FUNGSI HIPERBOLIK

Fungsi sinus hiperbolik dan cosinus hiperbolik didefinisikan sebagai berikut :

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad dan \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

Untuk fungsi hiperbolik yang lain :

1. $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

2. $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

3. $\sec h\, x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x - e^{-x}}$

4. $\csc h\, x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x + e^{-x}}$

Berikut beberapa identitas yang berlaku pada fungsi hiperbolik :

1. $\cosh^2 x - \sinh^2 x = 1$

2. $1 - \tanh^2 x = \mathrm{sech}^2 x$

3. $\coth^2 x - 1 = \mathrm{csch}^2 x$

4. $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

5. $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

6. $\cosh x + \sinh x = e^x$.

7. $\cosh x - \sinh x = e^{-x}$.

8. $\sinh 2x = 2 \sinh x \cosh x$

9. $\cosh 2x = \cosh^2 x + \sinh^2 x = 2 \sinh^2 x + 1 = 2 \cosh^2 x - 1$

10. $\cosh(-x) = \cosh x$

11. $\sinh(-x) = -\sinh x$

12. $\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$

13. $\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$

14. $\tanh(x + y) = \dfrac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

15. $\tanh(x - y) = \dfrac{\tanh x - \tanh y}{1 - \tanh x \tanh y}$

16. $\tanh 2x = \dfrac{2 \tanh x}{1 + \tanh x}$

17. $\cosh \dfrac{1}{2} x = \sqrt{\dfrac{1}{2}(\cosh x + 1)}$

18. $\sinh \dfrac{1}{2} x = \pm\sqrt{\dfrac{1}{2}(\cosh x - 1)}$

19. $\sinh x + \sinh y = 2 \sinh\left(\dfrac{x + y}{2}\right) \cosh\left(\dfrac{x - y}{2}\right)$

20. $\cosh x + \cosh y = 2 \cosh\left(\dfrac{x + y}{2}\right) \cosh\left(\dfrac{x - y}{2}\right)$

Turunan dan Integral Fungsi Hiperbolik

$$\text{Misal } y = \sinh u. \text{ Maka } y' = D_x\left(\frac{e^u - e^{-u}}{2}\right) = \frac{e^u + e^{-u}}{2} u' = \cosh u \ u'.$$

Jadi : $\displaystyle\int \cosh u \ du = \sinh u + C$

Untuk fungsi hiperbolik yang lain:

1. $y = \cosh u \Rightarrow y' = \sinh u \ u' \Leftrightarrow \displaystyle\int \sinh u \ du = \cosh u + C$

2. $y = \tanh u \Rightarrow y' = \sec h^2 u \ u' \Leftrightarrow \displaystyle\int \sec h^2 u \ du = \tanh u + C$

3. $y = \coth u \Rightarrow y' = -\csc h^2 u \ u' \Leftrightarrow \displaystyle\int \csc h^2 u \ du = -\coth u + C$

4. $y = \sec h\ u \Rightarrow y' = -\sec h\ u\ \tanh u\ u' \Leftrightarrow \int \sec h\ u\ \tanh u\ du = -\sec h\ u + C$

5. $y = \csc h\ u \Rightarrow y' = -\csc h\ u\ \coth u\ u' \Leftrightarrow \int \csc h\ u\ \coth u\ du = -\csc h\ u + C$

Soal Latihan

(Nomor 1 sd 5)Tentukan turunan pertama (y') dari :

1. $y = \cosh x^4$.

2. $y = \ln (\tanh 2x)$

3. $y = \cosh (1/x)$

4. $y = \sinh^3(2x)$

5. $y = \sqrt{4x + \cosh^2(5x)}$

(Nomor 6 sd 10) Hitung integral berikut :

6. $\int \cosh(2x - 3)\ dx$

7. $\int \csc h^2(3x)\ dx$

8. $\int \coth^2 x\ \csc h^2 x\ dx$

9. $\int \tanh x\ dx$

10. $\int \sinh^6 x\ \cosh x\ dx$