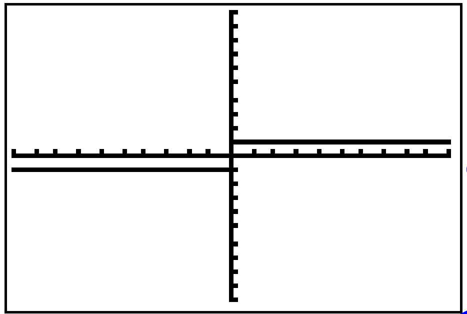
Determine whether the following limit exists.

Limit $|x|/x$

$x \to 0$



① $\lim_{x \to 0^-} \frac{|x|}{x} = -1$

② $\lim_{x \to 0^+} \frac{|x|}{x} = 1$

③ Since $\lim_{x \to 0^-} \frac{|x|}{x} \neq \lim_{x \to 0^+} \frac{|x|}{x}$

the $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

Example 3: Given the graph of *h(x)* shown below determine each following values:

1. $\lim_{x\to a} h(x)$
2. $\lim_{x\to g} h(x) = c$
3. $\lim_{x\to m} h(x) = p$
4. $\lim_{x\to j} h(x) = q$
5. $\lim_{x\to e} h(x) = h$
6. $\lim_{x\to 0} h(x)$ q
7. $\lim_{x\to \infty} h(x)$ c
8. $\lim_{x\to -\infty} h(x)$ c
9. $\lim_{x\to b} h(x)$ ∞

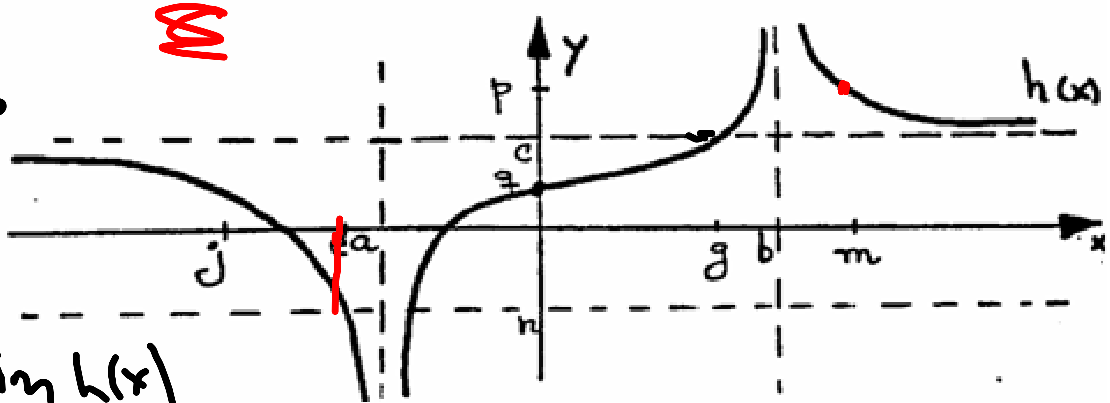
$\lim_{x\to a^-} h(x) = -\infty$

$\lim_{x\to a^+} h(x) = -\infty$

Lim

Since $\lim_{x\to a^+} h(x) = \lim_{x\to a^-} h(x)$

$\lim_{x\to a} h(x) = -\infty$



Limit Laws

Constant Limit:

$$\lim_{x \to c} C = C$$ (limit of the constant is the constant)

Addition Law

Difference Law

Product Law

Quotient Law

Constant Multiple

Power Law

* Limits of Polynomial Functions/ Rational

① If p is a polynomial function & c is a real number then

$$\lim_{x \to c} p(x) = p(c)$$

Ex: $\lim_{x \to 2} 2x^2 + 3x - 1 = 2(4) + 6 - 1 = 13$

② If R is a Rational function such that $R(x) = \frac{p(x)}{g(x)}$ & c is a real number such that $g(c) \neq 0$

$$\lim_{x \to c} r(x) = r(c) = \frac{p(c)}{r(c)}$$

** Limits that fail to exist

① Left behavior is different than right behavior Ex: $f(x) = \frac{|x|}{x}$

② Unbounded behavior

$\lim_{x \to 0} f(x)$ when $f(x) = \frac{1}{x}$

b/c $f(x)$ is not approaching a real number

as $x \to 0$, you can conclude that

the limit does not exist

③ Oscillating behavior - (very common)

$\lim_{x \to 0} \sin\left(\frac{1}{x}\right) =$ does not exist

More algebra techniques

Radicals $\to$ multiply by conjugate

$$\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}\left(\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}\right)$$

$$\lim_{x\to 0} \frac{\cancel{2}+x-\cancel{2}}{x\left(\sqrt{2+x}+\sqrt{2}\right)}$$

$$\lim_{x\to 0} \frac{\cancel{x}^{\,1}}{\cancel{x}\left(\sqrt{2+x}+\sqrt{2}\right)} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$