Intermediate Microeconomic Theory B
Undergraduate Lecture Notes

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Contents

Remarks

- Notes prepared during the 1st semester at the University of Haifa, March 2001 to June 2001 (Tash-Nach)
- For a Syllabus see a separate handout in Hebrew (summarized by the present Table of Content)
- Texts:
  1. Blumental, Levhari, Ofer, & Sheshinski. 1971. *Price Theory.* Academon Press.
  2. Varian H. 1987. *Intermediate Microeconomics.* W.W. Norton
  3. Shy, O. 1986. *Industrial Organization: Theory & Applications.* Cambridge, Mass.: The MIT Press
- Lecture is 3×45 minutes (given nonstop once a week

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Topic 1
Cost of Production

1.1 Cost Functions

- Let, W wage rate, R rental on capital
- $TC(W, R, y)$ maps factor-rental prices to $s
- Emphasize duality: cost function can derived from a production function, and vise versa.
- Marginal Cost: $\text{MC}(y) \stackrel{\text{def}}{=} \partial\text{TC}(y)/\partial y$
- Average Cost: $\text{AC}(y) \stackrel{\text{def}}{=} \text{TC}(y)/y$

1.2 Single-factor case: A demonstration

How to derive the cost function from a production function $y = \ell^\gamma$? Let, W wage rate and $\gamma > 0$.

1. Input-requirement function: $\ell = y^{1/\gamma}$
2. cost means payment to factors: $TC(W, y) = W\ell = Wy^{1/\gamma}$.
3. Note: return to scale (see Figure 1.1):

$$(\lambda\ell)^\gamma > \lambda\ell^\gamma \quad \text{iff} \quad \gamma > 1$$

1.3 Cost minimization and long-run cost functions

- Given W and R, find ℓ and k that minimize cost of producing y_0 units of output.

$$\min_{\ell,k} W\ell + Rk \quad \textbf{s.t.} \quad f(\ell, k) \geq y_0$$

- Discuss corner vs. interior solutions. If $\ell^{\min}$, $k^{\min} > 0$,

$$\frac{\text{MP}_\ell}{\text{MP}_k} = \frac{W}{R}$$

- The second equation is $y_0 = f(\ell^{\min}, k^{\min})$
- to get LRTC: $TC(W, R, y)$

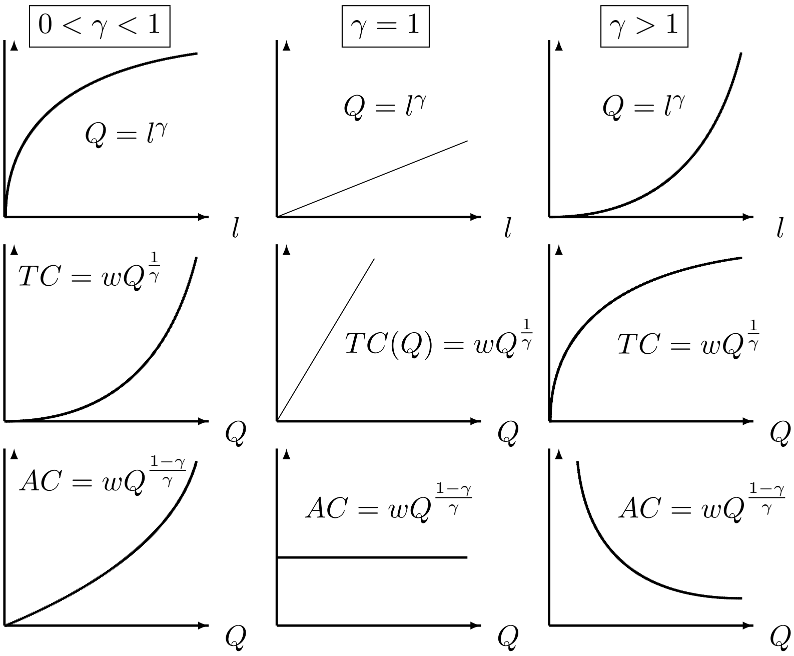

Figure 1.1: Duality between cost- and production functions

- Example: find LRTC for $y = \ell^\alpha k^{1-\alpha}$ (CRS).

$$k = \frac{1-\alpha}{\alpha}\frac{W}{R}\ell$$

yielding *conditional* demand functions

$$\begin{aligned} \ell(W,R,y) &= \left(\frac{\alpha}{1-\alpha}\frac{R}{W}\right)^{1-\alpha} y \\ k(W,R,y) &= \left(\frac{1-a}{\alpha}\frac{W}{R}\right)^{\alpha} y \end{aligned}$$

yielding

$$\text{LRTC}(W,R,y) = W\ell + Rk = \left[\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} R^{1-\alpha}W^{\alpha} + \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} R^{\alpha}W^{1-\alpha}\right] y$$

Note: MC(y) =AC(y) is constant

1.4 Properties of Cost Functions

1.4.1 Relation between TC, AC, MC

As an example, consider the total cost function given by $TC(Q) = F + cQ^2$, $F, c \geq 0$. This cost function is illustrated on the left part of Figure 1.2. We refer to F as the *fixed cost* parameter,

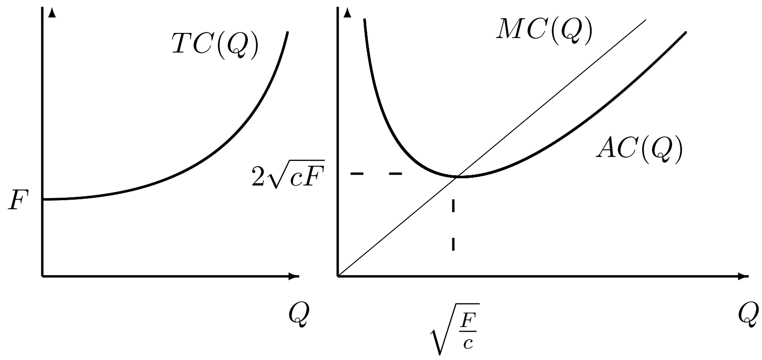

Figure 1.2: Total, average, and marginal cost functions

since the fixed cost is independent of the output level.

It is straightforward to calculate that $AC(Q) = F/Q + cQ$ and that $MC(Q) = 2cQ$. The average and marginal cost functions are drawn on the right part of Figure 1.2. The $MC(Q)$ curve is linear and rising with Q, and has a slope of $2c$. The $AC(Q)$ curve is falling with Q as long as the output level is sufficiently small ($Q < \sqrt{F/c}$), and is rising with Q for higher output levels ($Q > \sqrt{F/c}$). Thus, in this example the cost per unit of output reaches a minimum at an output level $Q = \sqrt{F/c}$.

We now demonstrate an "easy" method for finding the output level that minimizes the average cost.

Proposition 1.1 *If* $Q^{\min} > 0$ *minimizes* $AC(Q)$*, then* $AC(Q^{\min}) = MC(Q^{\min})$.

Proof. At the output level $Q^{\min}$, the slope of the $AC(Q)$ function must be zero. Hence,

$$0 = \frac{\partial AC(Q^{\min})}{\partial Q} = \frac{\partial \left(\frac{TC(Q^{\min})}{Q^{\min}}\right)}{\partial Q} = \frac{MC(Q^{\min})Q^{\min} - TC(Q^{\min})}{(Q^{\min})^2}.$$

Hence,

$$MC(Q^{\min}) = \frac{TC(Q^{\min})}{Q^{\min}} = AC(Q^{\min}).$$

■

We now return to our example illustrated in Figure 1.2, where $TC(Q) = F + cQ^2$. Proposition 1.1 states that in order to find the output level that minimizes the cost per unit, all that we need to do is extract $Q^{\min}$ from the equation $AC(Q^{\min}) = MC(Q^{\min})$. In our example,

$$AC(Q^{\min}) = \frac{F}{Q^{\min}} + cQ^{\min} = 2cQ^{\min} = MC(Q^{\min}).$$

Hence, $Q^{\min} = \sqrt{F/c}$, and $AC(Q^{\min}) = MC(Q^{\min}) = 2\sqrt{cF}$.

Do it in general (graphically only)

1.4.2 Another useful condition

$$\frac{W}{\mathrm{MP}_\ell} = MC = \frac{R}{\mathrm{MP}_k}$$

Proof.

$$\frac{dTC(y)}{d\ell} = \frac{dTC(f(\ell,k))}{d\ell} = \frac{\partial TC(y)}{\partial y}\frac{\partial y}{\partial \ell} = \mathrm{MC}(y) \times \mathrm{MP}_\ell$$

■

1.5 Optimal plant size

- Choosing the level of fixed costs (fixed factors)
- k denotes plant size (say k squared meters)
- $k(y)$ the optimal size plant given output level y
- Short run: $\mathrm{SRTC}(y, k)$
- $\mathrm{SRAC}(y, k) = \mathrm{SRTC}(y, k)/y$
- $\mathrm{LRTC}(y) = \mathrm{SRTC}(y, k(y))$
- Result: $\mathrm{LRTC}(y) \leq \mathrm{SRTC}(y, k)$
- Plot envelope long run optimal plant AC, and SRACs for given values of k.

Topic 2
Profit Maximization

- Profit definition: $\pi = \mathrm{TR} - \mathrm{TC}$
- Two methods:
  1. choose the profit-maximizing output, y, using $TC(y)$
  2. choose the profit-maximizing factor employment using $f(\ell, k)$

2.1 Choosing profit-maximizing output

$$\max_{y} \pi(y) = TR(y) - TC(y) = p_y y - TC(y)$$

If $y^* > 0$, $p_y = \mathrm{MC}(y^*)$
Condition needed: $p_y \geq ATC(y^*)$
Second order MC is declining with y.
Draw figures.

2.2 Choosing profit-maximizing factor employment

$$\pi = \mathrm{TR} - \mathrm{TC} = p_y f(\ell, k) - W\ell - Rk$$

yielding $W = p_y \mathrm{MP}_\ell = \mathrm{VMPL}$ and $R = \mathrm{MP}_k = \mathrm{VMPK}$
SOC:

$$f_{\ell\ell} < 0, \quad f_{kk} < 0, \quad \text{and } f_{\ell\ell} f_{kk} - (f_{\ell k})^2 > 0$$

Topic 3
Long-Run Supply and the Competitive Industry

3.1 Assumptions and Goals

- Each firm a competitive (price taker), thus faces a perfectly-elastic demand curve
- Long-run means free entry and exit
- Short-run, the number of firms is fixed (no entry or exit)
- Identical firms (not necessary, but will be assumed here)

The purpose of this analysis is

1. to calculate the long-run number of firms and aggregate output
2. to calculate the short run output level for a given number of firms

3.2 A Numerical Example

3.2.1 Data

- Industry faces the (inverse) demand curve: $p = 10000/Q$
- $n = 100$ identical firms: $\text{TC}(q_i) = 50 + (q_i)^2/2$

3.2.2 Long-run equilibrium

$$\text{MC} = q, \quad \text{AC} = \frac{50}{q} + \frac{q}{2}.$$

$$\text{MC} = \text{AC} \Longrightarrow q_i = 10 \quad \text{and} \quad \min_q \text{AC}(10) = 10.$$

Hence, the long-run industry supply is perfectly elastic at $p = 10$.

Intersecting demand and supply yields

$$p = \frac{10000}{Q} = 10 \Longrightarrow Q = 1000 \Longrightarrow q_i = \frac{Q}{n} = \frac{1000}{100} = 10.$$

3.2.3 The effect of a rise in fixed cost

Suppose that the government imposes a license fee of 15. We look for the new long-run equilibrium.

$$\text{TC}(q_i) = 65 + \frac{(q_i)^2}{2}.$$

$$\text{MC} = q, \quad \text{AC} = \frac{65}{q} + \frac{q}{2}.$$

$$\text{MC} = \text{AC} \Longrightarrow q_i = \sqrt{130} \Longrightarrow \min_q \text{AC}(q) = \frac{65}{130} + \frac{\sqrt{130}}{2} = \sqrt{130},$$

which is the industry's long-run supply curve.

$$p = \frac{10000}{Q} = \sqrt{130} \Longrightarrow Q = \frac{10000}{\sqrt{130}} \Longrightarrow n = \frac{Q}{q_i} = \frac{10000}{\sqrt{130}} = 76.92.$$

3.2.4 The effect of a rise in unit cost

Suppose that the government imposes a per-unit tax of 4.5. Calculate the *short-run* equilibrium (i.e., $n = 100$):

$$\text{TC}(q_i) = 50 + \frac{(q_i)^2}{2} + 4.5q_i.$$

$$\text{MC} = q + 4.5, \quad \text{AC} = \frac{50}{q} + \frac{q}{2} + 4.5.$$

Now,

$$p = \frac{10000}{100q} = \frac{100}{q} = q + 4.5 = \text{MC} \Longrightarrow q = 8 \Longrightarrow p = 12.5.$$

In the long run, the number of firms will decline.

$$q = 10 \Longrightarrow p = \text{MC} = 10 + 4.5 = 14.5 = \frac{10000}{10n} \Longrightarrow n = \frac{1000}{14.5} = 68.96.$$

3.2.5 The effect of a demand shock

Suppose that the Ministry of Health declared the product to be unhealthy. Formally, the demand drops to $p = 6400/Q$. In the *short run*, $n = 100$. So, solve

$$p = \frac{6400}{100q} = q = \text{MC} \Longrightarrow q = 8, \Longrightarrow Q = 800, \Longrightarrow p = \frac{6400}{800} = 8 < 10 = \min \text{AC}.$$

Therefore, in the *long run*, the number of firms must decline.

$$p = q = 10 \Longrightarrow 10 = p = \frac{6400}{Q} \Longrightarrow Q = 640, \Longrightarrow n = \frac{Q}{q} = \frac{640}{10} = 64.$$

Topic 4
The Monopoly

4.1 Demand Characterization

- Faces the entire market demand curve
- Characterize $\text{TR}(Q)$, $\text{MR}(Q)$, for the inverse demand curve $p = a - bQ$
- Relate $\max \text{TR}$ to elasticity
- If the demand function is linear, $p = a - bQ$, then the marginal-revenue function is also linear, has the same intercept as the demand, but has twice the (negative) slope. Formally, $MR(Q) = a - 2bQ$.
- $$MR(Q) = p(Q)\left[1 + \frac{1}{\eta_p(Q)}\right].$$

  Proof.

$$\begin{aligned} MR(Q) &\equiv \frac{\mathrm{d}\,TR(Q)}{\mathrm{d}Q} = \frac{\mathrm{d}[p(Q)Q]}{\mathrm{d}Q} = p + Q\frac{\mathrm{d}p(Q)}{\mathrm{d}Q} \\ &= p\left[1 + \frac{Q}{p}\frac{1}{\frac{\mathrm{d}Q(p)}{\mathrm{d}p}}\right] = p\left[1 + \frac{1}{\eta_p(Q)}\right]. \end{aligned}$$

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4.2 The Simple Monopoly

- Solve for the simply monopoly

$$\max_Q \pi = \text{TR}(Q) - \text{TC}(Q) \Longrightarrow \text{MR}(Q) = \text{MC}(Q) \quad \text{provided that} \quad p^m \geq \min \text{MC}(Q^m).$$

Example: Figure 4.1 illustrates the monopoly solution for the case where $\text{TC}(Q) = F + cQ^2$, and a linear demand function given by $p(Q) = a - bQ$. $\text{MR}(Q) = a - 2bQ$. Hence, if $Q^m > 0$, then Q^m solves

$$\text{MR}(Q) = a - 2bQ^m = 2cQ^m = \text{MC}(Q)$$

implying that

$$Q^m = \frac{a}{2(b+c)} \text{ and hence } p^m = a - bQ^m = \frac{a(b+2c)}{2(b+c)}.$$

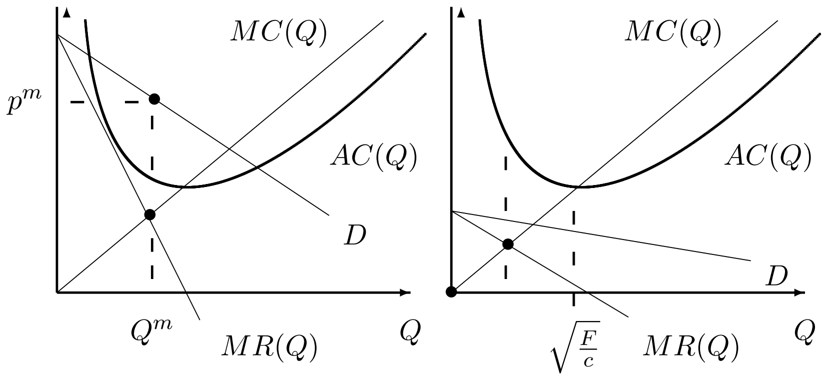

Figure 4.1: The monopoly's profit maximizing output

Consequently,

$$\begin{aligned} \pi(Q^m) &\equiv \text{TR}(Q^m) - \text{TC}(Q^m) \\ &= \frac{a^2(b+2c)}{4(b+c)^2} - F - c\left(\frac{a}{2(b+c)}\right)^2 = \frac{a^2}{4(b+c)} - F. \end{aligned}$$

Altogether, the monopoly's profit-maximizing output is given by

$$Q^m = \begin{cases} \frac{a}{2(b+c)} & \text{if } F \le \frac{a^2}{4(b+c)} \\ 0 & \text{otherwise .} \end{cases}$$

4.3 Discriminating Monopoly

- Selling to different markets (different demand curves)
- How to enforce anti-arbitrage measures (e.g., student discounts, senior citizens, hours of operation, late editions (book publishers))
- In some cases, it is profitable not to sell on some markets

Figure 4.2 illustrates the demand schedules in the two markets (market 1 and market 2).

- ΣMR is the horizontal sum of $\text{MR}_1 + \text{MR}_2$
- If it is profitable to serve *both* markets, then solution is found from $\Sigma\text{MR} = \text{MC}(q_1 + q_2)$
- Find q_1 and q_2 from $\text{MC}(q_1 + q_2) = \text{MR}_1(q_1) = \text{MR}_2(q_2)$
- Find p_1 and p_2 from each market demand curve
- It is NOT clear that it is profitable to serve market 1 (must be checked!)

Example: Two *segmented* markets: $q_1 = 2 - p_1$, and $q_2 = 4 - p_2$. Marginal cost is $c = 1$.

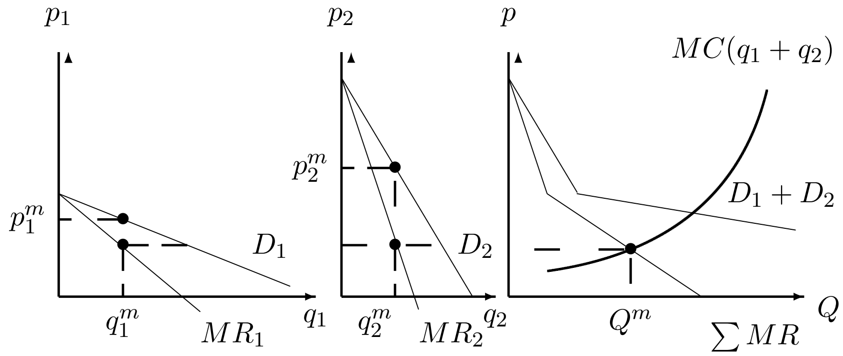

Figure 4.2: Monopoly discriminating between two markets

1. In market 1, $p_1 = 2 - q_1$. Hence, $\text{MR}_1(q_1) = 2 - 2q_1$. Equating $\text{MR}_1(q_1) = c = 1$ yields $q_1 = 0.5$. Hence, $p_1 = 1.5$.

   In market 2, $p_2 = 4 - q_2$. Hence, $\text{MR}_2(q_2) = 4 - 2q_2$. Equating $\text{MR}_2(q_2) = c = 1$ yields $q_2 = 1.5$. Hence, $p_2 = 2.5$.

2. $\pi_1 = (p_1 - c)q_1 = (0.5)^2 = 0.25$, and $\pi_2 = (p_2 - c)q_2 = (1.5)^2 = 2.25$. Summing up, the monopoly's profit under price discrimination is $\pi = 2.5$.

3. Suppose now that price discrimination is infeasible (markets are open). There are two cases to be considered: (i) The monopoly sets a uniform price $p \geq 2$ thereby selling only in market 2, or (ii) setting $p < 2$, thereby selling a strictly positive amount in both markets. Let us consider these two cases:

   (a) If $p \geq 2$, then $q_1 = 0$. Therefore, in this case the monopoly will set q_2 maximize its profit in market 2 only. By subquestion 1 above, $\pi = \pi_2 = 2.25$.

   (b) Here, if $p < 2$, $q_1 > 0$ and $q_2 > 0$. Therefore, aggregate demand is given by $Q(p) = q_1 + q_2 = 2 - p + 4 - p = 6 - 2p$, or $p(Q) = 3 - 0.5Q$. Hence, $MR(Q) = 3 - Q$. Equating $MR(Q) = c = 1$ yields $Q = 2$, hence, $p = 2$. Hence, in this case $\pi = (p - c)2 = 2 < 2.25$.

   Altogether, the monopoly will set a uniform price of $p = 2.5$ and will sell $Q = 1.5$ units in market 2 only.[1]

Finally, to find the relationship between the price charged in each market and the demand elasticities,

$$p_1^m(1 + 1/\eta_1) = p_2^m(1 + 1/\eta_2).$$

Hence, $p_2^m > p_1^m$ if $\eta_2 > \eta_1$, (or $|\eta_2| < |\eta_1|$, recalling that elasticity is a negative number). Hence, a discriminating monopoly selling a strictly positive amount in each market will charge a higher price at the market with the less elastic demand.

[1]Note that consumers in market 1 are better off under price discrimination than without it, since under no discrimination no output is purchased in market 1. Given that the price in market 2 is the same under price discrimination and without it, we can conclude that in this example, price discrimination is Pareto superior to nonprice discrimination, since both consumer surplus and the monopoly profit are higher under price discrimination.

4.4 The Cartel

- Contract among N competing firms
- Agreement on price or quantity quota (our focus)
- Examples: OPEC, IATA

The objective of the cartel is to choose $q_1, q_2, \ldots, q_N$ to

$$\max_{q_1,q_2,\ldots,q_N} \Pi(q_1, q_2, \ldots, q_N) \equiv \sum_{i=1}^{N} \pi_i(q_i) = \left[a - b\sum_{i=1}^{N} q_i\right]\left(\sum_{i=1}^{N} q_i\right) - \sum_{i=1}^{N} TC_i(q_i). \tag{4.1}$$

The cartel has to solve for N quantities, so, after some manipulations, the N first-order conditions are given by

$$0 = \frac{\partial \Pi}{\partial q_j} = a - 2b\sum_{i=1}^{N} q_i - MC_j(q_j) = MR(Q) - MC_j(q_j), \;\; j = 1, 2, \ldots, N. \tag{4.2}$$

4.4.0.1 A simple cartel example

- 10 firms, each has $\text{TC}(q_i) = 200 + 2(q_i)^2$
- Market demand: $p = 140 - Q$
- Solve for the cartel's output level, market price, and profit

$$\text{MR}(Q) = 140 - 2Q = 140 - 2 \cdot 10 \cdot q = 4q = \text{MC}(Q) \Longrightarrow q = \frac{35}{6}$$

Hence,

$$Q = 10 \cdot q = \frac{175}{3} \Longrightarrow p = 140 - Q = \frac{245}{3}.$$

Hence,

$$\pi_i = \left(\frac{245}{3}\right)\left(\frac{35}{6}\right) - 200 - 2\left(\frac{35}{6}\right)^2 = \frac{625}{3}. \Longrightarrow \Pi = 10\pi_i = \frac{6250}{3} \approx 208.$$

4.4.0.2 A more general example

- Our calculations will rely on $\text{TC}(q_i) = F + c(q_i)^2$
- Hence, $\text{MC}(q_i) = 2cq_i$ and $\text{AC}(q_i) = F/q_i + cq_i$
- Industry market demand: $p = a - bQ$

Since all plants have identical cost functions, we search for a symmetric equilibrium $q_1 = q_2 = \ldots = q_N \equiv q$. Hence,

$$a - 2bNq = 2cq \quad \text{implying that} \quad q = \frac{a}{2(bN+c)}. \tag{4.3}$$

The total cartel's output and the market price are given by

$$Q = Nq = \frac{Na}{2(bN+c)} \quad \text{and} \quad p = a - bQ = \frac{a(bN+2c)}{2(bN+c)}. \tag{4.4}$$

4.5 Multiplant Monopoly

- Same as a cartel, but can adjust N (the number of producers/plants), since all under the same ownership.
- i.e., $\text{MR}(Q) = \text{MC}(q_i)$ for all $i = 1, \ldots, N$
- Choose q_i that minimizes $\text{AC}(q_i)$

4.5.0.3 The simple multiplant-monopoly example

- Variable (controlled) number of firms, each has $\text{TC}(q_i) = 200 + 2(q_i)^2$
- Market demand: $p = 140 - Q$
- Solve for # firms, output levels, market price, and profit
- Key issue: Here the monopoly adjusts output by changing the number of plants. In contrast, a cartel adjusts output by putting production quotas on member firms.

$$\text{MC} = 4q_i, \quad \text{AC} = \frac{200}{q_i} + 2q_i \Longrightarrow \arg\min \text{AC}(q_i) = 10, \quad \min \text{AC} = 40$$

Now,

$$\text{MR} = 140 - 2Nq = 140 - 20N = 40 = \text{MC} \Longrightarrow N = 5$$

Hence,

$$Q = 50, \Longrightarrow p = 90 \Longrightarrow \Pi = 90 \cdot 50 - 5 \cdot 400 = 2500$$

4.5.0.4 The more general example

- Hence, $q_i = \sqrt{F/c}$
- Solve $\text{MR}(Q) = \text{MC}(q_i)$
- Hence, $q_i = a/[2(bN+c)]$
- Altogether, $\sqrt{F/c} = a/[2(bN+c)]$, Hence,

$$N = \frac{a\sqrt{c}}{2b\sqrt{F}} - \frac{c}{b}$$

Topic 5
Oligopoly: Competition Among Few Firms

5.1 Noncooperative Game Theory: Nash Equilibrium

See Shy (1996), Chapter 2: pp.12–15, 18–20.

5.2 The Cournot Market Structure: Quantity Competition

5.2.1 Example of Cournot equilibrium

- Market demand: $Q = 3200 - 1600p$, Hence,

$$p = 2 - \frac{Q}{1600} = 2 - \frac{q_1 + q_2}{1600}$$

- 2 firms, firm 1 has a cost advantage:

$$\mathrm{TC}_1(q_1) = 0.25q_1 \qquad \mathrm{TC}_2(q_2) = 0.5q_2$$

- Solve for the Cournot output levels, market price, and profit levels

Firm 1 solves:

$$\max_{q_1} \pi_1 = \frac{3200 - q_1 - q_2}{1600} q_1 - 0.25q_1 \tag{5.1}$$

yielding a *best-response* function given by

$$q_1(q_2) = 900 - \frac{1}{2}q_2$$

Firm 1 solves:

$$\max_{q_2} \pi_2 = \frac{3200 - q_1 - q_2}{1600} q_2 - 0.5q_2 \tag{5.2}$$

yielding a *best-response* function given by

$$q_2(q_1) = 1200 - \frac{1}{2}q_1 \tag{5.3}$$

Solving the two best-response functions yield

$$q_1 = \frac{3200}{3} \quad q_2 = \frac{2000}{3} \Longrightarrow Q = \frac{5200}{3} \approx 1733 \Longrightarrow p = \frac{11}{12}$$

Hence,

$$\pi_1 = \frac{6400}{9} \approx 711 > 278 \approx \frac{2500}{9} = \pi_2$$

5.2.2 General Cournot Theory

See Shy (1996), Chapter 6: pp.98–103.

5.3 Stackelberg Equilibrium: Sequential Moves

5.3.1 Example

- Two-stage game (two periods)
- Suppose now that firm 1 sets q_1 (stage I) before firm 2 sets q_2 (stage II)
- Firm 1 is called a *leader*
- Firm 2 is called a *follower* (choosing q_2 by taking q_1 as given)
- Solving the game *backwards* starting in the 2nd stage
- Stage II: Firm 2 takes q_1 as given and chooses q_2 to solve $\max_{q_2} \pi_2$ which is essentially the same as (5.2),
- yielding firm 2's best response function: (5.3)
- Stage I: Firm 1, knowing that firm 2 reacts according to (5.3)
- Thus, substitute (5.3) into (5.1), firm 1 solves

$$\max_{q_1} \pi_1 = \frac{3200 - q_1 - q_2(q_1)}{1600} q_1 - 0.25 q_1 == \frac{q_1(3200 - q_1)}{3200} \tag{5.4}$$

yielding

$$q_1 = 1600 > \frac{3200}{3} \Longrightarrow q_2 = 400 < \frac{2000}{3} \Longrightarrow Q = 2000 \Longrightarrow p = \frac{3200 - Q}{1600} = \frac{3}{4} < \frac{11}{12} \tag{5.5}$$

Also

$$\pi_1 = 800 > \frac{6400}{9} \qquad \pi_2 = 100 < \frac{2500}{9}$$

- Thus, the output and profit levels of firm 1 are higher than under Cournot
- The output and profit levels of firm 2 are lower than under Cournot
- Aggregate output is higher (hence, equilibrium price is lower)

5.3.2 General Stackelberg Theory

See Shy (1996), Chapter 6: pp.104–106

5.4 Dominant Firm

- An industry having a single dominant firm and many competitive firm
- Two-stage game: Stage I, leader sets output, q_d, or price p
- Stage II: Competitive firms take p as given and set competitive output level, q_i

Take the following example:

- One dominant firm with zero production cost
- 50 competitive firms, each with cost $\mathrm{TC}_i(q_i) = (q_i)^2/2$
- Market demand: $Q = 1000 - 50p$
- Calculate the price set by the dominant firm
- Calculate output levels of all firms

Second stage: Given p, find the competitive firms' aggregate supply curve:

$$p = \mathrm{MC}_i(q_i) = q_i \Longrightarrow q_i = p \Longrightarrow Q_c = 50p$$

First stage: The *residual* demand facing the dominant firm is:

$$q_d = Q - Q_c = 1000 - 50p - Q_c = 1000 - 50p - 50p = 1000 - 100p$$

The dominant firm, therefore, chooses p to solve

$$\max_p = pQ_d = p(1000 - 100p) \Longrightarrow p = 5$$

Hence,

$$q_d = 1000 - 500 = 500, \quad q_i = p = 5, \quad Q_c = 50 \cdot 5 = 250$$

Topic 6
Welfare Economics

- Define efficiency in the "weakest" possible sense (i.e., not to mix with a political definition)
- Characterize *market failure* situations where an outcome (equilibrium) is inefficient from a social view point
- Propose policy instruments (e.g., taxation) that will restore efficiency.

6.1 Pure Exchange Economy: Basic Definitions

- Pure exchange economy means no production (to be added later on)
- Prices are irrelevant for these definitions
- 2 persons: A and B
- 2 goods: X and Y
- x_A^0 initial *endowment* of good X to individual A. y_A^0, x_B^0, and y_B^0 are similarly defined
- Aggregate economy endowment (manna from heaven): $\bar{x} \stackrel{\text{def}}{=} x_A^0 + x_B^0$ and $\bar{y} \stackrel{\text{def}}{=} y_A^0 + y_B^0$
- x_A *allocation* of good X to individual A. x_B, y_A and y_B are similarly defined
- In class, draw Edgeworth Box

Definition 6.1 *An allocation is said to be **feasible** if $x_A + x_B = \bar{x}$ and $y_A + y_B = \bar{y}$.*

Definition 6.2 *A feasible allocation is said to be **Pareto Efficient** (Pareto Optimal) if there does not exist a different feasible allocation which makes at least one consumer better off and does not make any consumer worse off.*

Alternative definition:

Definition 6.3 *(a) A feasible allocation (x_A, y_A, x_B, y_B) is said to be **Pareto Superior** to allocation $(\hat{x}_A, \hat{y}_A, \hat{x}_B, \hat{y}_B)$ if*

$$U_A(x_A, y_A) \geq U_A(\hat{x}_A, \hat{y}_A) \quad (6.1)$$
$$U_B(x_B, y_B) \geq U_B(\hat{x}_B, \hat{y}_B) \quad (6.2)$$

where at least one strict inequality $>$ must hold.

*(b) A feasible allocation is said to be **Pareto Optimal** if there does not exist an allocation which is Pareto superior to it.*

6.2 Contract Curves

- Draw the **contrast curve** for different preferences
- Prove that on any interior allocation on the contract curve,

$$\mathrm{RCS}^A = \frac{\mathrm{MU}_X^A}{\mathrm{MU}_Y^A} = \frac{\mathrm{MU}_X^B}{\mathrm{MU}_Y^B} = \mathrm{RCS}^B$$

- Example (interior contract curve): $U_A = x_A \cdot y_A$ endowed with $(3, 2)$ and $U_B = x_B \cdot y_B$ endowed with $(1, 6)$. Solution:

$$\frac{y_A}{x_A} = \frac{y_B}{x_B} = \frac{8 - y_A}{4 - x_A} \Longrightarrow y_A = 2x_A$$

- Example (non-interior contract curve, perfect substitutes): $U_A = x_A + 2y_A$, $U_B = 2x_B + 2y_B$, with $\bar{x} = 4$ and $\bar{y} = 8$. *Note:* Only the aggregate endowment matters for drawing the contract curve. Solution: 2 sides: $x_A = 0$ and $y_A + y_B = 8$; and $y_A = 8$ and $x_A + y_A = 4$.
- Example (perfect complements and Cobb-Douglas): $U_A = x_A \cdot y_A$ endowed with $(0, 10)$, and $U_B = \min\{x_B, y_B\}$ endowed with $(20, 5)$. Solution:

$$y_A = \begin{cases} 0 & \text{if } x_A \leq 5 \\ -5 + x_A & \text{if } 5 \leq x_A \leq 20 \end{cases}$$

- Example (perfect substitutes and Cobb-Douglas): $U_A = x_A + y_A$ endowed with $(60, 10)$; and $U_B = x_B \cdot y_B$ endowed with $(20, 30)$. Solution:

$$y_A = \begin{cases} 0 & \text{if } x_A \leq 40 \\ -40 + x_A & \text{if } 40 \leq x_A \leq 80 \end{cases}$$

- Difficult example (both consumes have perfect complements preferences): $U_A = \min\{x_A, y_A\}$ and $U_B = \min\{x_B, y_B\}$ with aggregate endowment of $\bar{x} = 20$ and $\bar{y} = 10$.
- As above but $U_A = \min\{2x_A, y_A\}$ and $U_B = \min\{x_B, y_B\}$
- As above buy $U_A = x_A + y_A$ and $U_B = x_B + y_B$. Solution: The contract curve is the entire box.

6.3 Efficient production

INSERT

6.4 Integrated economy (production economy)

INSERT

Topic 7
Competitive Equilibrium and the Neoclassical Welfare Theorems

7.1 Competitive equilibrium

INSERT

7.2 The First-Welfare Theorem

INSERT

7.3 The Second-Welfare Theorem

INSERT

7.4 Monopoly in Edgeworth Box

Demonstrate.

Topic 8
Public Goods

8.1 Definition

INSERT

8.2 Samuelson's Efficiency Condition

INSERT

8.3 Market failure associated with competitive equilibrium

INSERT

8.4 Government Policy (e.g., taxation)

INSERT

8.5 The Tragedy of the Commons

INSERT

Topic 9
Externalities

9.1 Production Externalities

INSERT

9.2 Consumption Externalities

INSERT