The Official SAT Question of the Day™

Read the following SAT test question, then click on a button to select your answer.

3, 6, 11, 18, ...

The first four terms of a sequence are shown above. Which of the following could be the formula that gives the nth term of this sequence for all positive integers n?

A. $2n$

B. $2n + 1$

C. $3n$

D. $n^2 + 1$

E. $n^2 + 2$

1.5 Inverse Functions

definition of an inverse

① Two functions are inverses if

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$

Show that f & g are inverses.

$f(x) = x^2$, $x \geq 0$ $\qquad$ $g(x) = \sqrt{x}$

$f(g(x)) = f(\sqrt{x})$

$= (\sqrt{x})^2$

$= x$ ✓

$g(f(x)) = g(x^2)$

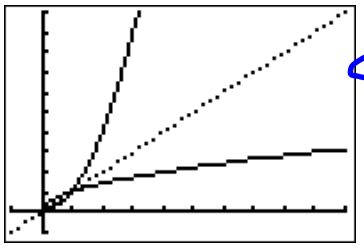
$= \sqrt{x^2}$

$= x$ ✓

$\therefore$ f & g are inverses

② Graphically

Two functions are inverses if they reflect across the line $y=x$



← Does the prove that 2 functions are inverses?

No ⇒ you have to show $f(g(x))$ & $g(f(x))$

③ Existence of an inverse function

Use horizontal line test ⇒ if a horizontal line passes through a function at most one time, the function has an inverse ⇒ you can limit the domain in order to get an inverse for many functions.

* To have an inverse a function must be "one-to-one" or the domain must be limited.

One-to-one ⇒ no two elements in the domain correspond to the same element in the range → each x has a unique y

How to find Inverses

1. Points

$\{(2,3)\ (-5,7)\ (-8,-9)\}$

switch x and y → which reflects the mirror $y = x$

$\{(3,2)\ (7,-5)\ (-9,-8)\}$

2. Algebraically

1. Decide if it has an inverse ⇒ restrict domain if necessary
2. Change $f(x)$ to y.
3. Switch x & y & solve for y.
4. Replace y w/ $f^{-1}(x)$

Ex: $f(x) = x^3 + 1$

$y = x^3 + 1$

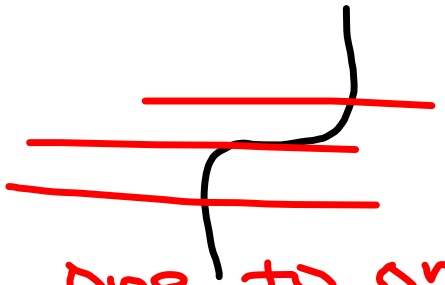
$x = y^3 + 1$

$x - 1 = y^3$

$y^3 = x - 1$

$y = \sqrt[3]{x-1}$

$f^{-1}(x) = \sqrt[3]{x-1}$

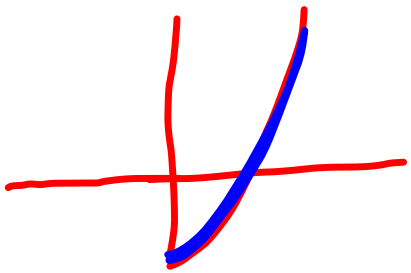


One to one

↓

$f(x) = x^3 + 1$ has an inverse

$f(x) = x^2 - 4 \quad x \geq 0$



$y = x^2 - 4$

$x = y^2 - 4$

$x + 4 = y^2$

$\sqrt{x+4} = y$

$f^{-1}(x) = \sqrt{x+4}$

$f(x) = 3x - 7$