(t) + 6\lambda(t)\kappa(t))Y^2 + 2\phi(t)\kappa(t)Y + \kappa^2(t)$$

and the utility maximization is then

$$\begin{aligned} 0 \equiv\ & 9\lambda^2(t)Y^4\left(-\frac{R}{2}DD^I\right) + \left(\dot{\lambda}(t) - \frac{R}{2}DD^I 12\lambda(t)\phi(t)\right)Y^3 \\ & + \left(\dot{\phi}(t) - \frac{R}{2}DD^I\left(4\phi^2(t) + 6\lambda(t)\kappa(t)\right)\right)Y^2 \\ & + \left(\dot{\kappa}(t) - \phi(t)\kappa(t)RDD^I + \frac{R^2}{2}6\lambda\right)Y + \dot{\psi}(t) + DD^I\phi(t) \\ & - \frac{R}{2}DD^I\kappa^2(t) + 2c_i'(\cdot)\left[c_i'(\cdot)\frac{R^2}{R+\frac{r}{2}}DD^I + a_1 + a_2\right] - 2c_i(\cdot) \end{aligned}$$

Collecting the terms yields a system of ordinary differential equations:

$$0 = -\frac{9}{2}\lambda^2(t)RDD^I;$$

$$0 = \dot{\lambda}(t) - \frac{R}{2}DD^I 12\lambda(t)\phi(t);$$

$$0 = \dot{\phi}(t) - \frac{R}{2} DD^{\top}\left(4\phi^2(t) + 6\lambda(t)\kappa(t)\right);$$

$$0 = \dot{\kappa}(t) - \phi(t)\kappa(t) R DD^{\top} + 3\lambda DD^{\top}$$

$$0 = \dot{\psi}(t) - \frac{R}{2}\kappa(t) DD^{\top} + 2c'_{-}(\cdot)\left(c'_{-}(\cdot)\frac{R^2}{R + \frac{r}{2}} DD^{\top} + a_1 + a_2\right) - 2c_{-}(\cdot).$$

The above ordinary differential equation system, together with the boundary conditions, yield a set of solutions: $\lambda(t) = 0$; $\phi(t) = 0$; $\kappa(t) = 1$; and $\dot{\psi}(t) = \frac{R}{2} DD^{\top}\kappa^2(t) - 2c'_{-}(\cdot)\left[c'_{-}(\cdot)\frac{R^2}{R+\frac{r}{2}} DD^{\top} + a_1 + a_2\right] + 2c_{-}(\cdot)$. Therefore the

$$\Psi(t;Y) = Y + \left\{\frac{R}{2} DD^{\top}\kappa^2(t) - 2c'_{-}(\cdot)\left(c'_{-}(\cdot)\frac{R^2}{R + \frac{r}{2}} DD^{\top} + a_1 + a_2\right) + 2c_{-}(\cdot)\right\}(t - 1). \tag{8.9}$$

Differentiating (8.9) with respect to Y gives

$$\frac{\partial \Psi(t;Y)}{\partial Y} = 1.$$

Recall the first order condition (8.7), we can have

$$c'_{-} = \frac{1}{2}. \tag{8.10}$$

By (8.10) and the first order condition (21.6), we then have

$$\beta_1 = \frac{R}{2R + r}. \tag{8.11}$$

To get α_1, simply substitute (14.10) into (8.4), and we have

$$S^* = S(0;Y) + 2\int_0 c(a_i)dt + \int_0 \frac{R}{2R+r}(dY - (a_1 + a_2)dt) + r\int_0 \left(\frac{R}{2R+r}\right)^2 \frac{DD^T}{4}dt.$$

We present the linear sharing rule result below.

Proposition 8 (Efficient sharing rule) *The solution to the principal's problem (1.1)-(1.4) is*

$$\begin{aligned} S^*(Y) = w_{CE} + 2\int_0^1 c(a_i)dt + \int_0^1 \frac{R}{2R+r}\left(dY - \sum_i a_i(t)dt\right) \\ + r\int_0^1 \left(\frac{R}{2R+r}\right)^2 \frac{DD^T}{4}dt \end{aligned} \tag{8.12}$$

where the effort level a_i is uniquely and implicitly defined by $c'(a_i) = 1$.

9 Optimal Individual Performance Evaluation

In this section, we study the optimal performance evaluation scheme in a two-person team where individual performance for each agent is observable. To make our analysis more vivid, we work on a more general setting where agents' control variables include the diffusion term related ones, and, the output processes are interrelated.

Let $X_1(t)$ and $X_2(t)$ be governed by the following stochastic differential equation system:

$$\begin{aligned} dX_1 &= (f(a_{11}(t))X_1(t) + f(a_{12}(t))X_2(t) + \mu_1)dt \\ &\quad + \sigma_1\sqrt{h(b_{11}(t))X_1(t) + h(b_{12}(t))X_2(t) + \varpi}\,dB_1(t); \end{aligned} \tag{9.1}$$

$$dX_2 = (g(a_{21}(t))X_1(t) + g(a_{22}(t))X_2(t) + \mu_2)dt \tag{9.2}$$
$$+\sigma_2 \sqrt{l(b_{21}(t))X_1(t) + l(b_{22}(t))X_2(t) + \varpi}\left(\rho dB_1(t) + \sqrt{1-\rho^2}dB_2(t)\right);$$

where $dB_1; dB_2(t)$ are once again independent Brownian motions. To save some notations, we define

$$Z_1 \equiv h(b_{11}(t))X_1(t) + h(b_{12}(t))X_2(t) + \varpi;$$

$$Z_2 \equiv l(b_{21}(t))X_1(t) + l(b_{22}(t))X_2(t) + \varpi;$$

and

$$dB_3(t) \equiv \rho dB_1(t) + \sqrt{1-\rho^2}dB_2(t):$$

We then immediately have

$$dX_1dX_1 = \sigma_1^2 Z_1 dt; \quad dX_1dX_2 = \rho\sigma_1\sigma_2\sqrt{Z_1Z_2}dt; \quad dX_2dX_2 = \sigma_2^2 Z_2 dt:$$

We know that under the optimal sharing rule, the agents will implement the effort level that the principal anticipates and they are constants over time. Therefore, (9:1) and (9:2) can be rewritten as

$$dX_1 = (f(a_{11})X_1(t) + f(a_{12})X_2(t) + \mu_1)dt + \sigma_1\sqrt{Z_1(t)}dB_1(t); \tag{9.3}$$

$$dX_2 = (g(a_{21})X_1(t) + g(a_{22})X_2(t) + \mu_2)dt + \sigma_2\sqrt{Z_2(t)}dB_3(t); \tag{9.4}$$

The value at time $t \le \infty$ of a contract executed from time 0, to the agent, is

$$E\left[-\exp\left\{-r\left(\int_t^\infty S_i(X_i(s),X_j(s))ds - \int_t^\infty c_i(a_{ij},b_{ij},a_{ji},b_{ji})ds\right)\right\} \Big| F(t)\right]$$

Since the coefficients in (9.3) and (9.4) are not time dependent but only the expected maturity $\tau \equiv \infty - t$, and the process pair (X_1, X_2) is of Markov, there exists a function $H(X_1, X_2, \infty - t)$ such that.

$$H(X_1, X_2, \infty - t) = E\left[-\exp\left\{-r\left(\int_t^\infty S_i(X_i(s),X_j(s))ds - \int_t^\infty c_i(a_{ij},b_{ij},a_{ji},b_{ji})ds\right)\right\} \Big| F(t)\right]$$

where $\{F(t)\}$ is a filtration in which $\{X_1\}$ and $\{X_2\}$ are adapted.

By the tower property, we have that

$$-\exp\left\{-r\left(\int_0^t S_i(X_i(s),X_j(s))ds - \int_0^t c_i(a_{ij},b_{ij},a_{ji},b_{ji})ds\right)\right\} H(X_1, X_2, \infty - t)$$

is a martingale. Likewise,

$$-\exp\left\{-r\left(\int_0^t S_i(X_i(s),X_j(s))ds\right)\right\} M(X_1, X_2, \infty - t)$$

is also a martingale where

$$M(X_1, X_2, \infty - t) \equiv -\exp\left\{-r\left(\int_t^\infty S_i(X_i(s),X_j(s))ds\right)\right\}.$$

Therefore, we have

$$d\left(-\exp\left\{-r\int_0^t S_i(X_i(s),X_j(s))ds\right\}M(X_1,X_2,1-t)\right)$$

$$= r\exp\left\{-r\int_0^t S_i(X_i(s),X_j(s))ds\right\}[X_1Mdt - M_{X_1}dX_1 - M_{X_2}dX_2 + M_\tau dt$$

$$-\frac{1}{2}rM_{X_1X_1}(dX_1)^2 - \frac{1}{2}rM_{X_1X_2}dX_1dX_2 - \frac{1}{2}rM_{X_2X_2}(dX_2)^2]$$

$$= r\exp\left\{-r\int_0^t S_i(X_i(s),X_j(s))ds\right\}[(X_1M - (f(a_{11}(t))X_1(t) + f(a_{12}(t))X_2(t)$$

$$+\mu_1)M_{X_1} - (g(a_{21}(t))X_1(t) + g(a_{22}(t))X_2(t) + \mu_2)M_{X_2} + M_\tau$$

$$-\frac{1}{2}r\sigma_1^2(h(b_{11}(t))X_1(t) + h(b_{12}(t))X_2(t) + \varpi)M_{X_1X_1} - r\rho\sigma_1\sigma_2\sqrt{Z_1Z_2}M_{X_1X_2}$$

$$-\frac{1}{2}r\sigma_2^2(l(b_{21}(t))X_1(t) + l(b_{22}(t))X_2(t) + \varpi)M_{X_2X_2})dt + \sigma_1\sqrt{Z_1}M_{X_1}dB_1$$

$$+\sigma_2\sqrt{Z_2}M_{X_2}dB_3]$$

and

$$0 = X_1M - (f(a_{11}(t))X_1(t) + f(a_{12}(t))X_2(t) + \mu_1)M_{X_1} - (g(a_{21}(t))X_1(t) + M_\tau \tag{9.5}$$

$$+g(a_{22}(t))X_2(t) + \mu_2)M_{X_2} - \frac{1}{2}r\sigma_1^2(h(b_{11}(t))X_1(t) + h(b_{12}(t))X_2(t) + \varpi)M_{X_1X_1}$$

$$-\sqrt{h(b_{11}(t))X_1(t) + h(b_{12}(t))X_2(t) + \varpi}\sqrt{l(b_{21}(t))X_1(t) + l(b_{22}(t))X_2(t) + \varpi}$$

$$\times r\rho\sigma_1\sigma_2M_{X_1X_2} - \frac{1}{2}r\sigma_2^2(l(b_{21}(t))X_1(t) + l(b_{22}(t))X_2(t) + \varpi)M_{X_2X_2}.$$

Our premise for the solution is of the form

$$M(X_1,X_2,1-t) = -\exp\{-r(X_1^2C_1(\tau) + X_2^2C_2(\tau) + X_1C_3(\tau) + X_2C_4(\tau) + A(\tau))], \tag{9.6}$$

The initial conditions must be incentive compatible, that is,

$$C_1(0) = C_2(0) = C_3(0) = C_4(0) = A(0) = 0, \tag{9.7}$$

as $\tau = 0$ corresponds to $t = 1$.

In order to find $C_1(\tau), C_2(\tau), C_3(\tau), C_4(\tau)$, and $A(\tau)$ for $\tau > 0$, first we find the derivatives for $M(\cdot)$ as follows

$$M_\tau(\cdot) = -r(X_1^2 C_1'(\tau) + X_2^2 C_2'(\tau) + X_1 C_3'(\tau) + X_2 C_4'(\tau) + A'(\tau))M(\cdot), \tag{9.8}$$

$$M_{X_1}(\cdot) = -r(2X_1 C_1(\tau) + C_3(\tau))M(\cdot), \tag{9.9}$$

$$M_{X_2}(\cdot) = -r(2X_2 C_2(\tau) + C_4(\tau))M(\cdot), \tag{9.10}$$

$$M_{X_1X_1}(\cdot) = 2r^2 C_1(\tau)(2X_1 C_1(\tau) + C_3(\tau))M(\cdot), \tag{9.11}$$

$$M_{X_2X_2}(\cdot) = 2r^2 C_2(\tau)(2X_2 C_2(\tau) + C_4(\tau))M(\cdot), \tag{9.12}$$

$$M_{X_1X_2}(\cdot) = r^2(2X_2 C_2(\tau) + C_4(\tau))(2X_1 C_1(\tau) + C_3(\tau))M(\cdot). \tag{9.13}$$

Therefore (9.5) can be rewritten into

$$
\begin{aligned}
0 \;=\; & M(\cdot)\{X_1 - r(X_1^2C_1'(\tau) + X_2^2C_2'(\tau) + X_1C_3'(\tau) + X_2C_4'(\tau) + A'(\tau)) \qquad (9.14)\\
& + r[f(a_{11}(t))X_1(t) + f(a_{12}(t))X_2(t) + \mu_1](2X_1C_1(\tau) + C_3(\tau))\\
& + r[(g(a_{21}(t))X_1(t) + g(a_{22}(t))X_2(t) + \mu_2)(2X_2C_2(\tau) + C_4(\tau))\\
& - \frac{1}{2}r^{\zeta}\sigma_1^2(h(b_{11}(t))X_1(t) + h(b_{12}(t))X_2(t) + \varpi)2C_1(\tau)(2X_1C_1(\tau) + C_3(\tau))\\
& - r^{\zeta}\rho\sigma_1\sigma_2\sqrt{h_{11}X_1 + h_{12}X_2 + \varpi}\sqrt{l_{21}X_1 + l_{22}X_2 + \varpi}\\
& \times (2X_2C_2(\tau) + C_4(\tau))(2X_1C_1(\tau) + C_3(\tau))\\
& - \frac{1}{2}r^{\zeta}\sigma_2^2[l(b_{21}(t))X_1(t) + l(b_{22}(t))X_2(t) + \varpi)2C_2(\tau)(2X_2C_2(\tau) + C_4(\tau)]\}.
\end{aligned}
$$

Without loss of generality, let f_{11}; f_{12}; g_{11}; g_{12}; h_{11}; h_{12}; l_{11}; l_{12} denote $f(a_{11}(t))$; $f(a_{12}(t))$; $g(a_{21}(t))$; $g(a_{22}(t))$; $h(b_{11}(t))$; $h(b_{12}(t))$; $l(b_{21}(t)$; $l(b_{22}(t))$; respectively. Then we can further rewrite (9:14) into

$$
\begin{aligned}
0 \;=\;& X_1 M(\cdot)\{1 + r f_{11}C_3 - rC_3' + 2r\mu_1 C_1 + r g_{11}C_4 + 2r\mu_2 C_2 - r^3\sigma_1^2 h_{11}C_1C_3 \\
& - 2r^3\sigma_1^2 \varpi C_1^2 - 2r^3\rho\sigma_1\sigma_2 \sqrt{h_{11}X_1 + h_{12}X_2 + \varpi}\sqrt{l_{21}X_1 + l_{22}X_2 + \varpi}\, C_1C_4 \\
& - r^3\sigma_2^2 l_{11}C_2C_4\} + rX_2 M(\cdot)\{-C_4' + f_{12}C_3 + g_{22}C_4 + 2\mu_2 C_2 - r^2\sigma_1^2 h_{12}C_1C_3 \\
& - 2r^2\rho\sigma_1\sigma_2 \sqrt{h_{11}X_1 + h_{12}X_2 + \varpi}\sqrt{l_{21}X_1 + l_{22}X_2 + \varpi}\, C_2C_3 - 2r^2\sigma_2^2 C_2^2 \varpi \\
& - r^2\sigma_2^2 C_4C_2 l_{22}\} + rX_1^2 M(\cdot)\{-C_1' + 2f_{11}C_1 - 2r^2\sigma_1^2 h_{11}C_1^2\} \\
& + rX_2^2 M(\cdot)\{-C_2' + 2g_{22}C_2 - 2r^2\sigma_2^2 l_{22}C_2^2\} + rX_1X_2 M(\cdot)\{2f_{12}C_1 + g_{11}C_2 \\
& - 2r^2\sigma_1^2 h_{22}C_1^2 - 4r^2\rho\sigma_1\sigma_2 \sqrt{h_{11}X_1 + h_{12}X_2 + \varpi}\sqrt{l_{21}X_1 + l_{22}X_2 + \varpi}\, C_1C_2 \\
& - 2r^2\sigma_2^2 l_{21}C_2^2\} + rM(\cdot)\{-A' + \mu_1 C_3 + \mu_2 C_4 - r^2\sigma_1^2 C_1C_3\varpi \\
& - r^2\rho\sigma_1\sigma_2 \sqrt{h_{11}X_1 + h_{12}X_2 + \varpi}\sqrt{l_{21}X_1 + l_{22}X_2 + \varpi}\, C_3C_4 \\
& - r^2\sigma_2^2 C_2C_4\varpi\}.
\end{aligned}
\tag{9.15}
$$

We then from (9:15) have five equations:

$$C_1' = 2f_{11}C_1 - 2r^2\sigma_1^2 h_{11}C_1^2; \tag{9.16}$$

$$C_2' = 2g_{22}C_2 - 2r^2\sigma_2^2 l_{22}C_2^2; \tag{9.17}$$

$$
\begin{aligned}
C_3' \;=\;& r^{-1} + f_{11}C_3 + 2\mu_1 C_1 + g_{11}C_4 + 2\mu_2 C_2 - r^2\sigma_1^2 h_{11}C_1C_3 \\
& - 2r^2\sigma_1^2 \varpi C_1^2 - 2r^2\rho\sigma_1\sigma_2 \sqrt{h_{11}X_1 + h_{12}X_2 + \varpi}\sqrt{l_{21}X_1 + l_{22}X_2 + \varpi}\, C_1C_4 \\
& - r^2\sigma_2^2 l_{11}C_2C_4;
\end{aligned}
\tag{9.18}
$$

$$
\begin{aligned}
C_4' = {} & f_{12}C_3 + g_{22}C_4 + 2\mu_2 C_2 - r^2\sigma_1^2 h_{12}C_1C_3 - r^2\sigma_2^2 C_4C_2 l_{22} \\
& - 2r^2\rho\sigma_1\sigma_2 \sqrt{h_{11}X_1 + h_{12}X_2 + \varpi}\sqrt{l_{21}X_1 + l_{22}X_2 + \varpi}\, C_2C_3 - 2r^2\sigma_2^2 C_2^2 \varpi;
\end{aligned}
\tag{9.19}
$$

$$
\begin{aligned}
& 2r^2\rho\sigma_1\sigma_2 \sqrt{h_{11}X_1 + h_{12}X_2 + \varpi}\sqrt{l_{21}X_1 + l_{22}X_2 + \varpi}\, C_1C_2 \\
& = f_{12}C_1 + g_{11}C_2 - r^2\sigma_1^2 h_{22}C_1^2 - r^2\sigma_2^2 l_{21}C_2^2.
\end{aligned}
\tag{9.20}
$$

$$
\begin{aligned}
A' = {} & \mu_1 C_3 + \mu_2 C_4 - r^2\sigma_1^2 C_1C_3\varpi - r^2\sigma_2^2 C_2C_4\varpi \\
& - r^2\rho\sigma_1\sigma_2 \sqrt{h_{11}X_1 + h_{12}X_2 + \varpi}\sqrt{l_{21}X_1 + l_{22}X_2 + \varpi}\, C_3C_4;
\end{aligned}
\tag{9.21}
$$

Combined with initial conditions (9.7); we solve (9.16) to (9.20) simultaneously, and then integrate (9.21) to obtain $A(\tau)$.

We propose a solution that is compatible with the differential equation system (9.16) to (9.21) in the following proposition. This solution is analogous to its' single agent counterpart.

Proposition 9 Suppose both drift and diffusion terms are control variables to the agents; agents' disutility of effort $c(\cdot)$ is convex in drift enhancing efforts and diffusion reducing efforts; and let $(f_{11}^*, f_{12}^*, h_{11}^*, h_{12}^*)$ and $(g_{21}^*, g_{22}^*, l_{21}^*, l_{22}^*)$ be the control vectors of agent 1 and 2, respectively, that maximize the principal's expected utility and also incentive compatible. Then a linear optimal compensation scheme to agent 1 is given by

$$S^{*} = S(0; X_1; X_2) + c_1(f^{*}_{11}; f^{*}_{12}; h^{*}_{11}; h^{*}_{12}) + \frac{c^{*}_{f_{11}}}{f'(a^{*}_{11})}[X_1 - f^{*}_{11} - g^{*}_{21} - \mu_1] + \frac{c^{*}_{f_{12}}}{f'(a^{*}_{12})}[X_2 - f^{*}_{12} - g^{*}_{22} - \mu_2] + \frac{r}{2}\left[\left(\frac{c^{*}_{f_{11}}}{f'(a_{11})}\right)^2 \sigma_1^2 Z^{*}_1 + \left(\frac{c^{*}_{f_{12}}}{f'(a^{*}_{12})}\right)^2 \sigma_2^2 Z^{*}_2\right]: \quad (9.22)$$

The scheme to agent 2 is similar.

This optimal linear sharing rule can be interpreted as follows: the first two terms in (9:22) remunerate the agent a certainty opportunity cost of working in a team for the effort exerted in the unit time period; the third term provides the agent with appropriate incentives for value increasing activities in his own part of the work however it is net of the expectation, note the mathematical expectation for this term is 0; the fourth term provides proper incentives for the agent to exert effort in helping his team mate in project NPV mean improving activities, also net of the expectation; the last term is a risk premium paid to the agent for participating two risky projects with zero means. Note if the agent is risk neutral, in other words, $r = 0$; then the last term drops.

10 Extension and Discussion (to be extended)

In this section we will present a number of results in the forms of lemmas and corollaries of Proposition 2 and sequels of martingale analysis in the previous section. The content is categorized into the following:

1. Comparison between some other non-linear schemes and linear scheme. This is complicate by having di®usion as an extra control variable. We will show that although some nonlinear schemes can slightly outperform the linear scheme, the unnecessary risk added by nonlinear contracts will reduce their credential substantially therefore making the linear or nearly linear scheme thrive in practice.

2. Comparison between competition oriented and attribution oriented linear schemes. Previous research emphasizes the competition amongst the agents within an agency where yardstick or tournament like schemes are proclaimed to be e±-cient. Yet yardstick competition and tournaments have their virtues when the information is coarse and existing literature has very well exploited it. However our suspicion aroused when we do not ¯nd such extreme schemes that could severely penalize an agent given peer performance °uctuates. In a fast paced techno-metabolic and changing business world nowadays, sharp outliers of business performance either individual personal or ¯rm level could be very possibly happening, however not all of the team mates are laid-o®, nor are the companies going bankruptcy. In fact, small sized teams emphasizing cooperation, synchronization and synergy are favoured by many industries. Heterogeneity among the agents is recognized easily by the principal but instead of aligning the heterogeneities, ¯rms prefer to best utilize the idiosyncracy of the employ-

ees by assigning them di®erent jobs to bring their comparative advantages into play. Therefore, in our modeling, even if the sensitivity of the scheme is similar to that in other analyses, our interpretation di®ers. Furthermore, since cooperation and synergy are encouraged here, our policy suggestion on job allocation would be di®erent.

3. Comparative statics. The slope of the sensitivity of the optimal incentive scheme is the major concern. We calculate its' derivatives on own and cross control variables, its' derivatives on agents' idiosyncratic parameters or functions. Some illustrative and intuitive conjectures are tested. The optimal linear incentive scheme proposed can instruct the agents to best help their counterparts reciprocally.

4. Finally, grouping is once again a typical issue we will visit. The fundamental question here is, given the production technology is additive, should a team constituted by homogeneous or heterogeneous agents?

11 Concluding Remarks

This section will be based on the results in section 3, 4 and 5.

Our analysis shows that either individual output is observable or not, linear incentive schemes can always outperform other schemes or approximate optimal schemes.

If individual performance is observable, we discuss some interesting topics including best relative performance comparison schemes, own and cross e®ort e®ect on incentive sensitivity in optimal linear schemes, teaming etc. We ¯nd that the main theme in a team where the team members and the owner are all risk averse, cooperation is fundamental to the team's prosperity.

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A Game Theory Approach on Managerial Teams: Career Concerns and Informativeness Revisited

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[7] This thesis was motivated by a PhD reading course on dynamic modelling that I took with Prof. LeBlanc.

Abstract

In this essay, we show that in an Alchian-Demsetz firm, even in a finite period game setting, cooperative effort level amongst team members can be achieved when the output depends not only agents' effort but also their ability and that may explain why teams merit to sustain.

Résumé

Dans cet essai, nous montrons cela dans une entreprise Alchian-Demsetz, même dans un cadre du jeu de la période fini, le niveau de l'effort coopératif parmi équipiers peut être accompli quand la production dépend l'effort d'agents pas seul mais aussi leur capacité et cela peut expliquer pourquoi mérite des équipes soutenir.

12 Introduction

The study of teams can be traced back in Alchian and Demsetz (1972), where in order to eliminate or minimize free-riding, monitoring is essential and this role is performed by a monitor who turns to a residual claimer. Alchian and Demsetz (1972) show how team production results in organizations commonly called as the \classic ¯rm". The point they emphasize is that in team production, the jobs done by workers are not perfectly separable. It is hence not feasible to compensate workers based on their marginal contributions. The workers therefore have an incentive to shirk, to free-ride on other team members' e®ort. The well known reason is that the cost of shirking is born by the entire team. The result is that market contracting with individuals is not possible, rather, this free-riding problem is overcome only by the \classic ¯rm" in which worker are paid by wage and a principal supervises or monitors the workers. It is then optimal to let the principal be the residual claimant. By this arrangement, the incentive compatibility of the residual claimant is satis¯ed and team members' free-riding is attenuated as the monitor is assigned with the authority to dismiss any slackers. They are right by awaring the free-riding but it seems subjective to conclude that monitoring the agents by the principal is the only remedy.

Holmstrom (1982) o®ers alternative reasons for a residual claimant to be exist to prevent the team from falling apart by free-riding, is that it is the credible group penalty that induces efficient effort level but not the monitoring per se. Holmstrom shows that collective punishments and rewards may provide the necessary incentives for workers to exert the desired level of effort without the need of a monitor. The feature is that the penalties or rewards required to ensure efficient production are not budget-balancing. Thus the role of the principal of a firm or the owner of it is to break the budget-balancing constraint rather than monitoring. The conclusion is that the capitalistic firms in which the owners do not provide labor services have advantages over proprietorship because of the owners' ability to finance budget-breaking schemes. It offers a deeper understanding on the difference between a team and a commune. Moreover, it explains why the residual claimant generally does not involve in supervision while the supervisor who fulfills the monitoring role is commonly salaried. Rasmusen (1987), McAfee and McMillan (1991) follow this approach. However, the organizational design suggestions and pay structure recommendations are sometimes very weird and rarely used. For instance, scapegoat penalty in Rasmusen (1987) or large lump-sum upfront commitment fee payment in McAfee and McMillan (1991) were almost never seen in practice[8].

In neither of these approaches, mutual monitoring among the agents was addressed

[8] The main reason here that we understood is that the credit market is highly rationed so that despite an individual's risk attitude, persons who do not pose entrepreneurship can not acquire capital to finance himself upfront.

and discussed. In performance comparison literature[9], mutual monitoring is considered but the mechanism proposed to correct incentives in teams is to make each team member's pay not only depend on his own output but also on his peer's output if they are correlated. This is suspicious for two reasons, ¯rst, the individual contribution in a managerial team cannot usually be distinguished from an outsider if she does not involve in the production process; second, teams emphasize cooperation and synchronization instead of harsh competition, at least it appears to be. Moreover, tournament causes collusion and hence not coalition proof.

Amihai, Glazer and B. Segendor® (2001) considers reputational e®ect in teams and they conclude that if the production must be carried out by a two person team, it is better to have a low ability partner if one cares about his reputation. Their model is simple and illustrative but self-formed partnerships only take a small proportion of the whole lot of the ¯rms. In most cases, the managerial team is not self-selected, instead appointed by the Board of Directors and the agents are obligated to work together as a corporate norm.

The technical setup of Breton, Michele, P. St-Amour and D. Vencatachellum (2001) is mostly related to ours. In Breton et al (2001), the wages of the agents

9

1. See Meyer, Margeret (1994), Meyer, Margeret and J. Vickers (1997) for details.

are directly reputation-based in a dynamic stochastic framework. Three organizational forms, compulsory individual performance evaluation, compulsory teams, and elective teams are studied. Also the authors di®erentiate common shocks and idiosyncratic shocks in production. Welfare analysis is also presented. In some sense, Breton et al (2001) is comprehensive, especially when teams are formed endogenously and inter-generational grouping is available given de facto reputation for each candidate is common knowledge. For instance, in academic research in a university, no scheme is levied either to foster or impair team formation. However, in Breton et al (2001), all agents are risk neutral and the compensation is a random binary variable. These assumptions make the model di®erent from the facts documented or commonly seen and we argue risk aversion is important because team provides insurance e®ect against correlated shocks although our model is not solely based on risk aversion.

Relationship between managers or workers, in real world, appear much cooperative than portrayed by the existing theory on incentives, emphasizing competition among the agents or candidates. Whether this cooperative behaviours are induced or instructed by the contract provided by the principal or inherited from noncooperative, competing nature of the parallel positions, the answer is embedded in our model with moral hazard on top of adverse selection.

Secondly, much of the existing literature on incentives models the principal-agent relationship in a static setting, neglecting the repeated nature of most employment relationships, procurement and partnership. The relationship either between the owner of the business and the manager of it, or, the colleagues themselves, are long term and repeated rather than one-shot.

Furthermore, adverse selection and moral hazard are rarely considered together, in fact, a firm's output is a function of management's abilities and effort exerted, not one of those alone. Therefore, both hidden information and hidden actions taken by the managers as well as the interaction between the two will affect the aggregate output.

Finally, not only explicit incentives are at work but also numerous types of implicit incentives as well. Therefore a seemingly boring plain vanilla organizational form and compensation scheme is not as inefficient as one perceives. The low powered pay scheme and a simple multiagent firm may have the virtues to approximate first best outcome with implicit incentives in unwritten contract at work. Careless policy suggestions that neglect it would very possibly undermine the optimal or suboptimal incentive already provided. With these considerations, our model offers new insights in understanding the multiagent incentive problem.

This essay addresses the above four issues ensemble by focusing the mutual observability between peers and the repeated nature of working relationships in firms. These four elements: A. implicit side-contract behind the cooperative outlook of a team; B. repeated environment; C. combination of hidden action and hidden information; D. the advantage of passivity of the principal, combined together, enable us to explain how cooperative behaviors among agents can be sustained by a self-enforcing mechanism. Our model is consistent with many stylized features of the team-oriented pro¯t organizations observed. We consider this is largely due to the combined assumptions we imposed. We will ¯nd that reputational or career concerns play a major role in the implementation of optimal compensation scheme.

To this end, we consider a two period model of managerial teams where only the total output is observable to the principal but individual output for each period is known to the team members by the end of the period. This information structure essentially captures the nature of mutual monitoring and mutual observability in a small size team. Output is additive and is aggregate of e®ort, ability and random shocks. The team members have the option to quit and work on their own at the beginning of the second period if the information provided through output level indicates the her colleague shirked and/or is low ability type.

Unlike the usual two period model in which the subgame perfect Nash equilibrium outcome yields an ine± cient output level in the second period and free riding in the ¯rst period, these will not occur in our model because learning of the agents' ability and quitting as a credible treat make ¯rst period Pareto output level possible and teaming more attractive than separated because teaming provides more insurance e®ect relative to individual case.

We invoke the career concern model framework[10], but our model di®ers from others in the sense that career concerns here are for peers reciprocally, not for the principal who does not participate in the production but is just a residual claimant.

Our ¯ndings are summarized as follows.

First, we ¯nd group output based compensation scheme could approximate ¯rst best in earlier period. We eliminate technical synergy in teams to isolate the incentive in career concerns. By doing this, we are hoping to provide other robust reasons for teams to survive, treating teams just as aggregations of individual production. Many researchers realized that in a repeated setting, a competitive scheme, such as tournament, will not be optimal as coalition even without side-payment will arise. In our case, since individual output can not be observed, tournament is naturally ruled out, but similar schemes, such as a penalty introduced when output falls below some critical level, will not be optimal either[11]. Indeed, a moderate, passive scheme in which strategic collusion between agents will not be bene¯cial and hence muted, will do good. More importantly, joint performance evaluation in a repeated environment creates not only an incentive for agents to monitor each other but, with abilities attributed to the output, career concerns will force agents to work harder in their

[10] Bengt Holstrom (1999), Dewatripont et al (1999), Gibbons and Murphy (1992) etc.

[11] We say it is similar becuase agents can communicate and form a coalition where side-payment can be arranged and agents may shirk alternatively over time. Therefore tournament and penalty are similar in some sense.

earlier careers than that of a one-shot relationship.

Second, likewise in the single agent career concern models, we ¯nd that incentive provided by the optimal contract between the principal and the agents should increase over time to balance out the implicit incentives restored by reputational concerns between team members. This is a natural corollary from the ¯rst ¯nding. However, further comparisons between team setting and a single agent setting give us some other results. Our ¯ndings are: 1, both ¯rst and second period incentive power in team performance evaluation is lower than that of individual performance evaluation if output °uctuations are positively correlated; 2, team setting provides insurance e®ect compared with single agent case, and this is Pareto e± cient as the trade-o® of incentive and insurance between the principal and the agent is improved by adding another agent; 3, team equilibrium piece rate evolves faster than that of a single agent case. In a coarse language, one can say that team made everything modest except the incentive evolution speed.

Third, our ¯ndings imply that decentralizing the authority in a certain degree could be bene¯cial to the ¯rm. In Aghion and Tirole (1997), decentralization could be good if it facilitates the agent's investment in acquiring information about decision alternative if the principal and agent's interests are su± ciently congruent. We consider that decentralization of a broader authority could be good if their interests are su± ciently congruent. Here the problem is two-fold, ¯rst, how su± ciently congruent the interests are and do they diverge with decentralization, second, will moral hazard which is harmful for the firm's goal arise and get severe with the decentralization. The optimal level of decentralization can be then characterized for future research with these considerations.

Our innovation is powered by the combination of the four major considerations above mentioned. We believe that only a careful study with a more realistic and enriched environment can lead sensible policy recommendations on optimal compensation scheme design, grouping, multitask design, and best allocation of the authority. We do not provide bizarre or careless policy suggestions, rather, try to provide an alternative explanation for why an Alchian-Demsetz[12] firm would triumph in the real world and why the pay-to-performance ratio is low in explicit contracts. Nevertheless, our approach can be extended into a variety of directions including grouping and authority allocation and policy suggestions can be carried out in further research.

13 The Model

Team member i's contribution to output at period t is,

$$y_{it} = \eta_i + a_{it} + \xi_{it}; \qquad i = 1; 2 \tag{13.1}$$

[12]However the firm in our model differs slightly from Alchian-Demsetz (1972) in the sense that the role of monitoring the effort is replaced by peers instead of the principal.

where η_i is the agent's inborn entrepreneurial ability, a_{it} is the i's effort exerted in period t, ε_t is a common shock to all the team members at period t and ξ_{it} is the individual shock at t. Team members' utility is negative exponential, ie.,

$$u_i(\cdot) = -\exp\{-r\sum_{t=1}^{2}\delta^{t-1}[w_{it} - g(a_{it})]\}; \tag{13.2}$$

where w_{it} is the wealth of agent i at t and $g(a_{it})$ is the disutility for effort. The game is for two periods and we assume linear incentive scheme. The total expected utility for agent 1 is then

$$\begin{aligned} E[u_1(\cdot)] &= -E[\exp\{-r\{\alpha_1 + \beta_1 Y_1 - g(a_{11})] \\ &\quad - r\delta[\alpha_2 + \beta_2 Y_2 - g(a_{12})]\}]; \end{aligned} \tag{13.3}$$

[13]if no one quits in the second period. And the total output in each period is

$$Y_1 = y_{11} + y_{21} = \eta_1 + \eta_2 + a_{11} + a_{21} + \xi_{11} + \xi_{21}$$

$$Y_2 = y_{12} + y_{22} = \eta_1 + \eta_2 + a_{12} + a_{22} + \xi_{12} + \xi_{22};$$

[13]A more reasonable expression would be $E[u_i(\cdot)] = -E[\exp\{-r\{\alpha_1 + \beta_1 Y_1 - g(a_1)\}\}] + \delta\{-E[\exp\{-r\{\alpha_2 + \beta_2 Y_2 - g(a_2)\}\}]\}$ for single agent case. However the functional form of utility in our paper does not change our qualitative results as simple algebra shows that $g'(a_1) = \beta_1 + \delta\frac{\partial\alpha_2}{\partial a_1}$ in the first order condition of our setting where $g'(a_1) = \beta_1 + \delta\frac{u_2}{u_1}\frac{\partial\alpha_2}{\partial a_1}$ in the alternative setting mentioned above, where $u_1 = -E[\exp\{-r\{\alpha_1 + \beta_1 Y_1 - g(a_1)\}]$ and $u_2 = \delta\{-E[\exp\{-r\{\alpha_2 + \beta_2 Y_2 - g(a_2)\}]\}$:

where the prior belief on team members' entrepreneurial abilities follows

$$\eta_i \sim N(m_0; \sigma_0^2); \qquad \text{with correlation } corr(\eta_i; \eta_j) = 0; \text{ for } i \neq j;$$

and random shocks in each period follow

$$\xi_{it} \sim N(0; \sigma_\xi^2); \qquad \text{with correlation } corr(\xi_{it}; \xi_{jt}) = \rho; \text{ for } i \neq j:$$

One very important assumption about the timing on learning of ability is that managers themselves perfectly learn their own ability by the end of first period where it is still unknown to the principal. The agents also know how much effort they exerted, therefore they also perfectly learn random shocks in their individual output in the previous period.

The competitiveness of the managerial market induces the principal (could be shareholders) to make zero expected profits, i.e., $\alpha_t = (\frac{1}{2} - \beta_t)E[Y_t]$: The intercept α and incentive term β are set by the principal but not arbitrarily, in fact, they are endogenous as we will see below. We use backwards induction to find subgame perfect equilibrium. The principal only observes aggregate output by the end of period one and she uses it in learning the aggregate ability; team members will exert effort noncooperatively in the second period and it is common knowledge. Assuming peers will observe individual output by the end of the first period but not the principal, quitting and working alone in the subsequent periods would be a credible punishment to their partner if the partner ever shirks in the first period or his ability is too low.

The principal desires cooperative production levels in the ¯rst period, because our setting of the game is not renegotiation proof, the principal can hold-up some stakes or payments once she observes at least one party quits in the second period. This is realistic because cash °ows in the ¯rm generally have time delays and court may in favorite to the ¯rm owner if one or some of the managers quit voluntarily however the penalty is limited[14].

Furthermore, because production technology is additive and team members are risk averse, working alone will expose too much risk relative to working in a team. Therefore the team members' ¯rst period choice on hidden actions is a trade-o® between free-riding and loss of control plus team insurance e®ect. It is then possible that team members exert e®orts at the level that the principal dictates through the incentive contract.

14 Equilibrium

We use backwards induction to derive the second period e®ort level and incentive contractual form. In the second period, which is also the last period, the team members will exert e®ort noncooperatively and they maximize the payo® for the that period. Because the decision is symmetric, we study the case for agent 1. Agent 1

[14]Generally the loss of control rights and other incumbent bene¯ts will su±ce to make the punishment credible.

maximizes the following when the second period commences and assuming he decides to stay in this team:

$$\max_{a_{12}} -E[\exp\{-r(\alpha_2 + \beta_2(\eta_1 + \eta_2 + a_{12} + a_{22} + \xi_{12} + \xi_{22}) - g(a_{12})|Y_1\}].$$

then the first order condition yields

$$\beta_2 = g'(a_{12}).$$

By the assumption that individual output can be observed to the agents by the end of each period, principal's learning could be only on the aggregate terms, ie.,

$$E\{\eta_1 + \eta_2|Y_1\} \equiv 2m_1 = \frac{2(1+\rho)\sigma_\xi^2 m_0 + \sigma_0^2(Y_1 - \hat{a}_{11} - \hat{a}_{21})}{2(1+\rho)\sigma_\xi^2 + \sigma_0^2}.$$

The conditional variance of $\eta_1 + \eta_2$ is then

$$Var[\eta_1 + \eta_2|Y_1] \equiv \frac{2(1+\rho)\sigma_\xi^2\sigma_0^2}{(1+\rho)\sigma_\xi^2 + \sigma_0^2},$$

Denote $\sigma_1^2 = \frac{2(1+\rho)\sigma_\xi^2\sigma_0^2}{(1+\rho)\sigma_\xi^2+\sigma_0^2}$, the variance of $(\eta_1 + \eta_2 + \xi_{12} + \xi_{22}|Y_1)$ is then

$$\sigma_2^2 \equiv \sigma_1^2 + 2(1+\rho)\sigma_\xi^2,$$

therefore the above maximand can be rewritten as

$$\max_{a_{12}} -E[\exp\{-r(\alpha_2(\beta_2) + \beta_2(\eta_1 + \eta_2 + a_{12} + a_{22} + \xi_{12} + \xi_{22}) - g(a_{12}^*(\beta_2))|Y_1\}]$$

$$= -\exp\{-r(m_1(Y_1, \hat{a}_{11}, \hat{a}_{21}) + \frac{1}{2}\hat{a}_{12}(\beta_2) + \frac{1}{2}\hat{a}_{22}(\beta_2) - g(\hat{a}_{12}(\beta_2)) - \frac{1}{2}r\beta_2^2\sigma_2^2)\}$$

where $\hat{a}_{12}(\beta_2)$ and $\hat{a}_{22}(\beta_2)$ are equilibrium levels and $\hat{a}_{12}(\beta_2) = \hat{a}_{22}(\beta_2)$.

First order condition of the UMP yields

$$\beta_2 = \frac{1}{2 + 2r\sigma_2^2 g''(a_{12})}. \tag{14.1}$$

By the end of first period, the team member's individual contribution to the output is revealed to himself and because production technology is additive, his peer's output is therefore observed as well. While the above analysis gives a seemingly efficient incentive contract for the team in the last period, the actual efficient contract is deduced from a subgame perfect equilibrium effort level through individual learning. By the virtue of individual observability, we have the following for agent 2,

$$\begin{aligned}
&\max_{a_{22}} -E[\exp\{-r(\alpha_2 + \beta_2(\eta_1 + \eta_2 + a_{12} + a_{22} + \xi_{12} + \xi_{22}) - g(a_{22}))|Y_1; y_i; y_j\}] \\
= \; &\max_{a_{22}} -E[\exp\{-r(\frac{1-2\beta_2}{2}E[Y_2|Y_1] + \beta_2(\eta_1 + \eta_2 + a_{12} + a_{22} + \xi_{12} + \xi_{22}) \\
&- g(a_{22}))|Y_1; y_i; y_j\}]
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
&-r\{\frac{1}{2}(\eta_2 + m_3) + E[\beta_2(\eta_1 - m_3)] + \frac{1}{2}(\hat{a}_{12} + \hat{a}_{22}) - g(\hat{a}_{22}) \\
&- \frac{1}{2}r\beta_2^2 var(\eta_1 + \xi_{12} + \xi_{22}|Y_1; y_i; y_j)\}
\end{aligned}$$

where

$$m_3 \equiv E[\eta_1|y_{11}; y_{21}] = \frac{(1-\rho)\sigma_\xi^2 m_0 + \sigma_0^2(y_{11} - \hat{a}_{11})}{(1-\rho)\sigma_\xi^2 + \sigma_0^2}; \tag{14.2}$$

and

$$\sigma_5^2 \equiv var(\eta_1 + \xi_{12} + \xi_{22}|Y_1; y_i; y_j) = \frac{(1-\rho)\sigma_\xi^2\sigma_0^2}{(1-\rho)\sigma_\xi^2 + \sigma_0^2} + 2(1+\rho)\sigma_\xi^2.$$

The first order condition to the certainty equivalent yields

$$\hat{\beta}_2 = \frac{1}{2 + 2r\sigma_5^2 g''(a_{22})}. \tag{14.3}$$

It is easy to show that $\sigma_2^2 > \sigma_5^2$ when $\rho > 0$; $\sigma_2^2 < \sigma_5^2$ when $\rho < 0$; therefore, obviously,

$$\beta_2 < \hat{\beta}_2 \qquad \text{if } \rho > 0.$$

Proposition 10 $\beta_2 < \hat{\beta}_2$ Team incentive is powered partially by mutual observability.

Now the expected utility for the representative agent, agent 1, at the first period perspective is

$$\begin{aligned} &- E[\exp\{-r(\alpha_1 + \beta_1(\eta_1 + \eta_2 + a_{11} + a_{21} + \xi_{11} + \xi_{21}) - g(a_{11})) \\ &- r\delta(\alpha_2(\hat{\beta}_2) + \hat{\beta}_2(\eta_1 + \eta_2 + \hat{a}_{12} + \hat{a}_{22} + \xi_{12} + \xi_{22}) - g(\hat{a}_{12}))\}] \end{aligned} \tag{14.4}$$

where

$$\alpha_2 = \frac{1 - 2\hat{\beta}_2}{2}\left[\frac{(1+\rho)\sigma_\xi^2 m_0 + \sigma_0^2(Y_1 - \hat{a}_{11} - \hat{a}_{21})}{(1+\rho)\sigma_\xi^2 + \sigma_0^2} + \hat{a}_{12} + \hat{a}_{22}\right].$$

Therefore, the first order condition with respect to a_{11} is then adjusted accordingly to the following,

$$g'(a_{11}) = \beta_1 + \delta(1 - 2\hat{\beta}_2)\frac{\sigma_0^2}{2(1+\rho)\sigma_\xi^2 + 2\sigma_0^2}$$

where the second term of RHS depicts the career concern effect. Denote $B_1 \equiv g'(a_{11}) = \beta_1 + \delta(1 - 2\hat{\beta}_2)\frac{\sigma_0^2}{2(1+\rho)\sigma_\xi^2 + 2\sigma_0^2}$.

The principal maximizes total welfare, which is a monotonic transformation of aggregate utility and can be written as the following,

$$
\begin{aligned}
& E[-\exp\{-r\{2m_0 + a_{11}(\beta_1) + a_{21}(\beta_1) - g(a_{11}(\beta_1)) - g(a_{21}(\beta_1)) && (14.5)\\
& + \xi_{11} + \xi_{21}) + \delta(2m_0 + \hat{a}_{12}(\hat{\beta}_2) + \hat{a}_{22}(\hat{\beta}_2) - g(\hat{a}_{12}(\hat{\beta}_2)) - g(\hat{a}_{22}(\hat{\beta}_2)) + \xi_{12} + \xi_{22}\}\}].
\end{aligned}
$$

Let σ_4^2 denote the variance of the expression inside the inner curly braces of (14.5). Then (14.5) can be rewritten as

$$
\begin{aligned}
& -\exp\{-r[2m_0 + a_{11}(\beta_1) + a_{21}(\beta_1) - g(a_{11}(\beta_1)) - g(a_{21}(\beta_1))] && (14.6)\\
& - r\delta(2m_0 + \hat{a}_{12}(\hat{\beta}_2) + \hat{a}_{22}(\hat{\beta}_2) - g(\hat{a}_{12}(\hat{\beta}_2)) - g(\hat{a}_{22}(\hat{\beta}_2))\} \cdot \exp\{\frac{1}{2}r^2\sigma_4^2\}.
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma_4^2 \equiv\ & var[\beta_1(\eta_1 + \eta_2) + \beta_1(\xi_{11} + \xi_{21}) + \delta\frac{(1 - 2\beta_2)}{2} \\
& \times \left(\frac{(1+\rho)\sigma_\xi^2 m_0 + \sigma_0^2(Y_1 - \hat{a}_{11} - \hat{a}_{21})}{(1+\rho)\sigma_\xi^2 + \sigma_0^2} + \hat{a}_{12} + \hat{a}_{22}\right) + 2\beta_2\delta(\xi_{12} + \xi_{22})] \\
& + \delta^2 Cov[\xi_{12}, \xi_{22}] + Cov[\xi_{11}, \xi_{21}].
\end{aligned}
$$

We assume that the random disturbances are not serial correlated, i.e., $Cov[\xi_{i1}, \xi_{i2}] = 0$ and $Cov[\xi_{i1}, \xi_{j1}] = 0$, where $i, j = 1, 2$ and $i \neq j$. Notice also $Y_1 - \hat{a}_{11} - \hat{a}_{21} =$

$\eta_1 + \eta_2 + \xi_{11} + \xi_{21}$; we have

$$\sigma_4^2 \equiv 2\left(2\beta_1 + 2\delta(1-2\beta_2)\frac{\sigma_0^2}{2(1+\rho)\sigma_\xi^2 + 2\sigma_0^2} + 2\delta\beta_2\right)^2(\sigma_\xi^2 + \sigma_0^2)$$
$$- 8\delta\beta_2\sigma_\xi^2 B_1 + 8\rho\beta_2^2\sigma_\xi^2 + 8\rho\sigma_\xi^2 B_1^2$$

where $B_1 = \beta_1 + \delta(1-2\beta_2)\frac{\sigma_0^2}{2(1+\rho)\sigma_\xi^2+2\sigma_0^2}$.

Maximizing (14.6) with respect to β_1 gives the following first order condition

$$a_1'(\beta_1) - g'(a_{11})a_{11}'(\beta_1) + a_{21}'(\beta_1) - g'(a_{21})a_{21}'(\beta_1) - \frac{1}{2}r\frac{\partial\sigma_4^2}{\partial\beta_1} = 0. \qquad (14.7)$$

Note in the first period, the principal and team members have the same knowledge over the team members abilities, therefore the only conflict between the principal and the team members is that the principal maximizes aggregate objectives while team members tend to maximize private objectives causing possible discrepancies in effort levels.

Evaluating the derivative of variance σ_4^2 with respect to β_1; we have

$$\frac{\partial\sigma_4^2}{\partial\beta_1} = 8\left(\beta_1 + \delta(1-2\beta_2)\frac{\sigma_0^2}{2(1+\rho)\sigma_\xi^2 + 2\sigma_0^2} + \beta_2\delta\right)(\sigma_\xi^2 + \sigma_0^2)] \qquad (14.8)$$
$$- 8\beta_2\delta\sigma_\xi^2 + 8\beta_1\rho\sigma_\xi^2 + 4\rho\delta\sigma_\xi^2(1-2\beta_2)\frac{\sigma_\xi^2\sigma_0^2}{(1+\rho)\sigma_\xi^2 + \sigma_0^2}.$$

Note the first order condition to (14.4) with respect to a_{i1} is

$$g'(a_{11}) = g'(a_{21}) = \beta_1 + \delta(1-2\beta_2)\frac{\sigma_0^2}{2(1+\rho)\sigma_\xi^2 + 2\sigma_0^2}. \qquad (14.9)$$

Substituting (14.8) and (14.9) into (14.7) and adjusting incentive power for each team member yield

$$\hat{\beta}_1 = \frac{1}{2+4r[\sigma_\xi^2+\sigma_0^2+\rho\sigma_\xi^2]g''(a_{11})} - \frac{\delta(1-2\beta_2)\sigma_0^2}{2(1+\rho)\sigma_\xi^2+2\sigma_0^2} - \frac{r\delta[\frac{1}{2}\sigma_0^2+2\beta_2\sigma_\xi^2]g''(a_{11})}{1+2r[\sigma_\xi^2+\sigma_0^2+\rho\sigma_\xi^2]g''(a_{11})}. \tag{14.10}$$

Proposition 11 $\hat{\beta}_1 < \hat{\beta}_2$.

Proof. $\beta_2 < \hat{\beta}_2$ is proven earlier. To prove $\hat{\beta}_1 < \hat{\beta}_2$, it suffices to prove $\hat{\beta}_1 < \beta_2$, and it is therefore sufficient to show

$$2\left(\sigma_\xi^2+\sigma_0^2+\rho\sigma_\xi^2\right) \geq \sigma_2^2$$

given the assumption that $g'''(\cdot) \geq 0$. The relationship that $2\left(\sigma_\xi^2+\sigma_0^2+\rho\sigma_\xi^2\right) \geq \frac{(1-\rho)\sigma_\xi^2\sigma_0^2}{(1-\rho)\sigma_\xi^2+\sigma_0^2} + 2(1+\rho)\sigma_\xi^2$ is obvious. ■

15 Comparison with Classical Capitalistic Firms

We refer the term classical capitalism as individual performance evaluation where team based evaluation and any teamwork are abandoned.

Proposition 12 Second period incentive power in team performance evaluation is lower than that of individual performance evaluation if output fluctuations are positively correlated, i.e., $2\hat{\beta}_2 < b_2$ if $\rho > 0$.

Proof. Notice that

$$b_2 = \frac{1}{1 + r\left(\sigma_\xi^2\right) g''(a_2)};$$

then

$$\sigma_\xi^2 < \frac{(1-\rho)\sigma_\xi^2\sigma_0^2}{(1-\rho)\sigma_\xi^2 + \sigma_0^2} + 2(1+\rho)\sigma_\xi^2.$$

■

Lemma 13 First period incentive power in team performance evaluation is lower than that of individual performance evaluation if output fluctuations are positively correlated and team insurance effect is greater than that of the single agent case , $2\hat{b}_1 < b_1$ if $\rho > 0$ and

$$\frac{r\delta[\frac{1}{2}\sigma_0^2 + 2\beta_2\sigma_\xi^2]g''(a_{11})}{1 + 2r[\sigma_\xi^2 + \sigma_0^2 + \rho\sigma_\xi^2]g''(a_{11})} > \frac{r\delta b_2\sigma_0^2 g''(a_1)}{1 + r[\sigma_\xi^2 + \sigma_0^2]g''(a_1)}$$

where $b_2; a_1$ is the incentive term and effort level of the first period in single agent case, respectively.

Proof. Note that in the single agent case,

$$b_1 = \frac{1}{1 + r[\sigma_\xi^2 + \sigma_0^2]g''(a_1)} - \frac{\delta(1-b_2)\sigma_0^2}{\sigma_\xi^2 + \sigma_0^2} - \frac{r\delta b_2\sigma_0^2 g''(a_1)}{1 + r[\sigma_\xi^2 + \sigma_0^2]g''(a_1)}. \qquad (15.1)$$

If $\rho > 0$; then,

$$2\frac{\delta(1-2\beta_2)\sigma_0^2}{2(1+\rho)\sigma_\xi^2 + 2\sigma_0^2} > \frac{\delta(1-b_2)\sigma_0^2}{\sigma_\xi^2 + \sigma_0^2}$$

and $2r[\sigma_\xi^2 + \sigma_0^2 + \rho\sigma_\xi^2] > r[\sigma_\xi^2 + \sigma_0^2]$: Therefore, given that

$$\frac{r\delta[\frac{1}{2}\sigma_0^2 + 2\beta_2\sigma_\xi^2]g''(a_{11})}{1 + 2r[\sigma_\xi^2 + \sigma_0^2 + \rho\sigma_\xi^2]g''(a_{11})} > \frac{r\delta b_2\sigma_0^2 g''(a_1)}{1 + r[\sigma_\xi^2 + \sigma_0^2]g''(a_1)},$$

we have $2\hat{b}_1 < b_1$: ■

Corollary 14 The evolution of equilibrium piece rate in a managerial team is faster than that of a single manage case, i.e.,

$$\frac{\hat{b}_2 - \hat{b}_1}{\hat{b}_2 + \hat{b}_1} > \frac{b_2 - b_1}{b_2 + b_1}:$$

Proof. (Sketch) To prove $\frac{\hat{b}_2 - \hat{b}_1}{\hat{b}_2 + \hat{b}_1} > \frac{b_2 - b_1}{b_2 + b_1}$; it is sufficient to prove $2(\hat{b}_2 - \hat{b}_1) > b_2 - b_1$ as $2(\hat{b}_2 + \hat{b}_1) < b_2 - b_1$: After a few algebraic manipulations, we can further simplify the condition into

$$4\frac{2\sigma_0^2 + (1 - \rho)\sigma_\xi^2}{\sigma_0^2 + (1 - \rho)\sigma_\xi^2} > \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_0^2}$$

and it obviously holds. ■

Corollary 5 is consistent and even strengthens the claim that an efficient managerial incentive contractual arrangement should put more explicit incentives in their earlier careers relative to those in their later careers.

16 Endogenous Cooperative Behaviours

So far we completed the derivation of equilibrium strategy set and gained some intuition about career concern and informativeness in a dynamic managerial team.

However these presentations are still fairly \static". In this section, as we discuss the o®-equilibrium paths, we will make our point more clear.

Consider for a certain player with some particular entrepreneurial ability, he faces 3 strategies in the ¯rst period: to exert e®ort according to what $\hat{b}_1$ implicitly instructs; deviating by shirking in the ¯rst period; deviating by overworking in the ¯rst period. In either of the latter two cases, he is hoping the gain from deviation in the ¯rst period could cover the loss in the second period, in expected terms.

Suppose agent 1 is the one who deviates. We consider the case that agent 1 shirks in the ¯rst period in an attempt for instant bene¯t. Because adverse selection only occurs when the second period begins, agent 2 does not change his learning rule and therefore his estimate on his peer's ability after the ¯rst period is lower than if he would not have shirked. Agent 2 then might ¯nd working alone may bene¯cial than staying in this team with agent 1. Therefore there is an incentive for agent 2 to leave if agent 1 shirks in the ¯rst period. Agent 2 will break up with his partner if

$$CE_{22}|_{stay} > CE_{22}|_{quit};$$

where $CE_{it}|_k$ is agent i's certainty equivalent at period t given event(or decision) k:

Given that any of the two managers decided to break up happened, the principal instantly know that very possibly someone has shirked, therefore, she will not even apply learning on gross output. Hence, there is another contract for individual evaluation in the second period in the case of breaking up. It is obtained by the following maximand,

$$
\begin{aligned}
&\max_{a_{22}} -E[\exp\{-r(\widetilde{\alpha}_2 + \widetilde{\beta}_2(\eta_2 + a_{22} + \xi_{22}) - g(a_{22}) | y_{11} \text{ lower than agent 2 expected}\}] \\
= &\max_{a_{22}} -r\left[m_0 + \widetilde{a}_{22} - g(\widetilde{a}_{22}) - \frac{1}{2} r \widetilde{\beta}_2^2 \sigma_\xi^2\right]:
\end{aligned}
$$

Because

$$\sigma_\xi^2 < \frac{(1-\rho)\sigma_\xi^2\sigma_0^2}{(1-\rho)\sigma_\xi^2 + \sigma_0^2} + 2(1+\rho)\sigma_\xi^2;$$

we easily establish

$$\widetilde{a}_{22} > \widehat{a}_{22};$$

where $\widetilde{a}_{22}$ is the effort level that agent 2 exerts if he decides to work on his own in the second period.

Team member 2's decision on whether to leave to stay with his peer depends on if $CE_{22}|_{stay} \gtreqless CE_{22}|_{quit}$ and it is equivalent to the following,

$$\frac{\eta_2}{2} + \frac{m_3}{2} + \widehat{a}_{22} - g(\widehat{a}_{22}) - \frac{1}{2} r \widehat{\beta}_2^2 \sigma_5^2 - \eta_2 - \widetilde{\alpha}_{22} - g(\widetilde{a}_{22}) - \frac{1}{2} r \widetilde{\beta}_2^2 \sigma_\xi^2 \gtreqless 0: \qquad (16.1)$$

where

$$m_3 \equiv E[\eta_1 | y_{11}; y_{21}] = \frac{(1-\rho)\sigma_\xi^2 m_0 + \sigma_0^2(y_{11} - \widehat{a}_{11})}{(1-\rho)\sigma_\xi^2 + \sigma_0^2}:$$

Therefore depending on agent 1's first period performance, agent 2 learns if his peer's ability is too low to work with in the sense that his partner takes too much of his expected contribution to the output.

The short run expected gain from myopic behaviour(shirking in the first period) for agent 1 is $CE_{11}(\underline{a}_{11}) - CE_{11}(\hat{a}_{11})$ which is equivalent to

$$\frac{\underline{a}_{11}}{2} - \frac{\hat{a}_{11}}{2} - g(\underline{a}_{11}) + g(\hat{a}_{11})$$

where $\underline{a}_{11}$ is the short run optimal private effort level for agent 1 and it is obtained from the following UMP,

$$\max_{a_{11}} -E[\exp\{-r(\alpha_1 + \beta_1(\epsilon_1 + \epsilon_2 + a_{11} + \hat{a}_{21} + \xi_{11} + \xi_{21}) - g(a_{11})\}] \quad (16.2)$$

$$= \max_{a_{11}} -[\exp\{-r(m_0 + \frac{a_{11}}{2} + \frac{\hat{a}_{21}}{2} - g(a_{11}) - \frac{1}{2} r\hat{\beta}_1^2(2(1+\rho)\sigma_\xi^2 + \sigma_0^2)\}]:$$

$\underline{a}_{11}$; the solution to (16:2) ; solves the following,

$$a_{11} = \frac{r\hat{\beta}_1[(1+\rho)\sigma_\xi^2 + \sigma_0^2]}{1 - 2g'(a_{11})}:$$

The expected loss to agent 1 at the first period's perspective is

$$-P(CE_{22}|_{stay} < CE_{22}|_{quit})\, E[\frac{\epsilon_1}{2} + \frac{m_0}{2} + \hat{a}_{12} - g(\hat{a}_{12}) - \frac{1}{2} r\hat{\beta}_2^2\sigma_5^2 - \epsilon_1 - \tilde{a}_{12} - g(\tilde{a}_{12}) - \frac{1}{2} r\tilde{\beta}_2^2\sigma_\xi^2]$$

where $P(CE_{22}|_{stay} < CE_{22}|_{quit})$ is the probability of $CE_{22}|_{stay} < CE_{22}|_{quit}$; and it is obtained by converting ϵ into a standard normal random variable and checking cumulative Z table for (16:1) :

As we discussed in this chapter, cooperative behaviours are endogenous in a dynamic team. This may explain why team as an organizational choice, with apparent drawbacks in the first glance, like exposure to free-riding, still robust in real corporate life. Indeed, we show that team is a device to eliminate free-riding through mutual monitoring by its members and the illusion of potential free-riding in teams is because previous research neglected the dynamic nature of business partnerships. In fact, team could be more efficient than ordinary principal supervision in the existence of mutual observability as people in the play have a more precise vision of the business and his partner's entrepreneurship than outsiders or even some pseudo insiders like the members of the board of directors or major shareholders.

17 Discussion (to be extended)

We consider another case where agents know their own abilities upfront, in other words, adverse selection emerges at the very beginning of the game.

18 Concluding Remarks

Apart from career concerns that we discussed throughout, we emphasize the informativeness and authority allocation within a firm. Our findings are consistent with Kim(1995)'s claim that complete retrieval of information is not always necessary. Individual output, even though available to the principal, may not be desired as if the correlation of the random shocks are not known to the principal, it will levy more risk to the agents which worsen the trade-off between incentive and insurance effect. Therefore, although detailed accounting information may be acquired under a low cost, it may not be efficient as making the pay contingent on coarser information might induce the team members to work harder. Our model also suggests that if the principal's and agents' objectives or interests are sufficiently congruent, it may be beneficial to let the agents have more control rights, in particular, mutual monitoring might be Pareto superior than supervision in a traditional capitalistic firm. Our findings are robust because our model does not rely on synergy or synchronization, nor any other strong conditions imposed. We find that the repeated nature of partnership is the reason why teams sustain.

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A Rent Seeking Theory on CEOs' Compensation

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Abstract

It is shown in this paper that wealth and entrepreneurial ability have positive effects on CEOs' expenditures in obtaining and enlarging the rent if long term (call) stock options are granted with uncertainty. While rent seeking within the firms is becoming increasingly popular, we argue that it is an efficient screening mechanism in authority allocation.

Résumé

Il est montré dans ce papier que les CEOs ont richesse et la capacité d'entrepreneur effectue positive sur leurs dépenses dans obtenir et agrandir la location si long terme (call) stock options sont allouées avec incertitude. Pendant que la location qui cherche dans les entreprises devient de plus en plus populaire, nous discutons que c'est un mécanisme de la sélection effectif dans allocation de l'autorité.

19 Introduction

In contemporary CEO compensation schemes, large sum of stock call options are granted. In Hall and Murphy (2000a), they found 97% of the US companies grant stock options to their CEOs. In ¯scal 1998, the grant-date value of stock options accounted for 40% of total pay for S&P 500 CEOs; stock-options become increasingly important for workers as well: 45 % of the US companies awarded options to their exempt salaried employees in the same year, documented in Hall and Murphy (2000a).

One fact about executive stock options is that almost all options were at-the-money, in 1998, 94% of the option grants to S&P 500 CEOs were at-the-money grants. In 1997, Henry R. Silverman of Cendant Corp. received approximately 257 million US$ in stock-option alone. The total income, including earnings from stock options, for S&P 500 CEOs were quadrupled in real terms between 1970 and 1996 however the median salary was just doubled. Therefore the increase in total pay can be largely attributed to the increase in stock options issued. In the survey by Beer and Katz (2000), senior executives are themselves quite skeptical with regard to the expected positive motivation e®ects of bonuses and stock option plans. We consider this apparent di®erence can be explained by rent seeking theory, in other words, these overwhelming stock options granted to the CEOs are essentially some economic rent to them.

The board of directors, as the representatives of the shareholders, is in charge of deciding on management compensation schemes. These so-called compensation committees only consist of external members. However the process is rather quite informal and the committee's proposal is based on a revision of rati¯cation of compensation plans worked out by the management of the human resource department. Even the advocates of stock option plans concede that the CEOs and other top managers exert some in°uence on both the level and the structure of their pay (Murphy(1990)).

The purpose for granting long term call options is to resolve or at least mitigate CEOs' managerial myopia. Hall and Liebman (1997) empirically examined the adjusted level and sensitivity of the compensation with options considered and it is found that measures of CEO pay-to-performance sensitivity increased by factor of 2 to 7 during the period from early 80's to late 90's. Again as we concerned, the level of direct cpompensation, including salary and stock options, had grown by 209% during the same period of time.

Therefore it is natural to formulate this senario into a rent seeking model and CEOs will have expenditures on in°uence costs and will receive some rent, here in this case, stock options, with uncertainty. Our model di®ers from the existing literature in that we are the ¯rst to study the wealth and ability e®ect on this rent-seeking like behaviours within a ¯rm and hence the optimality of ex ante authority allocation compared with e± ciency arguement and related comparative statics studied in the existing body of the literature.

20 The Model

Let the CEO's end of period wealth be $W_1 = E[w_i(1 + r_f) + sP_1 - x_i + \alpha_i \max(0; P_1 - P_0)]$ when the CEO obtains the rent, i.e., the large block of stock options, and $W_2 = E[w_i(1 + r_f) + sP_1 - x_i]$ otherwise, where w_i is the CEO's initial wealth at time 0, x_i is the monetary influence cost expenditure, in other words, investment in getting or enlarging the rent; s is the cash compensation sensitivity; α_i is the contracts of call options CEO i entitled and r_f is the risk free interest rate. The options are long term call options which can not be exercised for a fixed time after the grant-date, usually 10 to 20 years. We normalize time into unit.

The company's stock price at the end of the period, P_1 is

$$P_1 = P_0 + \eta + \varepsilon; \qquad \varepsilon \sim N(0; \sigma^2) \tag{20.1}$$

where P_0 is the grant-date stock price; η is the CEO's entrpreneurial ability and ε is the random shock.

As the certainty equivalent can be represented through the following way,

$$\begin{aligned} E[u(x + \xi)] &= E\{u(x) + \xi u'(x) + \frac{1}{2}\xi^2 u''(x) + O(\xi^3)\} \qquad (20.2) \\ &= u(x) + \frac{1}{2}\sigma_\xi^2 u''(x) + O(\xi^3); \end{aligned}$$

Then the CEO i's utility maximization problem is,

$$\max E[u(w)] = \pi_i \times E\left[u(w_i(1+r_f) + sP_1 - x_i + \alpha_i \max(0; P_1 - P_0))\right] \quad (20.3)$$
$$+(1-\pi_i) \times E\left[u(w_i(1+r_f) + sP_1 - x_i)\right].$$

where π_i is the probability of getting the block of options and $\pi_i' \equiv \frac{\partial \pi_i}{\partial x_i} > 0$; $\pi_i'' \equiv \frac{\partial^2 \pi_i}{\partial x_i^2} < 0$. Throughout the paper, we assume $u'''' \to 0$.

21 Wealth Effect

We distinguish two kinds of behaviours in rent-obtaining: rent-seeking behaviour which increases the probability that the rent is obtained and rent-augmenting behaviour which increases the size of the rent to be obtained. The wealth effect on rent seeking behaviour is discussed in this section.

21.1 The Option Valuation

We approximate the option value by Black-Scholes formula[16]. The option value is then

$$c = \max(0; P_1 - P_0)) = P_1 \Phi(D_1) - P_0 \exp(-r) \cdot \Phi(D_2). \quad (21.1)$$

[16] The options granted to the CEOs can not be exercised for a very long period, normally 10 to 20 years, to align CEOs' incentives be congruent to the firm's long term objectives. Therefore it is natural to approximate it by European call option valuation.

where $D_1 = \frac{\ln(\frac{P_1}{P_0}) + (r_f + \frac{1}{2}\sigma^2)}{\sigma}$; $D_2 = D_1 - \sigma$ and $\Phi(\cdot)$ is the cumulative distribution function of normal distribution. The asymptotic distribution of the Black-Scholes call-option price estimator $\hat{P}_1$ is

$$\sqrt{n}(\hat{P}_1 - P_1) \overset{a}{\sim} N(0; V_c); \quad V_c = \frac{1}{2}P_1^2\sigma^2\phi^2(D_1)D_2. \tag{21.2}$$

where $\phi(\cdot)$ is the probability density function of normal distribution. Based on some existing results, we have

$$\frac{\partial c}{\partial \theta} = \Phi(d_1); \tag{21.3}$$

and

$$\frac{\partial V_c}{\partial \theta} = -\theta\sigma\phi(d_1)d_2; \tag{21.4}$$

where $d_1 \equiv E[D_1] = \frac{\ln(\frac{\theta}{P_0}) + (r_f + \frac{1}{2}\sigma^2)}{\sigma}$; $d_2 = d_1 - \sigma$.

21.2 The Maximization Problem

The expected utility maximization problem can be rewritten as

$$\begin{aligned} \max_{x_i} E[u(w)] &= \pi_i \times \{u(w_i(1 + r_f) + sE[P_1] - x_i + \alpha_i \max(0; P_1 - P_0)) \\ &\quad + \frac{1}{2}u''(W_1)\left[\alpha_i^2\frac{1}{2}P_1^2\sigma^2\phi^2(d_1) + s^2\sigma^2 + s\alpha_i\Phi(d_1)\sigma^2\right]\} \\ &\quad + (1 - \pi_i) \times \{u(w_i(1 + r_f) + sE[P_1] - x_i) + \frac{1}{2}u''(W_2)s^2\sigma^2\}. \end{aligned} \tag{21.5}$$

The first order condition is

$$\frac{\partial E[u(w)]}{\partial x_i} \equiv 0 = \pi_i' \times \left\{u_1 + \frac{1}{2}u_1'' \left[\frac{\alpha_i^2}{2}P_1^2\sigma^2\phi^2(d_1) + s^2\sigma^2 + s\alpha_i\gamma(d_1)\sigma^2\right] - (u_2 + \frac{1}{2}s^2\sigma^2u_2'')\right\} - Eu'; \tag{21.6}$$

where $u_1 = u(W_1); u_2 = u(W_2); Eu' = \pi_i \times \left\{u_1' + \frac{1}{2}u_1''' \left[\frac{\alpha_i^2}{2}P_1^2\sigma^2\phi^2(d_1) + s^2\sigma^2 + s\alpha_i\gamma(d_1)\sigma^2\right]\right\} + (1 - \pi_i) \times \left\{u_2' + \frac{1}{2}s^2\sigma^2u_2'''\right\}.$

Let $\Sigma_1 = \left[\frac{\alpha_i^2}{2}EP_1^2\sigma^2\phi^2(d_1) + s^2\sigma^2 + s\alpha_i\gamma(d_1)\sigma^2\right].$ By total differentiating (21.6), we have

$$\begin{aligned} 0 &= \{\pi_i''\{u_1 + \frac{1}{2}u_1''\Sigma_1 - (u_2 + \frac{1}{2}s^2\sigma^2u_2'')\} \\ &\quad - 2\pi_i'(u_1' + \frac{1}{2}u_1'''\Sigma_1 - (u_2' + \frac{1}{2}s^2\sigma^2u_2''')) + Eu''\}dx_i \\ &\quad + (1 + r_f)(\pi_i'\{u_1' + \frac{1}{2}u_1'''\Sigma_1 - (u_2' + \frac{1}{2}s^2\sigma^2u_2''')\} - Eu'')dw_i. \end{aligned} \tag{21.7}$$

Rewrite (21.7), we then have

$$\frac{dx_i}{dw_i} = -\frac{(1 + r_f)(\pi_i'\{u_1' + \frac{1}{2}u_1'''\Sigma_1 - (u_2' + \frac{1}{2}s^2\sigma^2u_2''')\} - Eu'')}{\pi_i''\{u_1 + \frac{1}{2}u_1''\Sigma_1 - (u_2 + \frac{1}{2}s^2\sigma^2u_2'')\} - 2\pi_i'(u_1' + \frac{1}{2}u_1'''\Sigma_1 - (u_2' + \frac{1}{2}s^2\sigma^2u_2''')) + Eu''} \tag{21.8}$$

By assumption the CEO's UMP has a determinated solution and the second-order condition is satisfied, the following holds,

$$\begin{aligned} \frac{\partial^2 Eu}{\partial x_i^2} &= \pi_i''\{u_1 + \frac{1}{2}u_1''\Sigma_1 - (u_2 + \frac{1}{2}s^2\sigma^2u_2'')\} - 2\pi_i'(u_1' + \frac{1}{2}u_1'''\Sigma_1 \\ &\quad - (u_2' + \frac{1}{2}s^2\sigma^2u_2''')) + Eu'' \\ &< 0. \end{aligned} \tag{21.9}$$

Therefore

$$\frac{dx_i}{dw_i} \geq 0$$

if and only if $(1 + r_f)(\pi_i' \{u_1' + \frac{1}{2}u_1'''\Sigma_1 - (u_2' + \frac{1}{2}s^2\sigma^2 u_2''')\} - Eu'') \geq 0$:

$$\begin{aligned} &\pi_i' \{u_1' + \frac{1}{2}u_1'''\Sigma_1 - (u_2' + \frac{1}{2}s^2\sigma^2 u_2''')\} - Eu'' \\ = \; &\pi_i'(u_1 - u_2)\frac{u_1' - u_2'}{u_1 - u_2} + \pi_i'\frac{1}{2}(\Sigma_1 u_1''' - s^2\sigma^2 u_2''') + A(W_1)\pi_i u_1' + A(W_2)(1 - \pi_i)u_2' \end{aligned} \tag{21.10}$$

where $A(\cdot)$ is the player's coefficient of absolute risk aversion. We assume that $dA(w)/dw \leq 0$; i.e., the absolute risk aversion is (weakly) decreasing in wealth. Consequently, the players have positive prudence, i.e., $u''' \geq 0$ and $-\frac{u_1'''}{u_1''} \geq A(W_1)$:

Denote $f(u_1; u_2) = -\frac{u_1' - u_2'}{u_1 - u_2} > 0$; then by (21:10) we have

$$\begin{aligned} &\pi_i'(u_1 - u_2)\frac{u_1' - u_2'}{u_1 - u_2} + \pi_i'\frac{1}{2}(\Sigma_1 u_1''' - s^2\sigma^2 u_2''') + A(W_1)\pi_i u_1' + A(W_2)(1 - \pi_i)u_2' \\ \geq \; &f(u_1; u_2)[\pi_i u_1' + (1 - \pi_i)u_2' - \pi_i'(u_1 - u_2)] + \frac{1}{2}\pi_i'\left[\frac{\alpha_i^2}{2}EP_1^2\sigma^2\phi^2(d_1) + s\alpha_i\beta(d_1)\sigma^2\right]u_1''' \end{aligned} \tag{21.11}$$

if

$$A(W_1) \geq A(W_2) + \frac{1}{t}\left(A(W_2) - \frac{u_1' - u_2'}{u_1 - u_2}\right); \tag{21.12}$$

where $t = \frac{\pi_i u_1'}{\pi_i u_1' + (1 - \pi_i)u_2'}$: By mean-value theorem, there is a $\theta \in [W_2; W_1]$ such that $u''(\theta) = \frac{u_1' - u_2'}{u_1 - u_2}$: So we can rewrite condition (21:12) as

$$A(W_1) \geq A(W_2) + \frac{1}{t}(A(W_2) - u''(\theta)): \tag{21.13}$$

We further rearrange the terms in the RHS of (21.11) as

RHS of (21.11) (21.14)

$$= f(u_1;u_2)[Eu' - \pi_i'(u_1 - u_2)] + \frac{1}{2}\pi_i' \left[\frac{\alpha_i^2}{2}P_1^2\sigma^2\phi^2(d_1) + s\alpha_i\Phi(d_1)\sigma^2\right] u_1'''$$

$$+\frac{1}{2}f(u_1;u_2)\pi_i'\left[\Sigma_1 u_1'' - s^2\sigma^2 u_2''\right] + \frac{f(u_1;u_2)}{2}\left(\pi_i\Sigma_1 u_1''' + (1-\pi_i)s^2\sigma^2 u_2'''\right)$$

$$\geq \frac{\pi_i'}{2}(\Sigma_1 - s^2\sigma^2)u_1''' - \frac{f(u_1;u_2)}{2}[\pi_i(\Sigma_1 - s^2\sigma^2) + s^2\sigma^2]u_1''' + \frac{\pi_i' f(u_1;u_2)}{2}(\Sigma_1 - s^2\sigma^2)u_1''$$

We find that a condition for (21.14) ≥ 0 to hold is

$$-\frac{u_1'''}{u_1''} \geq -u''(\theta) - \frac{\pi_i'}{\pi_i}.$$

Therefore under these conditions,

$$\frac{dx_i}{dw_i} \geq 0.$$

We then conclude the following proposition.

Proposition 15 If the rent seekers in the authority competition within a firm exhibit non-increasing absolute risk aversion, $d(-\frac{u''}{u'})\phi\, dw \leq 0$; the relative prudence is greater than the difference between risk aversion and likelihood ratio, i.e., $-\frac{u_1'''}{u_1''} \geq -u''(\theta) - \frac{\pi_i'}{\pi_i}$; and $A(W_1) \geq A(W_2) + \frac{1}{t}(A(W_2) - u''(\theta))$; then the rent seekers will have (weakly) monotonic increasing wealth effect on influent cost expenditures in rent-seeking sense, i.e.,

$$\frac{dx_i}{dw_i}\Big|_{\alpha_i = const} \geq 0.$$

Thus, it is shown that if the CEO's absolute risk aversion is decreasing in wealth, and he is prudent enough in an Knightian entrepreneur's sense, the initial wealth of the CEO will have unambiguously positive effect on his expenditure in rent-seeking activities.

22 Entrepreneurial Ability Effect

Total differentiating the first order condition (21:6) yields

$$\frac{dx_i}{d\theta_i} = -\frac{\pi_i'\{u_1'(s+\alpha_i\Phi(d_1)) + \frac{u_1'''}{2}(s+\alpha_i\Phi(d_1))\Sigma_1 + \frac{u_1''}{2}\frac{\partial\Sigma_1}{\partial\theta} - su_2' - \frac{1}{2}s^3\sigma^2u_2'''\} - \{\pi_i[u_1''(s+\alpha_i\Phi(d_1)) + \frac{u_1'''}{2}\frac{\partial\Sigma_1}{\partial\theta}] + (1-\pi_i)su_2''\}}{\pi_i''\{u_1 + \frac{1}{2}u_1''\Sigma_1 - (u_2 + \frac{1}{2}s^2\sigma^2u_2'')\} - 2\pi_i'(u_1' + \frac{1}{2}u_1'''\Sigma_1 - (u_2' + \frac{1}{2}s^2\sigma^2u_2''')) + Eu''} \tag{22.1}$$

where

$$\frac{\partial\Sigma_1}{\partial\theta} \equiv \frac{\partial\left[\frac{\alpha_i^2}{2}P_1^2\sigma^2\phi^2(d_1) + s^2\sigma^2 + s\alpha_i\Phi(d_1)\sigma^2\right]}{\partial\theta}.$$

Evaluating $\frac{\partial\Sigma_1}{\partial\theta}$; we have

$$\frac{\partial\Sigma_1}{\partial\theta} \equiv \alpha_i^2E[P_1]\sigma^2\phi(d_1)\left(\phi(d_1) + \frac{P_0}{\sigma}\right) + s\alpha_i\phi(d_1)\frac{P_0}{\sigma E[P_1]} \geq 0.$$

Assuming again that the second order condition (21:9) is satisfied, we need to show the numerator of (22.1) is nonnegative in order to have $\frac{dx_i}{d\eta_i} \geq 0$; that is,

$$
\begin{aligned}
&\pi'_i\left\{u'_1(s+\alpha_i\Phi(d_1)) + \frac{u'''_1}{2}(s+\alpha_i\Phi(d_1))\Sigma_1 + \frac{u''_1}{2}\frac{\partial\Sigma_1}{\partial\eta} - su'_2 - \frac{1}{2}s^3\sigma^2u'''_2\right\} \qquad (22.2)\\
&- \left\{\pi_i\left[u''_1(s+\alpha_i\Phi(d_1)) + \frac{u'''_1}{2}\frac{\partial\Sigma_1}{\partial\eta}\right] + (1-\pi_i)su''_2\right\}\\
= \;& \pi'_i(s+\alpha_i\Phi(d_1))u'_1 + \left(\begin{array}{c}\frac{\pi'_i}{2}\left(\alpha_i^2E[P_1]\sigma^2\phi(d_1)\left(\phi(d_1)+\frac{P_0}{\sigma}\right) + s\alpha_i\phi(d_1)\frac{P_0}{\sigma E[P_1]}\right)\\ -\pi_i(s+\alpha_i\Phi(d_1))\end{array}\right)u''_1\\
&+\frac{1}{2}\left(\begin{array}{c}\pi'_i\left[\frac{\alpha_i^2}{2}P_1^2\sigma^2\phi^2(d_1) + s^2\sigma^2 + s\alpha_i\Phi(d_1)\sigma^2\right](s+\alpha_i\Phi(d_1)) - s^3\sigma^2\pi'_i\\ -\pi_i\left(\alpha_i^2E[P_1]\sigma^2\phi(d_1)\left(\phi(d_1)+\frac{P_0}{\sigma}\right) + s\alpha_i\phi(d_1)\frac{P_0}{\sigma E[P_1]}\right)\end{array}\right)u'''_1\\
&-\left[\pi'_isu'_2 + (1-\pi_i)su''_2\right]\\
\geq \;& 0.
\end{aligned}
$$

A set of conditions for (22.2) to hold is

The rent seeker's absolute risk aversion at W_1, $A(W_1)$; satisfies

$$A(W_1) \leq \frac{\pi_i(s+\alpha_i\Phi(d_1))}{\frac{\pi'_i}{2}\left(\alpha_i^2E[P_1]\sigma^2\phi(d_1)\left(\phi(d_1)+\frac{P_0}{\sigma}\right) + s\alpha_i\phi(d_1)\frac{P_0}{\sigma E[P_1]}\right) - \pi_i(s+\alpha_i\Phi(d_1))}; \qquad (22.3)$$

the likelihood ratio of winning the rent, $\frac{\pi'_i}{\pi_i}$; satisfies

$$\frac{\pi'_i}{\pi_i} \geq \frac{\alpha_i^2E[P_1]\sigma^2\phi(d_1)\left(\phi(d_1)+\frac{P_0}{\sigma}\right) + s\alpha_i\phi(d_1)\frac{P_0}{\sigma E[P_1]}}{\left[\frac{\alpha_i^2}{2}P_1^2\sigma^2\phi^2(d_1) + s^2\sigma^2 + s\alpha_i\Phi(d_1)\sigma^2\right](s+\alpha_i\Phi(d_1)) - s^3\sigma^2}; \qquad (22.4)$$

and the rent seeker's absolute risk aversion at W_2, $A(W_2)$ is not less than the hazard rate, i.e.,

$$A(W_2) \geq \frac{\pi'_i}{1-\pi_i}. \qquad (22.5)$$

Therefore we conclude the following theorem,

Theorem 16 If Conditions (22:3), (22:4) and (22:5) are satisfied, then the the CEOs' entrepreneurial abilities will have unambiguous positive effect on rent seeking expenditures, i.e.,

$$\frac{dx_i}{d\eta_i} \geq 0. \tag{22.6}$$

While the condition (22:3) and (22:4) say that the player should be less risk averse when wealthy, condition (22:5) says that he shoud be relatively risk averse when he is less wealthy. It is not hard to find that the three conditions are interrelated, i.e., with a higher likelihood ratio, the player should have a higher risk aversion at W_2 and less risk averse at W_1. The intuition behind it is if the likelihood of winning the rent is relatively low, the more able CEO would spent more on influent costs as the expected marginal benefit from rent seeking expenditures, which is a risky investment, is higher relative to the risk cost while the conditions on risk aversion are more restictive otherwise.

23 Rent Augmenting Behaviours

In this section we assume that the probability of winning the rent is exogenously given. The fraction of the rent to be obtained, α_i; is endogenous instead. Moreover, $\alpha_i' \equiv \partial\alpha_i / \partial x_i > 0$; $\alpha_i'' \equiv \partial^2\alpha_i / \partial x_i^2 < 0$; $\alpha_i(0) = 0$ and $\partial\alpha_i / \partial x_j < 0$; $\forall i \neq j$:

The maximization problem is still

$$\max_{x_i} E[u(w)] = \pi_i E u_1 + (1 - \pi_i) E u_2. \tag{23.1}$$

However the first order condition now turns into

$$\begin{aligned}\frac{\partial EU}{\partial x_i} = {} & \pi_i \left[\alpha'_i c u'_1 + \frac{1}{2} \frac{\partial \Sigma_1}{\partial \alpha_i} u''_1 + \frac{1}{2} \Sigma_1 \alpha'_i c u'''_1 \right] \\ & - \pi_i \left(u'_1 + \frac{1}{2} u'''_1 \Sigma_1 \right) - (1 - \pi_i)\left(u'_2 + \frac{1}{2} s^2 \sigma^2 u'''_2\right)\end{aligned} \tag{23.2}$$

23.1 Wealth Effect

In order to get see the wealth effect on rent-augmenting behaviours, total differentiate the first order condition (23.2); we have the following,

$$\frac{dx_i}{dw_i} = - \frac{(1 + r_f)\left\{ \pi_i\left(\alpha'_i c u''_1 + \frac{1}{2}\frac{\partial \Sigma_1}{\partial \alpha_i} u'''_1\right) - \pi_i u''_1 - (1 - \pi_i) u''_2 \right\}}{\begin{aligned}&\{\pi_i \alpha'^2_i c^2 u''_1 - \pi_i u''_1 \alpha'_i c + \pi_i u'_i \frac{\partial(\alpha'_i c)}{\partial x_i} + \pi_i u''_1 (1 - \alpha'_i c) + \frac{\pi_i}{2}(E P_1^2 \sigma^2 \phi^2(d_1)) u''_1 \\ &+ \frac{1}{2}(\alpha_i E P_1^2 \sigma^2 \phi^2(d_1) + s\Phi(d_1)\sigma^2)\alpha'_i c u'''_1 + \frac{\pi_i}{2}\frac{\partial(\Sigma_1 \alpha'_i c u'''_1)}{\partial x_i} + (1 - \pi_i)(u'_2 + \frac{1}{2} s^2 \sigma^2 u'''_2)\end{aligned}};$$

(23.3)

where

$$\frac{\partial \Sigma_1}{\partial \alpha_i} = \alpha_i E P_1^2 \sigma^2 \phi^2(d_1) + s\Phi(d_1)\sigma^2 \geq 0.$$

The second order condition is satisfied, i.e.,

$$\begin{aligned}& \{\pi_i \alpha'^2_i c^2 u''_1 - \pi_i u''_1 \alpha'_i c + \pi_i u'_i \frac{\partial(\alpha'_i c)}{\partial x_i} + \pi_i u''_1 (1 - \alpha'_i c) + \frac{\pi_i}{2}(E P_1^2 \sigma^2 \phi^2(d_1)) u''_1 + \\ & \frac{1}{2}(\alpha_i E P_1^2 \sigma^2 \phi^2(d_1) + s\Phi(d_1)\sigma^2)\alpha'_i c u'''_1 + \frac{\pi_i}{2}\frac{\partial(\Sigma_1 \alpha'_i c u'''_1)}{\partial x_i} + (1 - \pi_i)(u'_2 + \frac{1}{2} s^2 \sigma^2 u'''_2) \\ & < 0.\end{aligned} \tag{23.4}$$

Therefore, we need to determine the sign of the numerator of (23.3). Given that $r_f > 0$, then

$$\begin{aligned} & \operatorname{sign}\left[(1+r_f)\left\{\pi_-\left(\alpha'_- c u''_1 + \frac{1}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u'''_1\right) - \pi_- u''_1 - (1-\pi_-)u''_2\right\}\right] \\ = \ & \operatorname{sign}\left[\pi_-\left(\alpha'_- c u''_1 + \frac{1}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u'''_1\right) - \pi_- u''_1 - (1-\pi_-)u''_2\right]. \end{aligned}$$

Rewritting the numerator of (23.3), we have

$$\begin{aligned} & \pi_-\left(\alpha'_- c u''_1 + \frac{1}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u'''_1\right) - \pi_- u''_1 - (1-\pi_-)u''_2 \qquad (23.5)\\ = \ & -A(W_1)\pi_- u'_1(\alpha'_- c - 1) + A(W_2)u'_2(1-\pi_-) + \frac{\pi_-}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u'''_1 \\ \geq \ & A(W_1)\left[u'_2(1-\pi_-) - \pi_- u'_1(\alpha'_- c - 1)\right] + \frac{\pi_-}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u'''_1 \\ = \ & A(W_1)\left[u'_2(1-\pi_-) - \pi_- u'_1(\alpha'_- c - 1) - \frac{1}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u''_1 - \frac{1}{2}\gamma_1\alpha'_- c u'''_1 + \frac{\pi_-}{2}u'''_1\gamma_1 + \frac{1-\pi_-}{2}s^2\sigma^2 u'''_2\right] \\ & + \frac{\pi_-}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u'''_1 - A(W_1)\left(\frac{\pi_-}{2}u'''_1\gamma_1 + \frac{1-\pi_-}{2}s^2\sigma^2 u'''_2 + \frac{\pi_-}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u''_1 + \frac{\pi_-}{2}\gamma_1\alpha'_- c u'''_1\right) \\ = \ & \frac{\pi_-}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u'''_1 - A(W_1)\left(\frac{\pi_-}{2}u'''_1\gamma_1 + \frac{1-\pi_-}{2}s^2\sigma^2 u'''_2 + \frac{\pi_-}{2}\frac{\partial \gamma_1}{\partial \alpha_-}u''_1 + \frac{\pi_-}{2}\gamma_1\alpha'_- c u'''_1\right) \\ \geq \ & \left[\frac{\pi_-}{2}\frac{\partial \gamma_1}{\partial \alpha_-} - A(W_1)\left(\frac{\pi_-}{2}\gamma_1 + \frac{1-\pi_-}{2}s^2\sigma^2 + \frac{\pi_-}{2}\gamma_1\alpha'_- c\right)\right]u'''_1. \qquad (23.6) \end{aligned}$$

where in the second step we exploited non-increasing absolute risk aversion and the third step we embedied first order condition (23.2).

Proposition 17 If the rent seeker's absolute risk aversion at wealth W_1, $A(W_1)$, satisfies,

$$A(W_1) \leq \frac{\pi_-\left(\alpha_- E P_1^2\sigma^2\phi^2(d_1) + s\Phi(d_1)\sigma^2\right)}{\pi_-(\gamma_1 + \gamma_1\alpha'_- c) + (1-\pi_-)s^2\sigma^2},$$

then the rent seeker's wealth will have unambiguous positive effect on his rent augmenting behaviours, i.e.,

$$\frac{dx_i}{dw_i} \geq 0.$$

Proof. It follows from (23.6) immediately. ■

In this paricular rent augmenting game; the conditions for positive wealth effect are less than restrictive relative to those in rent seeking games, though both require the entrepreneur who exhibits increasing influent costs on wealth to be less risk averse when he is wealthy.

23.2 Entrepreneurial Ability Effect

We consider the entrepreneurial ability effect in managerial rent augmenting games.

We are now interested in $\frac{\partial x_i}{\partial \eta_i}$ in the utility maximization problem (23.1). Total differentiating first order condition (23.2) yields

$$\frac{dx_i}{d\eta_i} = -\frac{\begin{array}{l}\pi_i \{u_1''(s+\alpha_i\Phi(d_1))a_i'c + \frac{u_1'''}{2}(s+\alpha_i\Phi(d_1))\frac{\partial \Sigma_1}{\partial \alpha_i} + \alpha_i'\Phi(d_1)u_1' - (1-\pi_i)su_2'' \\ + \frac{1}{2}\left[-2\alpha_i\eta_i\phi^2(d_1)\sigma d_2 + s\frac{\phi(d_1)}{EP_1}\right]u_1'' + \frac{1}{2}\left[-\alpha_i^2\eta_i\phi^2(d_1)\sigma d_2 + s\alpha_i\frac{\phi(d_1)}{EP_1}\right]\alpha_i' c u_1''' \\ + \frac{1}{2}\Sigma_1\alpha_i'\Phi(d_1)u_1'''\} - \pi_i\left(u_1''(s+\alpha_i\Phi(d_1)) + \frac{1}{2}\left[-\alpha_i^2\eta_i\phi^2(d_1)\sigma d_2 + s\alpha_i\frac{\phi(d_1)}{EP_1}\right]u_1'''\right)\end{array}}{\pi_i''\{u_1 + \frac{1}{2}u_1''\Sigma_1 - (u_2 + \frac{1}{2}s^2\sigma^2u_2'')\} - 2\pi_i'(u_1' + \frac{1}{2}u_1'''\Sigma_1 - (u_2' + \frac{1}{2}s^2\sigma^2u_2''')) + Eu''}. \tag{23.7}$$

Assuming that second order condition holds for the maximization problem, we only need to show that the numerator of (23.7) is nonnegtive. The numerator of

(23:7) can be written as

$$
\begin{aligned}
& -A(W_1)\pi_i u_1' \alpha_i' c(s+\alpha_i\Phi(d_1)) + \pi_i \alpha_i' \Phi(d_1) u_1' + \pi_i \frac{u_1'''}{2}(s+\alpha_i\Phi(d_1))\frac{\partial \Sigma_1}{\partial \alpha_i} + \frac{1}{2}\Sigma_1 \alpha_i' \Phi(d_1) u_1''' \\
& \quad + A(W_2)(1-\pi_i) s u_2' - \pi_i u_1''(s+\alpha_i\Phi(d_1)) - \frac{\pi_i}{2}\left[-\alpha_i^2 \eta_i \phi^2(d_1)\sigma d_2 + s\alpha_i \frac{\phi(d_1)}{EP_1}\right] u_1''' \\
& \quad + \frac{\pi_i}{2}\left[-2\alpha_i \eta_i \phi^2(d_1) d_2 + s\frac{\phi(d_1)}{EP_1}\right] u_1'' + \frac{\pi_i}{2}\left[-\alpha_i^2 \eta_i \phi^2(d_1)\sigma d_2 + s\alpha_i \frac{\phi(d_1)}{EP_1}\right] u_1''' \alpha_i' c \\
& -\pi_i u_1' \alpha_i' c + \pi_i u_1' + \pi_i \Sigma_1 \frac{u_1'''}{2} + (1-\pi_i) u_2' - \frac{\pi_i}{2}\left\{\alpha_i EP_1^2 \sigma^2 \phi^2(d_1) + s\sigma^2 \Phi(d_1)\right\} u_1'' \\
& \quad - \frac{\pi_i}{2}\Sigma u_1''' \alpha_i' c + \frac{1-\pi_i}{2} s^2 \sigma^2 u_2''' \\
= \; & 0;
\end{aligned}
$$

if

1. the entrepreneur is not too risk averse when he is wealthy, i.e.,

$$A(W_1) \le \frac{1+\alpha_i\Phi(d_1) - \alpha_i' c}{\alpha_i' c(s+\alpha_i\Phi(d_1))}; \qquad (23.8)$$

   and he should be prudent as well, i.e.,

$$A(W_2) \ge \frac{1}{s}; \qquad (23.9)$$

2. expected variance of the output on the project flow of the firm shall be not too large, i.e.,

$$\Sigma \le (s+\alpha_i\Phi(d_1))\frac{\partial \Sigma_1}{\partial \alpha_i}; \qquad (23.10)$$

   secondly, cross derivative of the variance on the size of the rent and entrepreneurial ability should be limited to a certain degree, i.e.,

$$\frac{\partial^2 \Sigma_1}{\partial \alpha_i \partial \eta_i} \le 2(s+\alpha_i\Phi(d_1)) - \frac{\partial \Sigma_1}{\partial \alpha_i}; \qquad (23.11)$$

in other words, variance does not increase significantly with respect to share of the rent and managerial abilities so that the CEO's utility will not lowered down substantially due to risk-taking. Moreover, variance does not diverge with respect to entrepreneurial abilities, i.e.,

$$\frac{\partial \Sigma_1}{\partial \theta_i} \le \Sigma \alpha_{i} c; \tag{23.12}$$

3. the probability ratio for this single Bernoulli trail should be not too small to making rent augmenting activities meaningful, i.e.,

$$\frac{\pi_i}{1 - \pi_i} \ge \frac{s^2 \sigma^2}{\left(\alpha'_i \frac{\partial \Sigma_1}{\partial \theta_i} c + \Sigma \psi(d_1) \right)}. \tag{23.13}$$

Proposition 18 If the utility maximization problem (23.1) is not trivial in the sense that rent augmenting activities do not bring overwhelming variance increment with respect to the share size and entrepreneurial abilities, and the prbability ratio is not too small, i.e., conditions (23.10) (23.11); (23.12) and (23.13) are satisfied, and also the rent seeker exhibits an Knightian entrepreneur's risk attitude, i.e., (23.8) and (23.9) hold, then the rent seeker's entrepreneurial ability has unambiguous positive effect on his influent cost expenditure in enlarging his share of the rent to be obtained, i.e.,

$$\frac{\partial x_i}{\partial \theta_i} \ge 0. \tag{23.14}$$

24 Concluding Remarks

Although rent seeking is long forgotten by economists in analysing incentive problems within the ¯rms, we ¯nd it still appealing in exploring candidate selecting mechanism and authority allocation in contemporary ¯rms. Our intention is not to explain the origin or the quantity of managerial rent per se but to verify if the mechanism is still e± cient within this rent seeking framework. This model, once again, is a combination of hidden information and hidden action, where the hidden information is CEOs' entrepreneurial abilities and the hidden action is his expenditure on in°uence costs.

Contrary to the widespreading nature of ine± ciency and welfare losses caused by rent seeking activities in governmental regulation, procurements and other collective problems, our model shows that the rent seeking like behaviours may be an e± cient mechanism in authority allocation competition within the ¯rms. The intuition behind these claims is that ¯rms are di®erent with governments in the way that entrepreneurship is essential for ¯rms' survival and prosperity, therefore rent-seeking improves the tradeo® between acquiring a productive manager but to lose some control rights and fractions of pro¯ts, and, having a low ability CEO but keep a larger share of the the pie he creates.

Our main results indicate that given initial wealth is known to all stakeholders, in°uence cost expenditure is a signal of his entrepreneurial abilities and risk attitude. We claim that second best outcome on rent dissipation, henceforth authority allocation within the ¯rms, will occur.

We study the wealth e®ect on rent augmenting like behaviours in CEOs' stock-option enlargement competition, and we found even looser conditions.

As in the real corporate life, rent seeking behaviour is coupled with rent augmenting behaviours, this model shows us that if an entrepreneur is cautious in investing on risky assets when his available liquid assets are limited but less risk averse when his personal wealth is high enough, then he will unambiguously spend more on rent seeking like activities in return for obtaining and enlarging the shares of company stock option (call) which will be granted to him, if he is wealthy and/or his entrepreneurial abilities are high. Indeed, these particular conditions on risk attitude imposed in Propositions 3 and 4 depict an Knightian entrepreneur's risk attitude in our understanding.

Further research can be focused on the di®erent e®ects with more complicated pay schemes as well as the relationship between an entrepreneurial ¯rm and outside investors. It should be promising not only because the current literature on this topic is nearly blank but also because of the increasing popularity of stock or option based compensation plans.

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