FUNGSI INVERS HIPERBOLIK

Tidak semua fungsi hiperbolik pada domainnya merupakan fungsi satu-satu sehingga tidak mempunyai invers. Oleh karena itu, agar didapatkan fungsi invers hiperbolik maka kita batasi domain fungsinya. Sedangkan untuk mencari turunan dari fungsi invers hiperbolik dilakukan terlebih dahulu cara sebagai berikut.

Misal $y = \sinh^{-1} u$. Maka $u = \sinh y$ [$\forall$ u, y].

Jadi : $u = \dfrac{e^y - e^{-y}}{2} \Leftrightarrow e^y - 2u - e^{-y} = 0 \Leftrightarrow e^{2y} - 2u e^y - 1 = 0$

$$\Leftrightarrow e^y = u \pm \sqrt{u^2+1} = u + \sqrt{u^2+1} \quad \left(sebab: e^y > 0, \forall y\right)$$

$$\Leftrightarrow y = \ln\left(u + \sqrt{u^2+1}\right)$$

Turunan Fungsi invers Hiperbolik.

Misal $y = \sinh^{-1} u = \ln\left(u + \sqrt{u^2+1}\right)$. Maka :

$$y' = \frac{1}{u + \sqrt{u^2+1}}\left(1 + \frac{u}{\sqrt{u^2+1}}\right)u' = \frac{u'}{\sqrt{u^2+1}}$$

Dari anti turunan fungsi invers sinus hiperbolik, didapatkan :

$$\int \frac{du}{\sqrt{u^2+1}} = \sinh^{-1} u + C$$

Dengan cara sama diperoleh turunan dan integral fungsi invers hiperbolik, sebagai berikut :

1. $y = \cosh^{-1} u = \ln\left(u + \sqrt{u^2-1}\right)$, $\{u \geq 1\}$

$$y' = \frac{u'}{\sqrt{u^2 - 1}} \Leftrightarrow \int \frac{du}{\sqrt{u^2 - 1}} = \cosh^{-1} u + C$$

2. $y = \tanh^{-1} u = \frac{1}{2} \ln\left(\frac{1+u}{1-u}\right)$, $\{|u| < 1\}$

$$y' = \frac{u'}{1-u^2} \Leftrightarrow \int \frac{du}{1-u^2} = \tanh^{-1} u + C \ , bila \ \ |u| < 1$$

3. $y = \coth^{-1} u = \frac{1}{2} \ln\left(\frac{u+1}{u-1}\right)$, $\{|u| > 1\}$

$$y' = \frac{u'}{1-u^2} \Leftrightarrow \int \frac{du}{1-u^2} = \coth^{-1} u + C \ , bila \ |u| > 1$$

4. $y = \sec h^{-1} u = \ln\left(\frac{1+\sqrt{1-u^2}}{u}\right), \{0 < u \leq 1\}$

$$y' = \frac{-u'}{u\sqrt{1-u^2}} \Leftrightarrow \int \frac{du}{u\sqrt{1-u^2}} = -\sec h^{-1}|u| + C$$

5. $y = \csc h^{-1} u = \ln\left(\frac{1}{u} + \frac{\sqrt{1+u^2}}{|u|}\right), \{u \neq 0\}$

$$y' = \frac{-u'}{|u|\sqrt{1+u^2}} \Leftrightarrow \int \frac{du}{u\sqrt{1+u^2}} = -\csc h^{-1}|u| + C$$

Soal Latihan

(Nomor 1 sd 12) Tentukan dy/dx dari :

1. $y = \cosh^{-1}(2x+1)$
2. $y = \coth^{-1}\left(\sqrt{x}\right)$
3. $y = \csc h^{-1}\left(e^{2x}\right)$
4. $y = \frac{1}{\tanh^{-1} x}$
5. $y = \sinh^{-1}\left(\frac{1}{x}\right)$
6. $y = \cosh^{-1}\left(\cosh x\right)$
7. $y = \ln\left(\cosh^{-1} x\right)$
8. $y = \sqrt{\coth^{-1} x}$

9. $y = \sinh^{-1}\left(\tanh x\right)$

10. $y = e^{x}\ \sec h^{-1}x$

11. $y = \tanh^{-1}\left(\dfrac{1-x}{1+x}\right)$

12. $y = \left(1 + x\ \csc h^{-1}x\right)^{10}$

(Nomor 13 sd 20) Hitung integral berikut :

13. $\int \dfrac{dx}{\sqrt{1+9x^2}}$

14. $\int \dfrac{dx}{\sqrt{x^2-2}}$

15. $\int \dfrac{dx}{\sqrt{9x^2-25}}$

16. $\int \dfrac{dx}{\sqrt{1-e^{2x}}}$

17. $\int \dfrac{\sin x\ dx}{\sqrt{1+\cos^2 x}}$

18. $\int \dfrac{dx}{x\sqrt{1+x^6}}$

19. $\int_0^{\sqrt{3}} \dfrac{dt}{\sqrt{t^2+1}}$

20. $\int_{1/4}^{1/2} \dfrac{dt}{\sqrt{t}\sqrt{1-t}}$

Danang Mursita
Sekolah Tinggi Teknologi Telkom, Bandung