$$(x^2y^3)^2 = x^4y^6$$

$3 \cdot 2$

$$(x+y)^2$$

$$(x-8)^{1/2}$$

$$f(x) = \sqrt{x-8}$$

$$= (x-8)^{1/2}$$

$$f'(x) = \frac{1}{2}(x-8)^{-1/2}$$

$$= \frac{1}{2(x-8)^{1/2}} = \frac{1}{2\sqrt{x-8}}$$

Product Rule / Quotient Rule / Higher Order Derivatives

Higher Order Derivatives

function	1st	2nd	etc
y	$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$	$\frac{d^3y}{dx^3}$	etc

Ex: $f(x) = x^7 - 3x^5 - \frac{6}{x} + \sqrt{x}$

$= x^7 - 3x^5 - 6x^{-1} + x^{1/2}$

$= x^7 - 3x^5 - 6x^{-1} + x^{1/2}$

Find the 2nd derivative

Ex: $f(x) = x^7 - 3x^5 - \frac{6}{x} + \sqrt{x}$

$= x^7 - 3x^5 - 6x^{-1} + x^{1/2}$

$f'(x) = 7x^6 - 15x^4 + 6x^{-2} + \frac{1}{2}x^{-\frac{1}{2}}$

$f''(x) = 42x^5 - 60x^3 - 12x^{-3} - \frac{1}{4}x^{-\frac{3}{2}}$

$= 42x^5 - 60x^3 - \frac{12}{x^3} - \frac{1}{4x^{3/2}}$ power

root

$= 42x^5 - 60x^3 - \frac{12}{x^3} - \frac{1}{4\sqrt{x^3}}$

Product Rule

If $y = f(x) \cdot g(x)$
$= u \cdot v$

$$\frac{dy}{dx} = \left(\text{derivative of 1st}\right)\left(\text{orginal 2nd}\right) + \left(\text{der. of 2nd}\right)\left(\text{original 1st}\right)$$

or $\dfrac{dy}{dx} = f'(x) \cdot g(x) + g'(x) \cdot f(x)$

$$\boxed{= u'v + v'u}$$

Ex: $f(x) = x^3 \cos x$

1st $u = x^3$ 2nd $v = \cos x$

$u' = 3x^2$ $v' = -\sin x$

$f'(x) = 3x^2 \cos x + (-\sin x)(x^3)$

$= 3x^2 \cos x - x^3 \sin x$

Answer

$$f(x) = (x+3)^8 (x-2)^7$$

$u = (x+3)^8 \qquad v = (x-2)^7$

$u' = 8(x+3)^7 \cdot 1 \qquad v' = 7(x-2)^6 \cdot 1$

$$f'(x) = \underbrace{8(x+3)^7(x-2)^7} + \underbrace{7(x-2)^6(x+3)^8}$$

$$= (x+3)^7(x-2)^6\left[8(x-2) + 7(x+3)\right]$$

$8x - 16 + 7x + 21$

$$= (x+3)^7(x-2)^6(15x+5)$$

Quotient Rule

If $y = \frac{u}{v}$, then

$$\frac{dy}{dx} = \frac{(\text{der of num} * \text{original den}) - (\text{der of den.} * \text{original num})}{(\text{denominator})^2}$$

$$\frac{dy}{dx} = \frac{u'v - v'u}{v^2}$$

$$y = \frac{x^3}{2x+3}$$

~~num~~ $u = x^3$ ~~den~~ $v = 2x+3$

$u' = 3x^2$ $v' = 2$

$$\frac{dy}{dx} = \frac{3x^2(2x+3) - 2x^3}{(2x+3)^2}$$

don't expand

$$= \frac{6x^3 + 9x^2 - 2x^3}{(2x+3)^2}$$

$$= \frac{4x^3 + 9x^2}{(2x+3)^2}$$

answer!