

Guideline for Solving Related Rate WP

- ① Draw + Label a diagram.
- ② Form algebraic expressions
- ③ Make a relationship between the two quantities involving the RATE to be determined.
- ④ Take the derivative of both sides with respect to time.
- ⑤ Solve for the desired RATE \Rightarrow substitute in values

- . A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius of the outer ripple is increasing at a rate of 1 ft/sec. When the radius is 4 feet, at what rate is the total area of the disturbed water increasing?

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\begin{aligned} \frac{dA}{dt} \Big|_{r=4} &= 2\pi(4\text{ft})\left(\frac{1\text{ft}}{\text{sec}}\right) \\ &= 8\pi \text{ft}^2/\text{sec} \end{aligned}$$

Rate will be
ft/sec
in/sec
m/min etc

$$\frac{dr}{dt} = 1\text{ft/sec}$$
$$r = 4\text{ft}$$

Air is being pumped into a spherical shaped balloon at a rate of $4.5 \text{ in}^3/\text{min}$. Find the rate of change of radius when the radius = 2 in.

$$\frac{dV}{dt} = 4.5 \text{ in}^3/\text{min} \quad r = 2 \text{ in}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

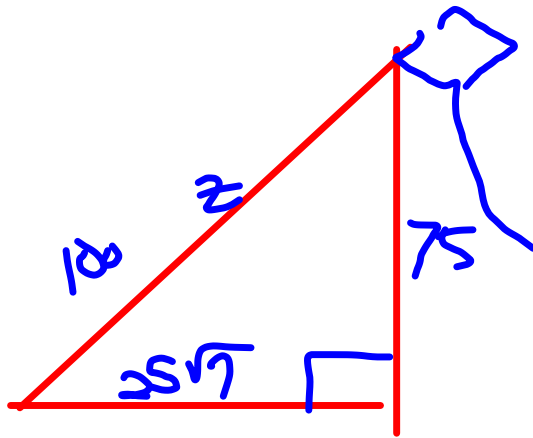
$$4.5 = 4\pi (2)^2 \frac{dr}{dt}$$

$$4.5 = 16\pi \frac{dr}{dt}$$

$$\frac{4.5}{16\pi} = \frac{dr}{dt}$$

$$\frac{9}{32\pi} \frac{\text{in}}{\text{min}} = \frac{dr}{dt}$$

A kite is at an altitude of 75 feet and is moving horizontally at 12 ft/sec. The length of the kite string is 100 feet. How fast is the string being paid out by the person holding the kite?



$$z = 100 \text{ @ some point}$$

$$\frac{dz}{dt} = 12 \text{ ft/sec}$$

$$x^2 + 75^2 = z^2$$

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$$\frac{dx}{dt} = \frac{z}{x} \frac{dz}{dt}$$

$$x^2 + 75^2 = 100^2$$

$$x = \sqrt{100^2 - 75^2} = 25\sqrt{7}$$

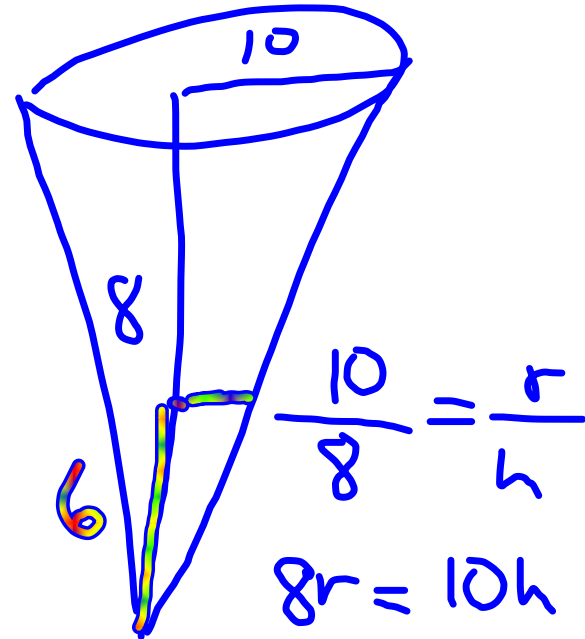
$$\frac{3\sqrt{7}}{100} = \frac{25\sqrt{7}(12)}{100} \frac{dz}{dt}$$

A conical tank of diameter 20 feet has a depth of 8 feet. The point of the cone is pointing down. If the water is leaking out at a rate of 5 ft³ per minute, when the depth of the water is 6 feet determine:

a. the velocity at the top of the water

$$\frac{dh}{dt} = ???$$

∴



c. how fast the water is moving down the tanks side.

$$\frac{10}{8} = \frac{r}{6} \implies r = 7.5$$

c. how fast the water is moving down the tanks side.