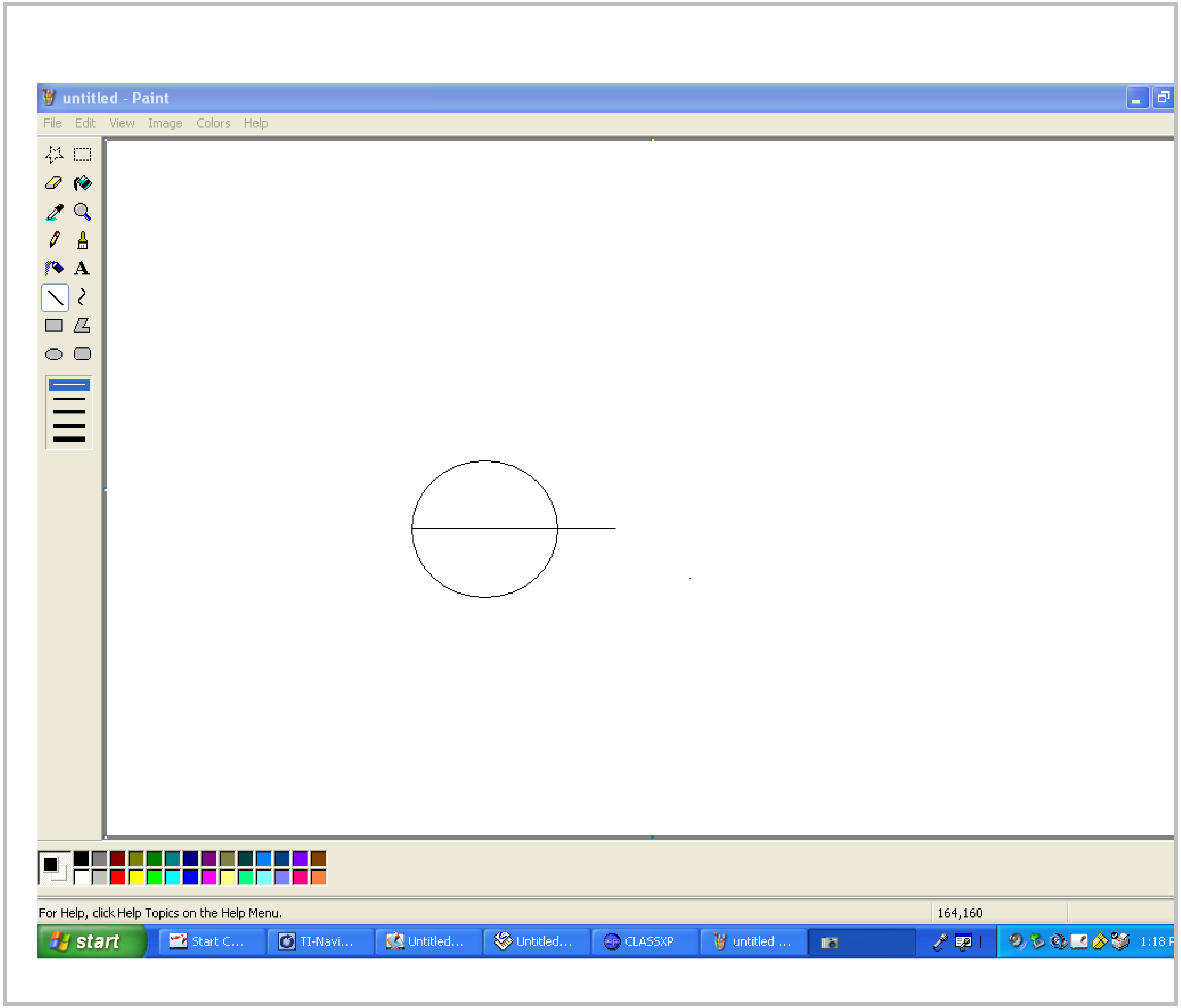
uha up

+ (mm



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Greatest Integer Function

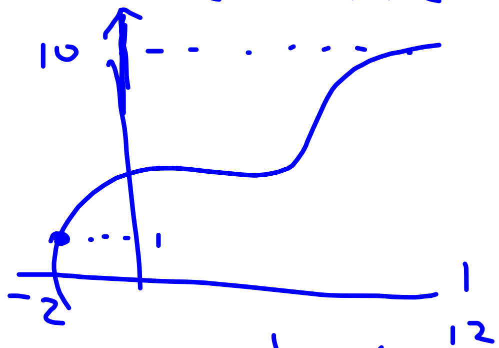
$[\![x]\!]$ = largest integer not greater than x

$[\![2.3]\!] = 2$ $[\![.9999]\!] = 0$

Existence of a Limit

Intermediate Value Theorem

If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c such that $f(c) = k$.



$[-2, 12]$ $k = 4$

Ex: Use IVT to show

$f(x) = x^3 + 2x - 1$ has a zero in $[0, 1]$

$$\left.\begin{array}{l} f(0) = 0^3 + 2(0) - 1 = -1 \\ f(1) = 1^3 + 2(1) - 1 = 2 \end{array}\right\}$$

Look at the y's => show a change from + to − or vice versa

Since 0 lies between −1 and 2, Intermediate Value Theorem applies. There must ~~be at~~ least one c such that $x^3 + 2x - 1 = 0$

use calculator to find the zero

$X = .453$

Maximum Value Property

If f is continous on the closed interval $[a,b]$, then f attains a maximum + minimum value at some point on that interval

Infinite Limits

= a limit in which $f(x)$ increases or decreases w/o bound

$\lim_{x \to c} f(x) = \infty$ or $\lim_{x \to c} f(x) = -\infty$

Vertical Asymptotes.

If $f(x)$ approaches ∞ or $-\infty$ as x approaches c from the right or the left, then the lim $x=c$ is a vertical asymptote

* If a function f has a vertical asymptote at $x=c$, f is not continuous @ $x=c$.

* Let f & g be continuous on an open interval containing c,
If $f(c)\neq 0$, $g(c)=0$,
there exists an open interval containing c such that $g(c)\neq 0$, and $x\neq c$ for that interval $\Rightarrow$
then the graph

$$h(x)=\frac{f(x)}{g(x)}$$ has a vertical asymptote at $x=c$.

Vertical asymptotes are nonremovable discontinuities.

Find the vertical asymptotes

$$f(x) = \frac{1}{2(x+1)}$$

$$\lim_{x \to -1^-} \frac{1}{2(x+1)} = -\infty$$

$\therefore x = -1$ is a vertical asymptote

Ex 2: $$f(x) = \frac{x^2+1}{x^2-1} = \frac{(x^2+1)}{(x-1)(x+1)}$$

$$\frac{+}{- \cdot +}$$

$$\lim_{x \to 1^-} \frac{x^2+1}{(x-1)(x+1)} = -\infty$$

$\therefore x = 1$ vertical asy

$$\lim_{x \to -1^+} \frac{x^2+1}{(x-1)(x+1)} = -\infty$$

$\therefore x = -1$ is a vert. asy.

$$f(x) = \frac{x^2+2x-8}{x^2-4}$$

$$= \frac{(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$$

Use original to find continuity

~~Removable discontinuity~~

$x = 2$

nonremovable:

$x = -2$

Vertical

$$\lim_{x \to -2^-} \frac{(x+4)}{(x+2)} = -\infty$$

$\therefore$ vertical asy @ $x = -2$