Economics 686
Assignment Two

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1. Question One

Using the data on file from assignment one, calculate the regression

$$G = \beta_0 + \beta_1 P_g + \beta_2 Y + \beta_3 P_{nc} + \beta_4 P_{pt} + \beta_5 P_d + \beta_6 P_n + \beta_7 P_s + \varepsilon$$

under the assumptions that the seven right-hand variables are fixed in repeated samples, that $E(x, \varepsilon) = (x, 0) = 0$; $\mathrm{Cov}[(x, \varepsilon), (z, \varepsilon)] = \sigma^2 (x, z)$ for all fixed x, z in R^n, and that $\varepsilon \sim N(0, I_n \sigma^2)$. Provide the following results but do not submit a computer output:

1. each $\hat{\beta}_i \pm SE\hat{\beta}_i$, $i = 1, 2, \cdots, 7$
2. $s^2 (X^\top X)^{-1}$;
3. R^2.

2. Question Two

Briefly explain the three different versions of the F-statistic for testing a linear hypothesis of r restrictions or $(k - r)$ independent coefficients $(k = 8, r = 1, 2, \cdots, 7)$:

$$F = \frac{\left\{ y^\top (P - P_0) y (n - k) \right\}}{\left\{ y^\top (I_n - P) y (r) \right\}} \tag{2.1}$$

$$= r^{-1}\left(B^\top\hat\beta\right)^\top\left[\hat D\left(B^\top\hat\beta\right)\right]^{-1}B^\top\hat\beta \tag{2.2}$$

$$= \frac{\left\{\left(\hat\varepsilon_0^\top\hat\varepsilon_0-\hat\varepsilon^\top\hat\varepsilon\right)(n-k)\right\}}{\hat\varepsilon^\top\hat\varepsilon\,(r)}. \tag{2.3}$$

Explain the relation between formula (2) and the Wald Principle and how this relates to the Lagrange Multiplier Principle and its small-sample specialization.

$$M = \frac{y^\top(P-P_0)\,y}{y^\top(I_n-P_0)\,y}. \tag{2.4}$$

3. Question Three

Illustrate your answer to question two by calculating the restricted regressions in each case for the following hypotheses:

$$\begin{array}{llll}
H_{01}: & \beta_3 = 0 & H_{a1}: & \beta_3 \neq 0 \\
H_{02}: & 7\beta_6 - \beta_8 = 0 & H_{a2}: & 7\beta_6 - \beta_8 \neq 0 \\
 & \beta_2+\beta_4 = 0 & & \beta_2+\beta_4 \neq 0 \\
H_{03}: & 7\beta_6 - \beta_8 = 0 & H_{a3}: & 7\beta_6 - \beta_8 \neq 0 \\
 & 2\beta_7+\beta_9 = 0 & & 2\beta_7+\beta_9 \neq 0.
\end{array}$$

Make sure that you provide the matrix $B^\top$ in $B^\top\beta = 0$ in your solutions in each case and check that the F- or t-test from your regression is the same as the calculation of (2.1), (2.2) or (2.3) you obtained from the restricted and unrestricted models.

4. Question Four

In question three, formulate H_{01}, H_{02} and H_{03} in the form of freedom equations. In the case of H_{03}, demonstrate that if $\beta = M\alpha$, then $B^\top M = 0$.