Electromagnetic Induction

Earlier we noted that if we moved a conductor in a magnetic field that we tended to make a current flow in the conductor. Faraday also observed that a changing magnetic field induced a voltage in a coil. Examining this process revealed the following behaviour

The voltage induced depended on how quickly the magnetic field changed.
The voltage induced depended on the number of turns on the coil.
The voltage induced depended on how much magnetic flux passed through (linked with) the coil.

This behaviour can be described by **Faraday's Law of electromagnetic induction**.

$$emf = N\frac{d\Phi}{dt}$$

Direction of EMF (Lenzs Law)

The EMF induced on the coil tends to make a current flow through the coil if an external circuit is present in the system. The direction of the current is such that it tends to reduce any changes in the flux linking the coil.

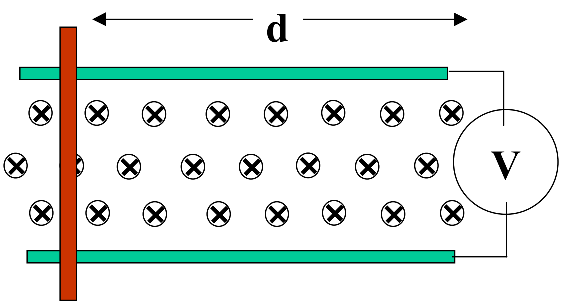
If we move a coil from a region of high magnetic field, (Large flux linkage) to a region of low magnetic field the flow of current will tend to increase the flux within the coil.

If we move a coil from a region of low magnetic field, (small flux linkage) to a region of high magnetic field the flow of current will tend to decrease the flux within the coil.

We can look at this behaviour by considering what happens if we treat our simple motor as a generator.

A simple generator

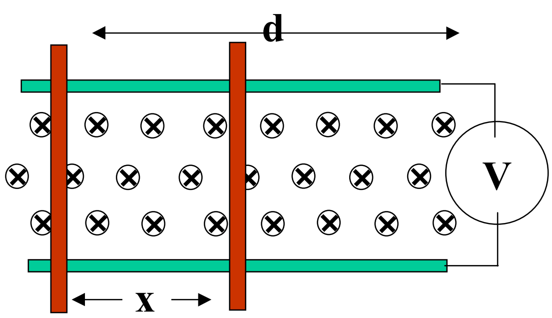
Imagine we take the motor from the previous example and disconnect the current source and replace it with a voltmeter.



The fixed rails, the sliding conductor and the voltmeter make up a single turn.
This turn is linked by a flux. Whose size is dependant on the flux density B and the area of the loop made up by the single turn in the system.

If we move the slider to the left, we increase the area of the turn and therefore increase the flux that links this turn.
If we move the slider to the right we decrease the area and therefore decrease the flux linking the turn.

Imagine we move the slider to the right at a constant velocity so that in one second the slider moves a distance x.



Original Area is

$$A_0 = ld$$

Original flux linking loop

$$\Phi_0 = BA_0 = Bld$$

After slider has moved distance x to the right the area of loop becomes:

$$A_x = l(d - x)$$

and the flux linking the loop become

$$\Phi_x = BA_x = Bl(d - x)$$

The change in the flux linking the coil is therefore given by

$$\Delta\Phi = \Phi_x - \Phi_0$$
$$= -Blx$$

Above we assumed that the slider had moved the distance x in one second
This allows us to write an expression for the rate of change of flux linking the coil.

$$\frac{d\Phi}{dt} = \frac{\Delta\Phi}{\Delta t} = \frac{-Blx}{1} = -Blx$$

We can now calculate a value for theEMF induced across the slider as it moves to the right

$$E = N\frac{d\Phi}{dt}$$

As there is only a single turn in the system, the loop made up of the slider, the fixed contacts and the voltmeter the expression for the EMF becomes

$$E = N\frac{d\Phi}{dt} = -Blx$$

Lets assume that the distance x we move the slider is 0.1m. We have figures for B=5 T and l=0.2 m from our original examination of the system as a motor.
We can therefore calculate the value of EMF that we would see on the system

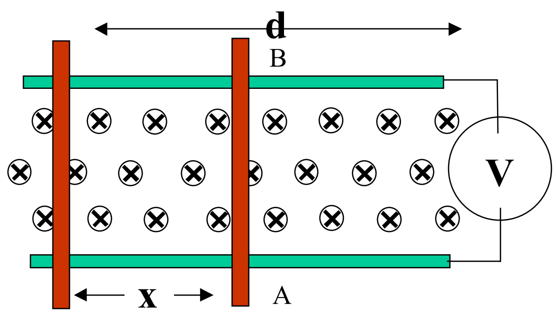
$$E = -Blx = -5 \times 0.1 \times 0.2 = -0.1V$$

We can determine the direction of the EMF by considering Lenz's law

The movement of the slider is reducing the magnetic flux linking the turn.

Therefore a current will want to flow in a direction that will increase the flux linking the coil.

To achieve this the current must flow from through the slider from the bottom of the diagram to the top of the diagram. This implies that the point A must be at a higher potential than the point B

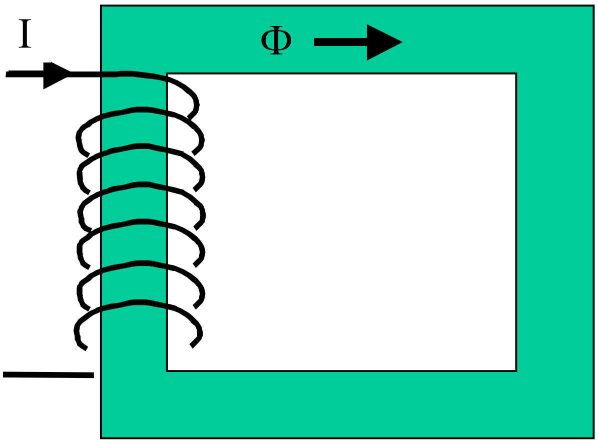


Self inductance

So far we have looked at changes in flux linkage caused by changing the area of our loop. Lets now look at how a coil carrying current interacts with itself.

Lets consider a coil of N turns carrying a current I wrapped round a ferromagnetic core with a reluctance $\Re$.

We can calculate the flux generated by the coil in the core.



$$\Phi = \frac{NI}{\Re}$$

The flux generated in the core by the coil passes through the coil. That means that it links with the coil.

Therefore if the flux in the core changes Faradays law of Electromagnetic Induction suggests that a voltage will appear across the coil

$$E = N\frac{d\Phi}{dt}$$

Combining the two equations above allows us to relate the voltage induced on the coil to the rate of change of current passing through the coil.

$$E = N\frac{d\frac{NI}{\Re}}{dt}$$

as the number of turns on the coil and the reluctance of the core are constants the equation can be rewritten in the form:

$$E = N\frac{N}{\Re}\frac{dI}{dt} = \frac{N^2}{\Re}\frac{dI}{dt}$$

Remember the equation that we stated in the section on AC circuits for the Inductor

$$V = L\frac{dI}{dt}$$

By inspection we can see that the two equations describe the same behaviour and we can therefore conclude that:

$$L = \frac{N^2}{\Re}$$

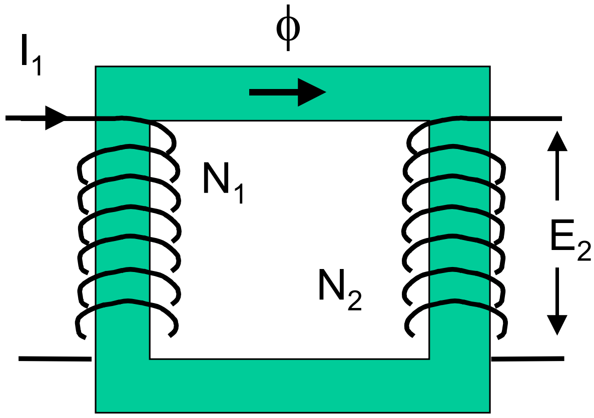
This relationship allows us to determine the value of inductance expected for known values of N and $\Re$.

Mutual Inductance

What happens if we have more than one coil wrapped around a ferromagnetic core.

Consider what happens if we have a core of reluctance $\Re$

Wrapped round the core we have two windings.
The first has N_1 turns
The second has N_2 turns



A current I_1 passes through the first winding.
This will generate a flux in the core.

$$\Phi = \frac{NI_1}{\Re}$$

There will be a self inductance associated with coil 1

$$L_1 = \frac{N_1^2}{\Re}$$

This flux generated by the current in winding 1 flows round the core and passes through (Links with) winding 2
Faradays Law of Electromagnetic Induction suggests that an EMF will appear across winding 2 if the flux linking it changes.

$$E_2 = N_2 \frac{d\Phi}{dt}$$

We can relate the EMF appearing across the second coil to the rate of change of current in the first coil.

$$E_2 = N_2 \frac{d\frac{NI_1}{\Re}}{dt}$$

As N_1 and $\Re$ are constants we can rearrange this expression to give:

$$E_2 = N_2 \frac{N_1}{\Re}\frac{dI}{dt} = \frac{N_1 N_2}{\Re}\frac{dI}{dt}$$

We define a term known as the Mutual Inductance to describe the relationship between the voltage induced on one coil by the change of current in the second coil.

$$E_2 = M_{21}\frac{dI_1}{dt} \qquad \text{where } M_{21} = \frac{N_1 N_2}{\Re}$$

Note if we pass a current I_2 through coil 2 we can define a self inductance for coil 2

$$L_2 = \frac{N_2^2}{\Re}$$

The flux generated by the current in coil 2 links with coil 1 therefore we can produce an equation relating the voltage induced in coil 1 by changes in the current in coil 2

$$E_1 = M_{12}\frac{dI_2}{dt} \qquad \text{where} \qquad M_{21} = \frac{N_2 N_1}{\Re}$$

Note as long as all the flux produced by each coil links with the other coil $M_{12} = M_{21}$ and

$$M_{12} = M_{21} = \sqrt{L_1 L_2}$$

<u>Transformer action</u>

For the system above we can derive two equations.

$$E_1 = L_1\frac{dI_1}{dt} = \frac{N_1 N_1}{\Re}\frac{dI_1}{dt} \Leftrightarrow \frac{E_1}{N_1} = \frac{N_1}{\Re}\frac{dI_1}{dt}$$

$$E_2 = M_{21}\frac{dI_1}{dt} = \frac{N_1 N_2}{\Re}\frac{dI_1}{dt} \Leftrightarrow \frac{E_2}{N_2} = \frac{N_1}{\Re}\frac{dI_1}{dt}$$

From inspection of these two equations we can see that

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} \Leftrightarrow \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

That is the ratio of the voltages on the two windings depends on the ratio of the turns on the two windings. This property forms the basis of the important electrical machine the transformer.