

Goldbach and the great Prime number Theorem

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Goldbach stated that every even number bigger or equal to six could be written as the sum of two prime numbers.

For example $6=3+3$, $8=5+3$, $10=5+5=7+3$ etc.

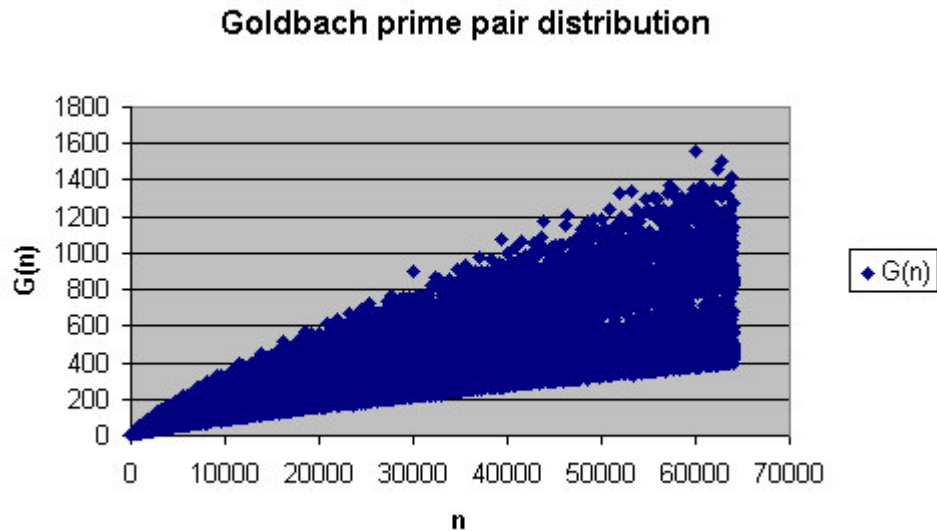
His conjecture could not be proven and is still not proven today. Proving his conjecture will equal you to the likes of Wiles who proved Fermat's last theorem.

I want to contribute something interesting towards solving this conjecture.

Let $G(n)$ be the number of prime pairs whose sums are equal to n where n even.

So $G(6)=1$ and $G(10)=2$ and $G(324)=20$ and so on.

Drawing a graph of $G(n)$ we see the following.

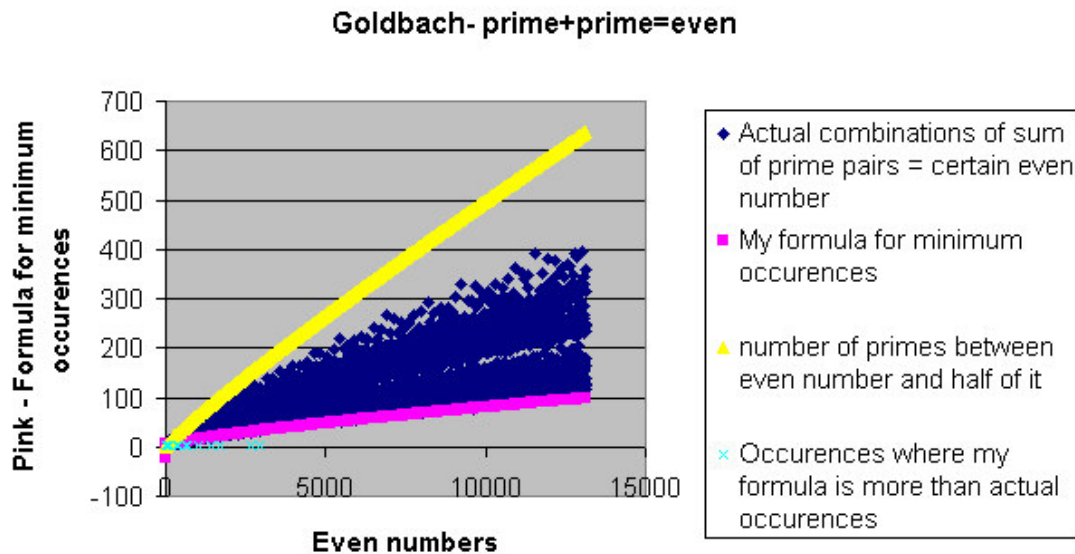


Interesting here is the fact that the minimum (bottom) values follow the $x/\ln(x)$ curve which can be quite clearly seen when maximum n somewhat smaller.

The great prime number theorem states that the primes less than n are approximately $n/\ln(n)$. Clearly the prime pairs per n cannot exceed the number of primes between n and $n/2$. The number of primes between n and $n/2$ is $h = n/\ln(n) - 0.5n/\ln(n/2)$

Because the bottom values of $G(n)$ follow a $x/\ln(x)$ function we should be able to create such a function such that this function follows the bottom of $G(n)$.

Intuitively we form $C(n) = h/\ln(h)$ where $h = n/\ln(n) - 0.5n/\ln(n/2)$
When plotting $C(n)$ on the same graph as $G(n)$ we get the following.



The pink area is $C(n)$. The dark blue area is $G(n)$ and the yellow area is h . The light blue area is where $G(n) < C(n)$. Note that for $n > 3000$ $C(n)$ follows the bottom of $G(n)$ perfectly and for big values of n it drops below $G(n)$. $C(n)$ tends to infinity as n tends to infinity. (Le Hospital's rule)

My conjecture thus as follows. $G(n) > C(n)$ for $n > 3000$ and $h = n/\ln(n) - 0.5n/\ln(n/2)$ Because values for $n < 3000$ has been confirmed my conjecture implies that every even number ≥ 6 can be written as the sum of two primes and furthermore that as n tends to infinity so will $C(n)$ and so will $G(n)$.

The even numbers for large n will therefore not only have a few prime pairs, but many and this is due to the fact that the number of primes between n and $n/2$ increases as well.

Also note the following

$$C(n) = h/\ln(h)$$

So the rate of change with increasing h is $[\ln(h) - 1] / [\ln(h) \ln(h)]$

For large h we have $\ln(h) / [\ln(h) \ln(h)]$

So the rate of change with increasing h is $1 / \ln(h)$

The rate of change for large h and thus for large n is thus proportional to the inverse log function and thus at infinity the rate of change will slow down to zero while the prime pairs will have increased to infinity.

The end.
