Spektralanalyse physiologischer Signale

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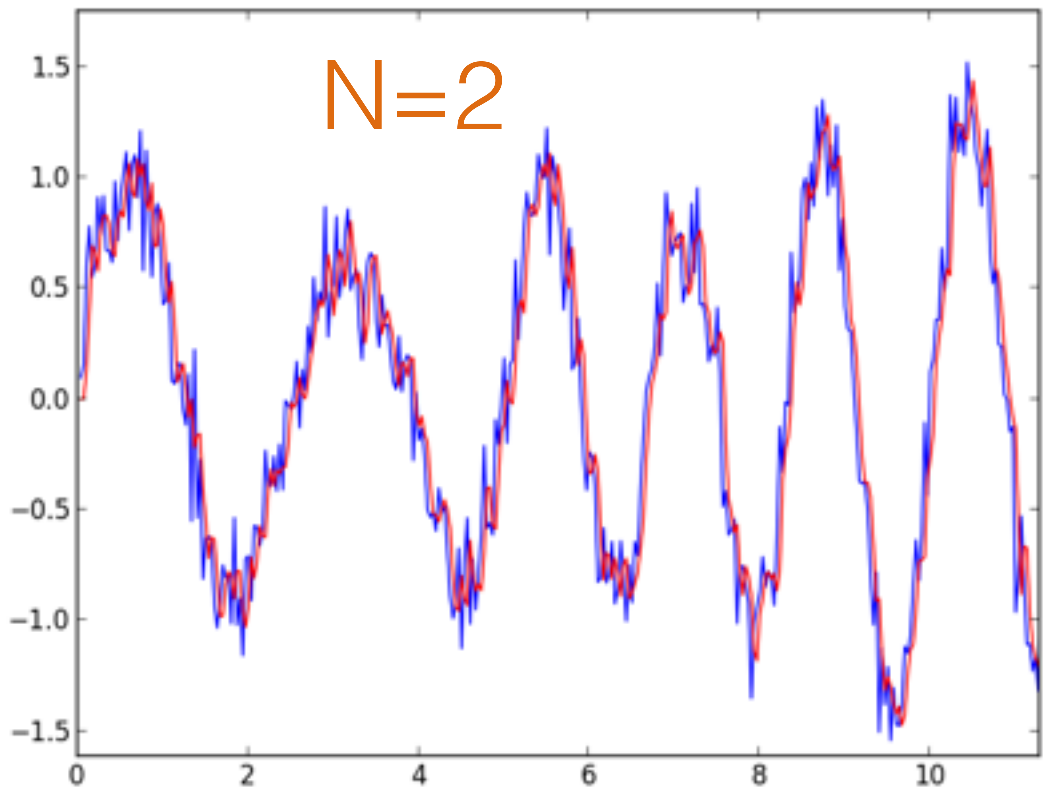
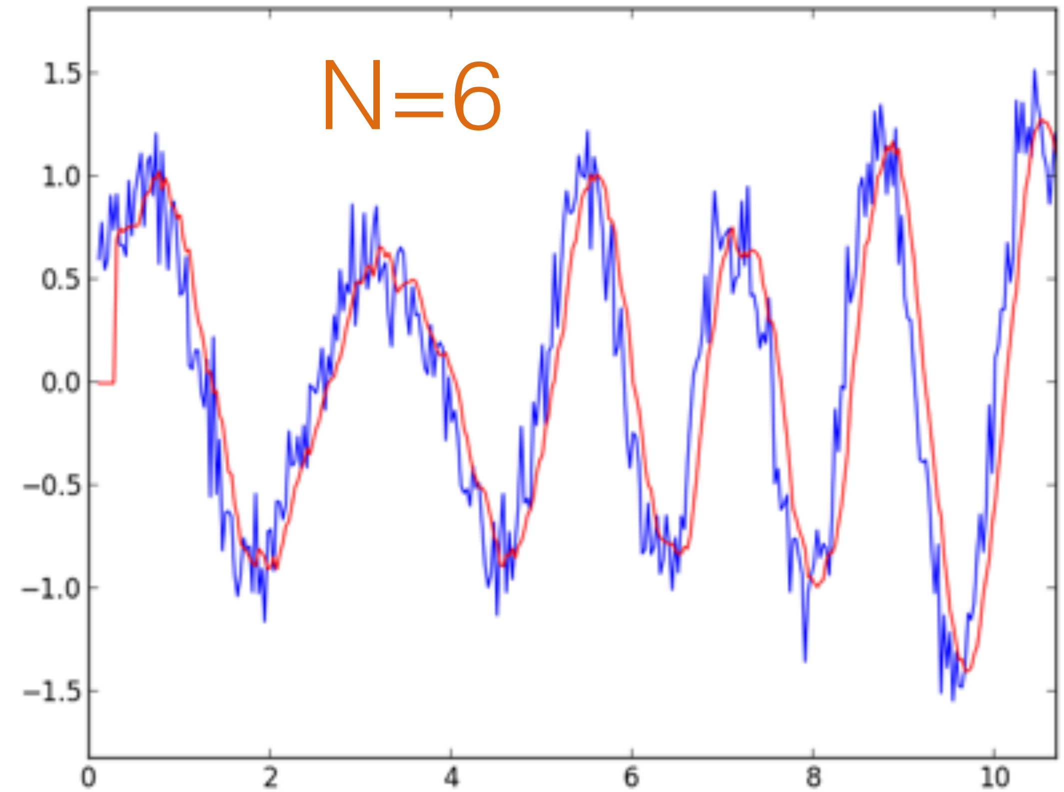
Vorlesung 9

zum Übungsblatt

1)

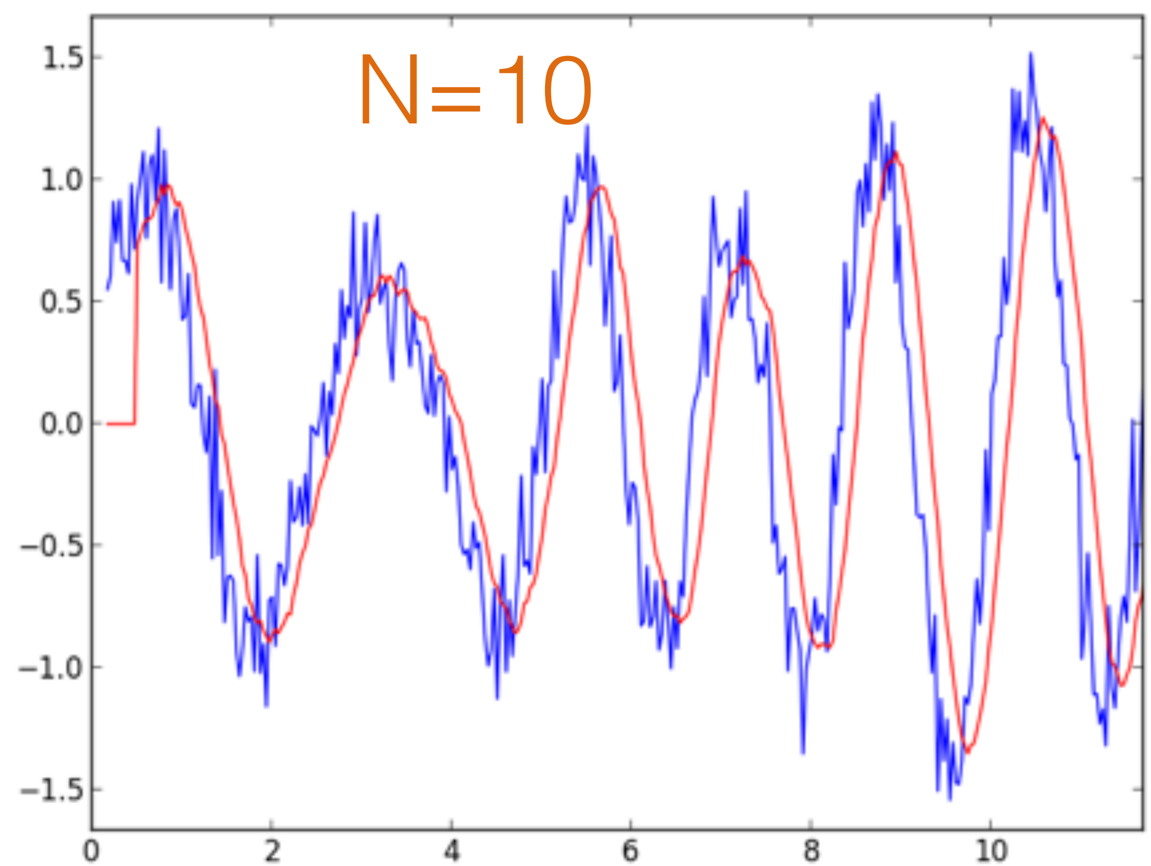


N=2
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8
10

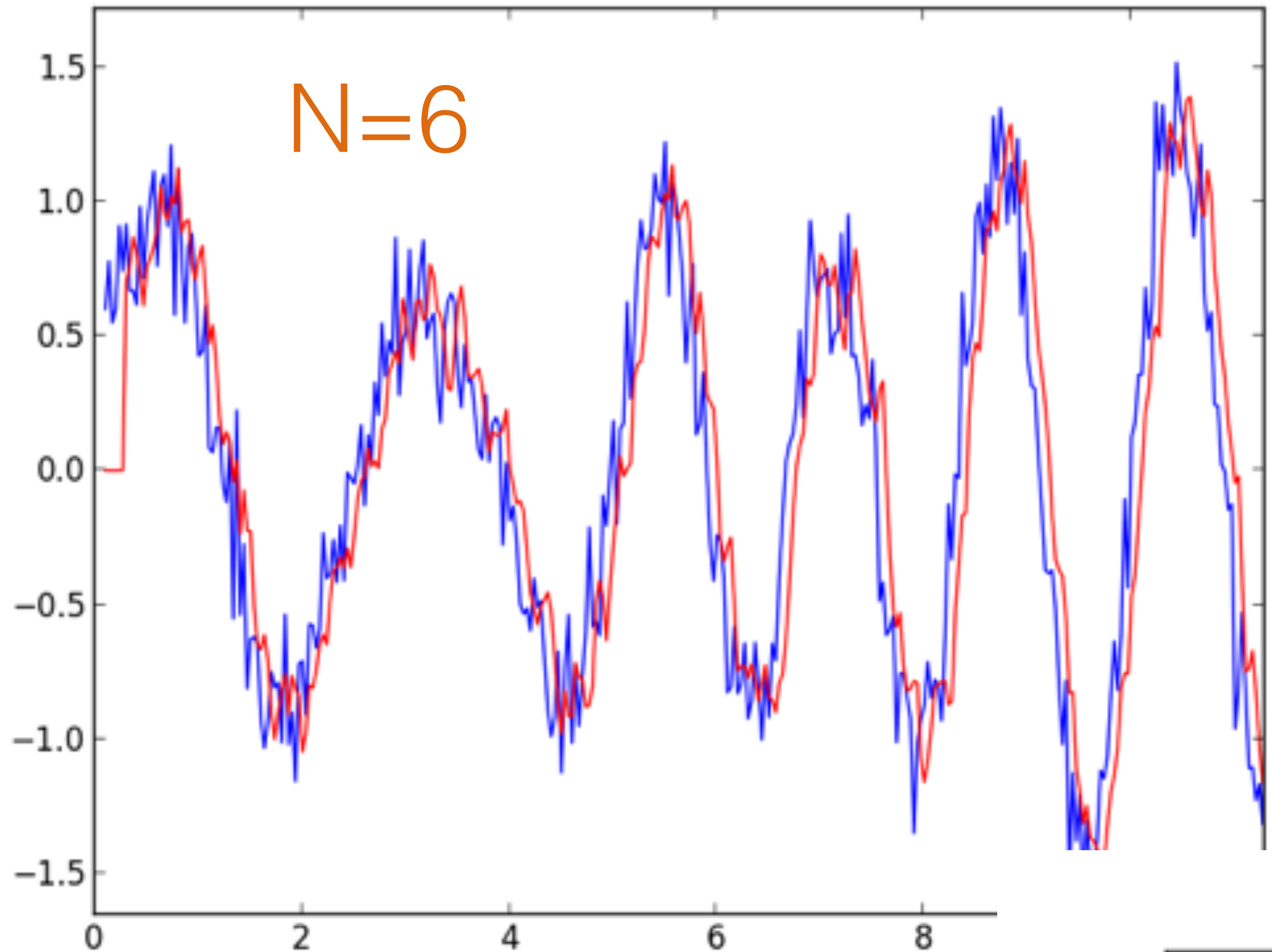


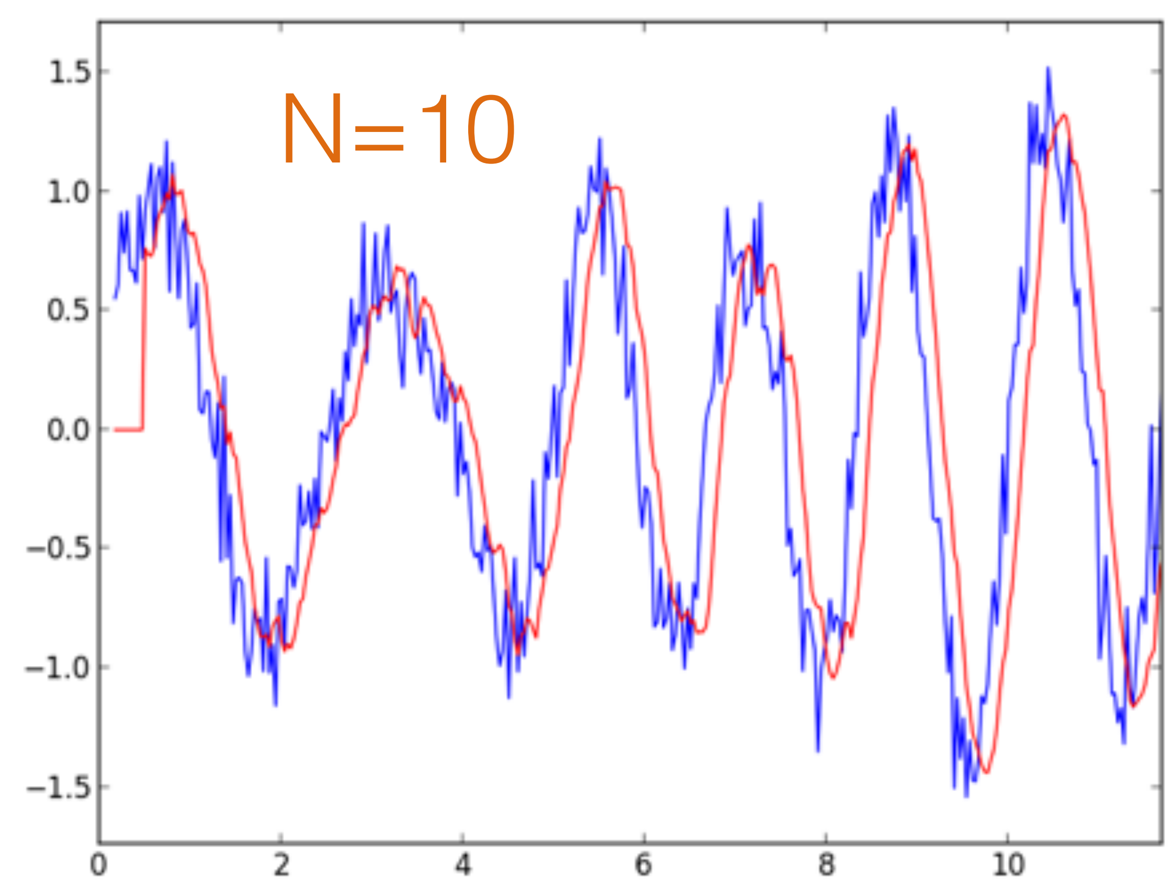
N=6
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8
10

m=0



N=10
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8
10





m=3

2)

wie klein darf Zeitfenster sein ?

T>1/Δf

3)

$$\Psi(t) = e^{-at^2} e^{i\sigma t}$$

$$\tilde{\Psi}(f) = \sqrt{\pi} e^{-(\sigma - 2\pi f)^2/4a}$$

$$f_m = \frac{\sigma}{2\pi} \qquad stddev = \sqrt{2}\sqrt{a}$$

komplexwertiges Morlet: $$f_m = \frac{\sigma}{2\pi} \qquad stddev = \sqrt{2}$$

realwertiges Morlet: $$f_m = \frac{2.5}{\pi} \qquad stddev = \sqrt{2}\sqrt{\sigma}$$

komplexwertiges Gauss: $$f_m = \frac{0.5}{\pi} \qquad stddev = \sqrt{2}\sqrt{\sigma}$$

weiter mit Vorlesung 9

$$\mathcal{F}^{-1}[sgn(\nu)](t) = \frac{i}{\pi t}$$

$$\mathcal{F}^{-1}[T[sgn(\nu)\tilde{s}(\nu)](t) = \frac{i}{\pi}\int_{-\infty}^{\infty}\frac{s(\tau)}{t-\tau}dt$$

für allgemeine Signale s(t):

$$\boxed{s_a(t) = s(t) + i\mathcal{H}[s](t)}$$

mit der Hilbert transform $$\mathcal{H}[s](t) = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau}\, d\tau$$

analytisches Signal is eine andere Darstellung des Signals,

jedoch mit **grossen Vorteilen**

im Allgemeinen:

für alle $x(t) \in \mathcal{C} \rightarrow \boxed{x(t) = Re^{i\phi}}$

R: amplitude

Φ: phase

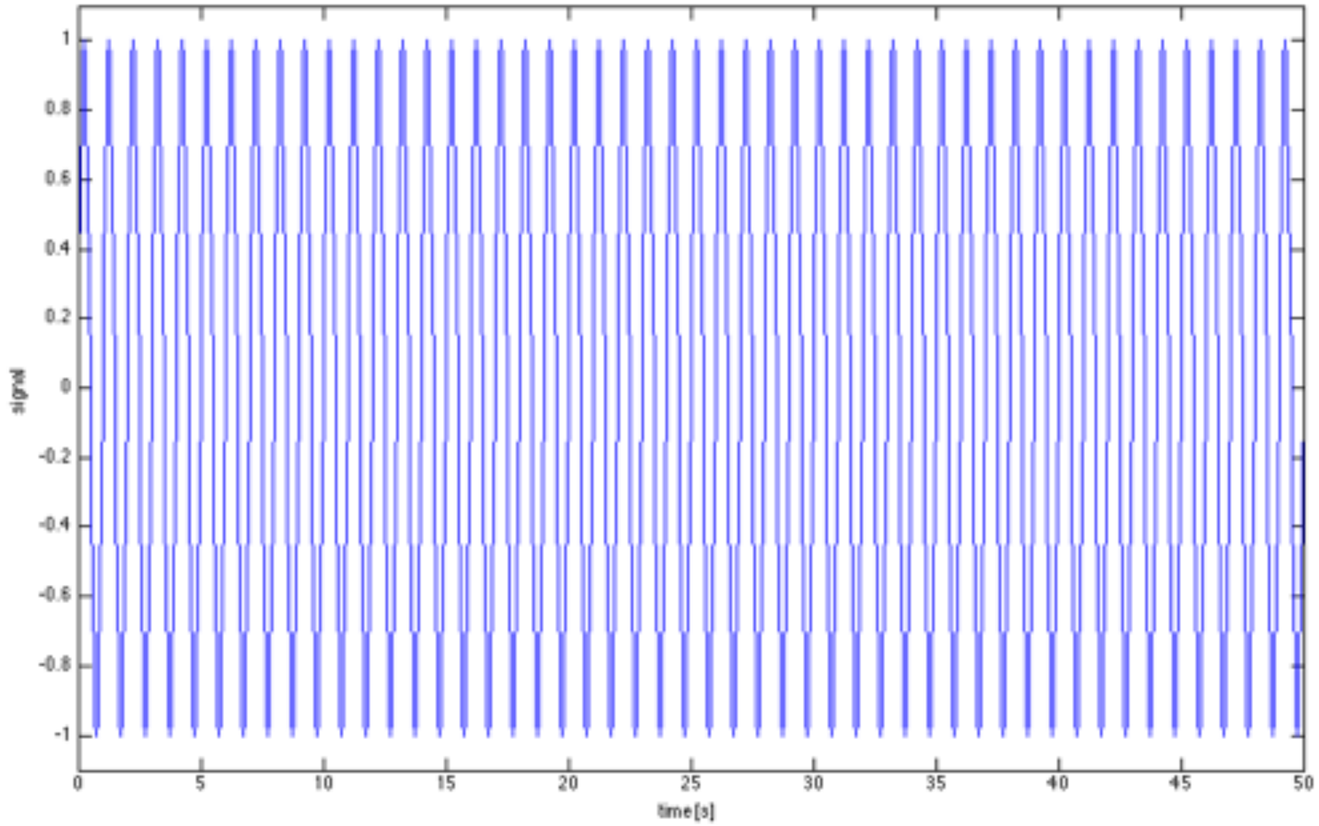
$$s_a(t) = R(t)e^{i\phi(t)}$$

R(t): instantane Amplitude

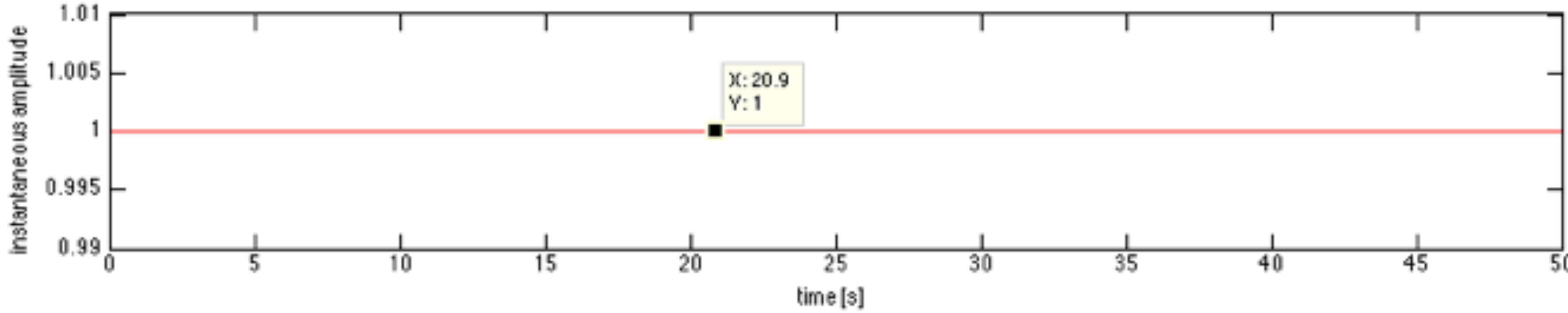
Φ: instantane Phase für $R \neq 0$

→ instantane Frequenz $f_a(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$

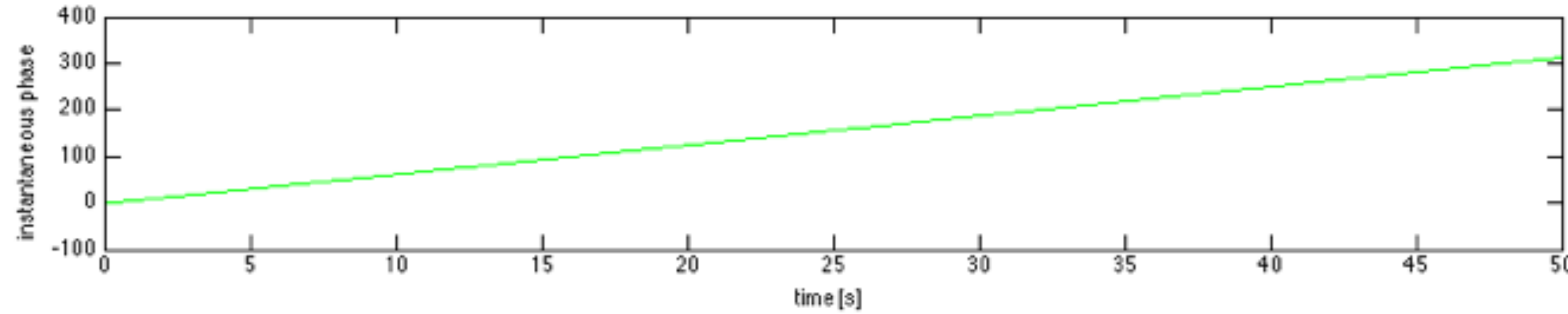
Beispiel



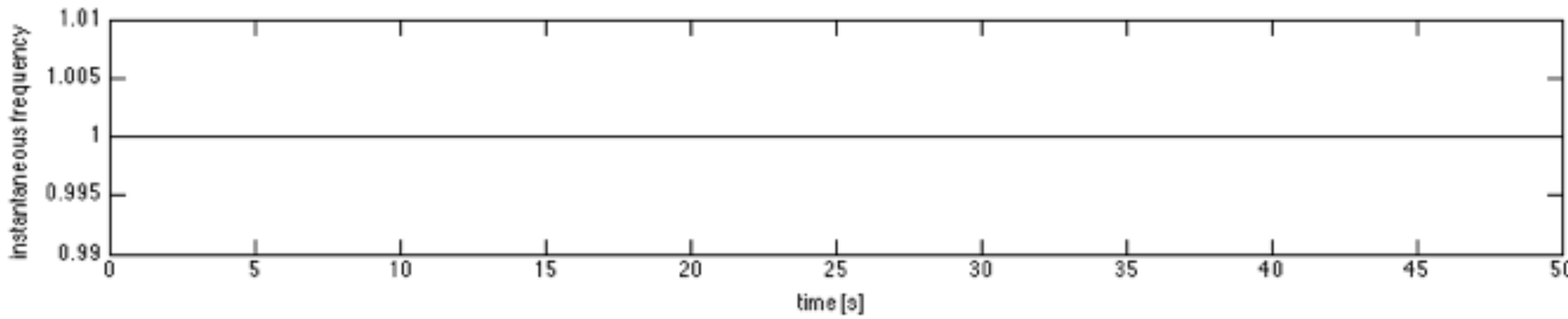
R(t)



Φ(t)



$f_a(t)$



Beispiel: amplituden-moduliertes Signal (AM)

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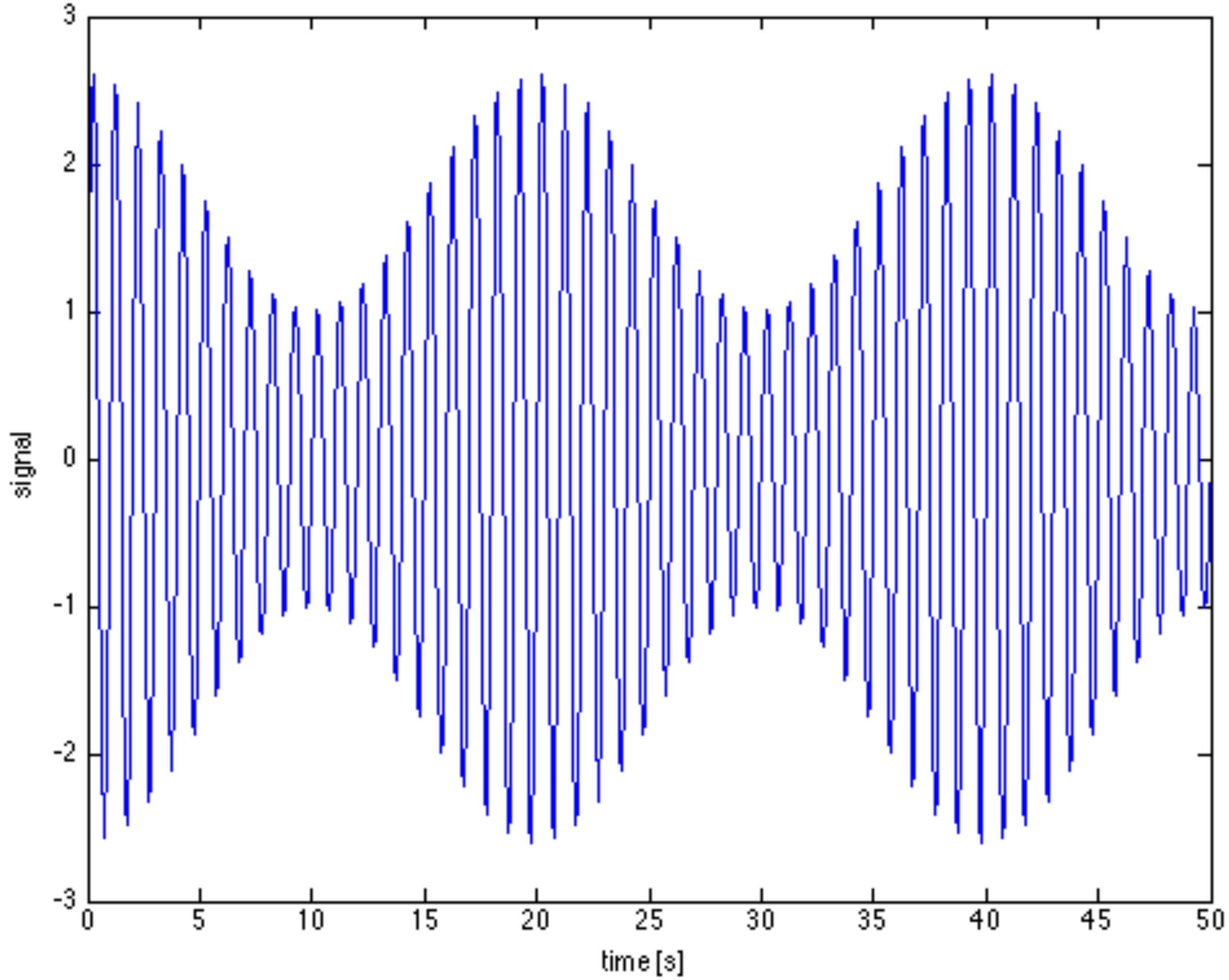
$$s(t) = [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t)$$

f_m=0.05Hz, f=1Hz

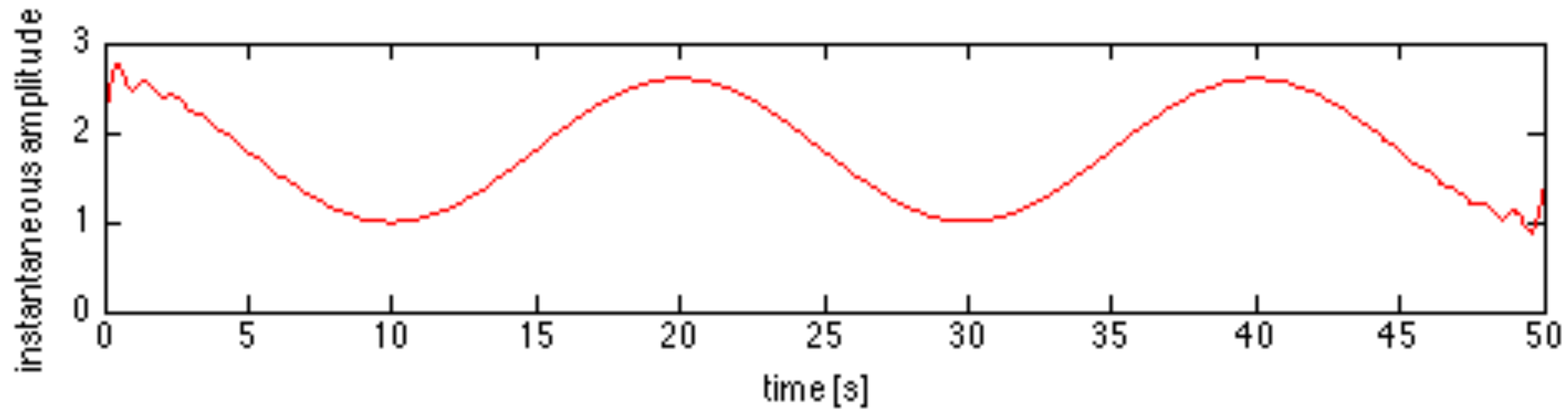
Schwebung



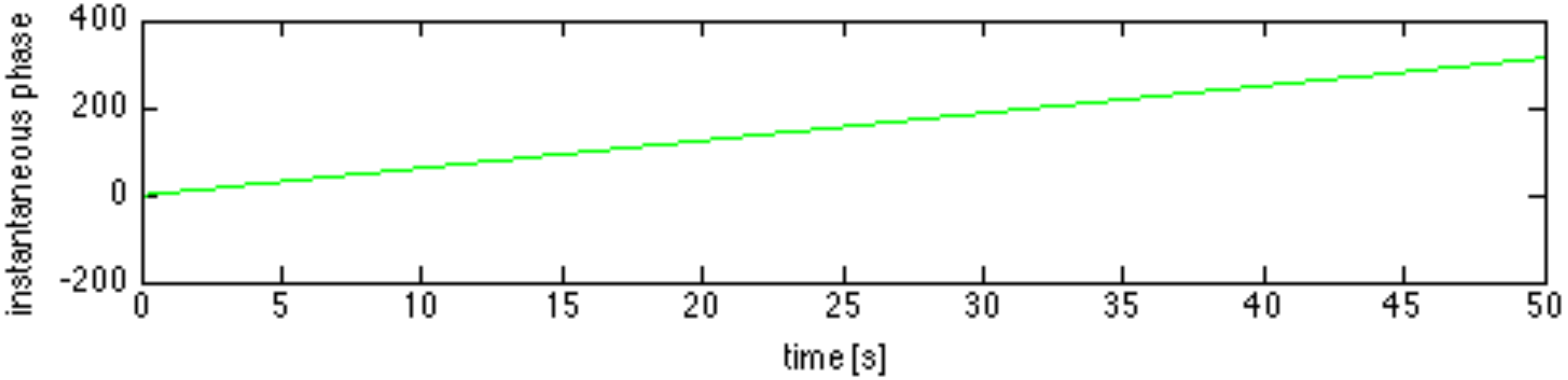
(TF_16.m)

(TF_16.m)

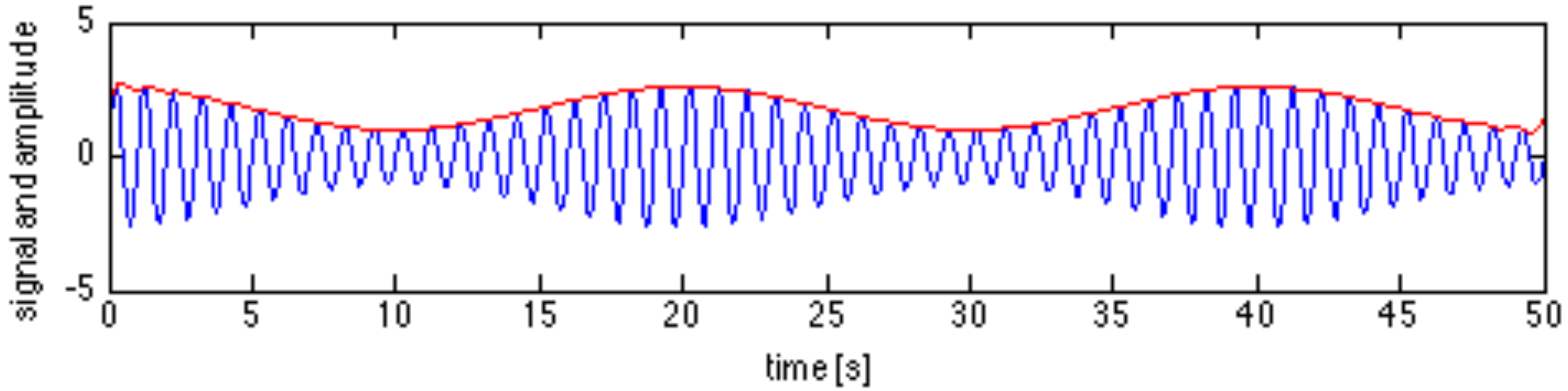
R(t)



Φ(t)



R(t)cos(Φ(t))



analytisches Signal:
Bestimmung der instantanen Amplitude and Phase möglich

Seitenbemerkung:

$$s(t) = [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t)$$

$$= \sin(2\pi f t) + 0.4\sin(2\pi(f + f_m)t) + 0.4\sin(2\pi(f - f_m)t)$$

3 Frequenzen: $f \;,\; f + f_m \;,\; f - f_m$

Amplitudenmodulation induziert Frequenz-Seitenbänder

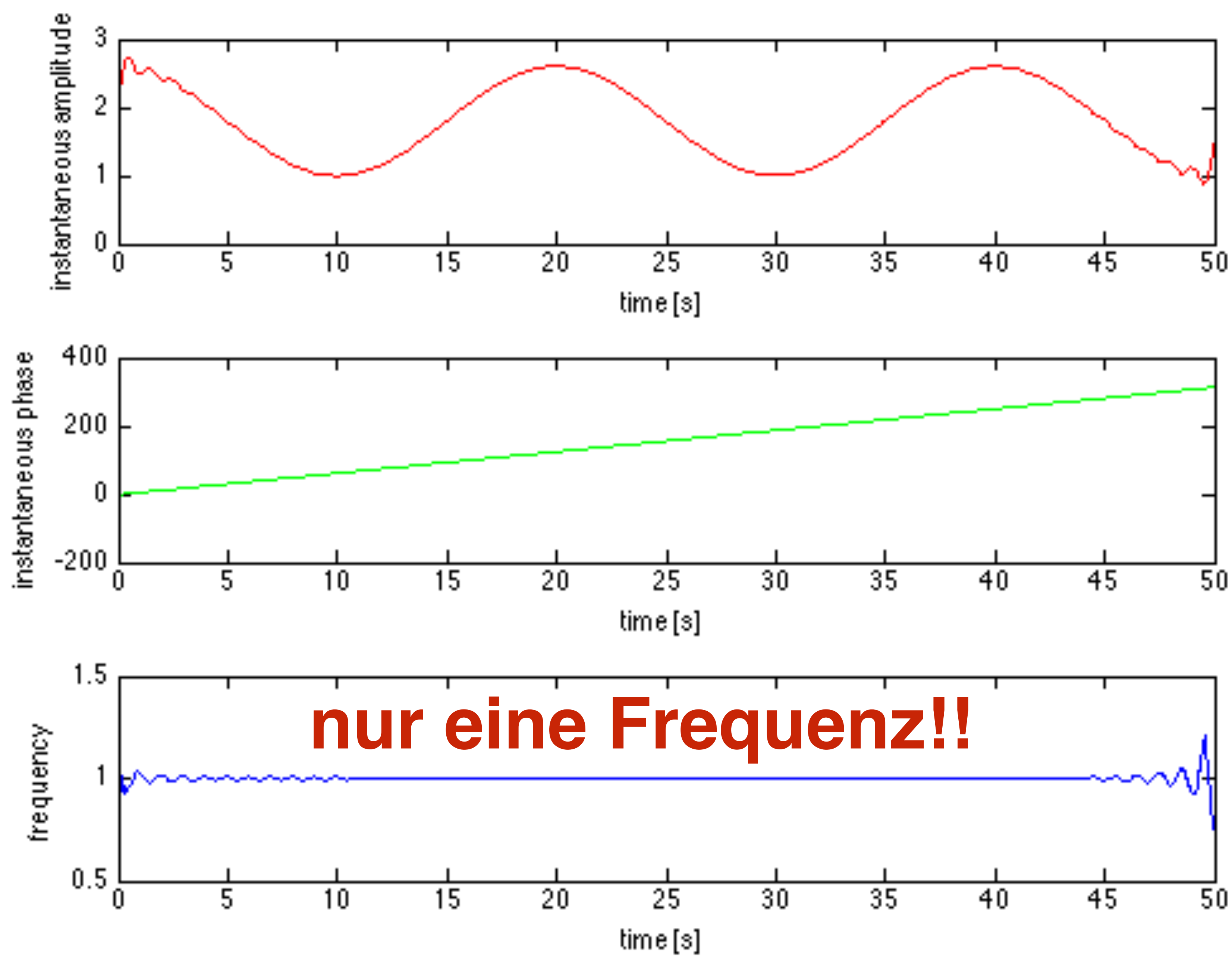
$$s(t) = [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t)$$
$$= \sin(2\pi f t) + 0.4\sin(2\pi (f + f_m) t) + 0.4\sin(2\pi (f - f_m) t)$$

Analyse:

R(t)

Φ(t)

$f_a(t)$



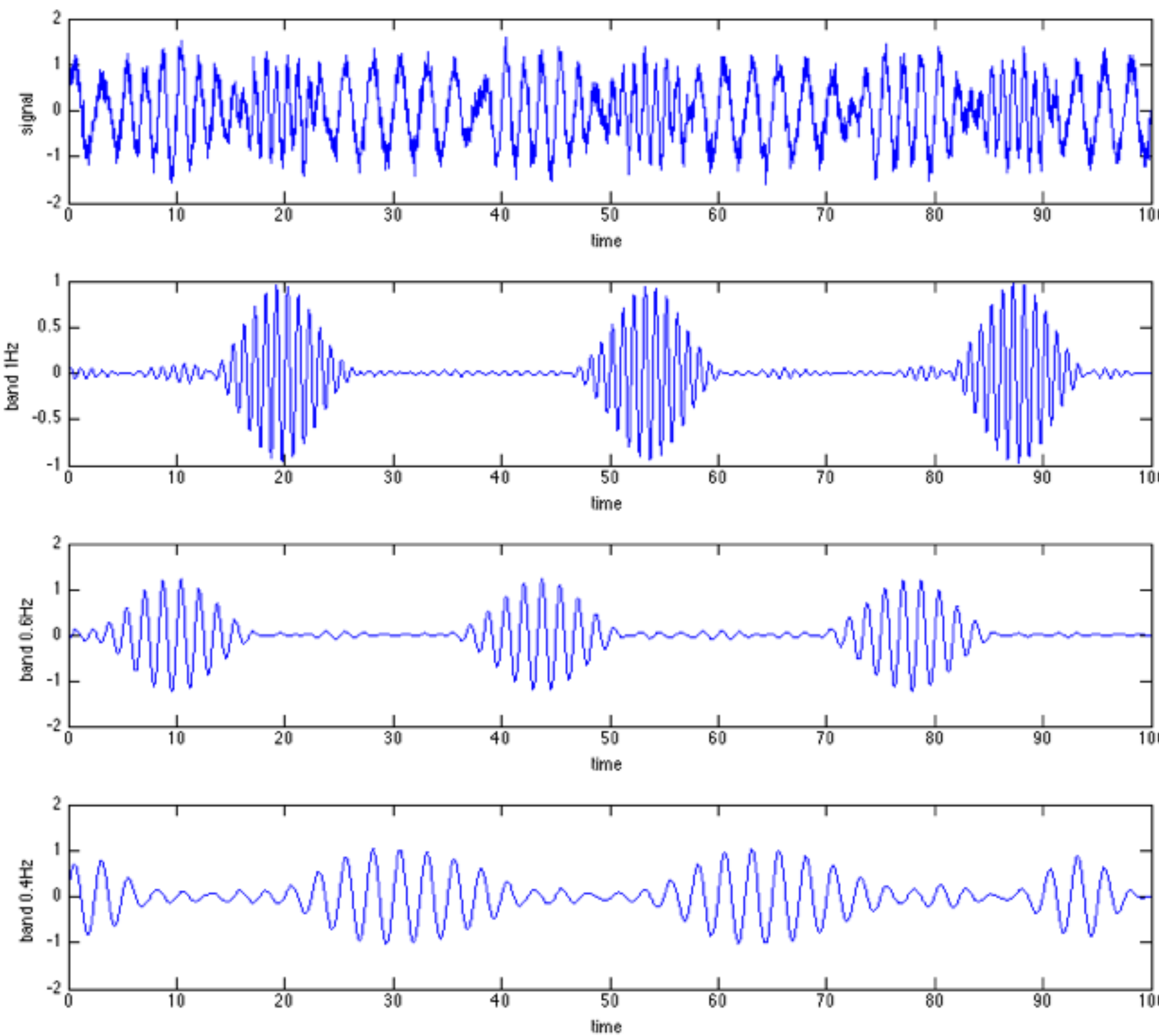
Analyse mittels Analytischem Signal zeigt ein Problem !!

Bemerkung: instantane Frequenz ist **nur** dann **interpretierbar**

falls sie aus einem bandpass-begrenztem Signal

mit **einer Leistungsspitze** bestimmt wird.

Anwendung auf bandpass-gefiltertes Signal

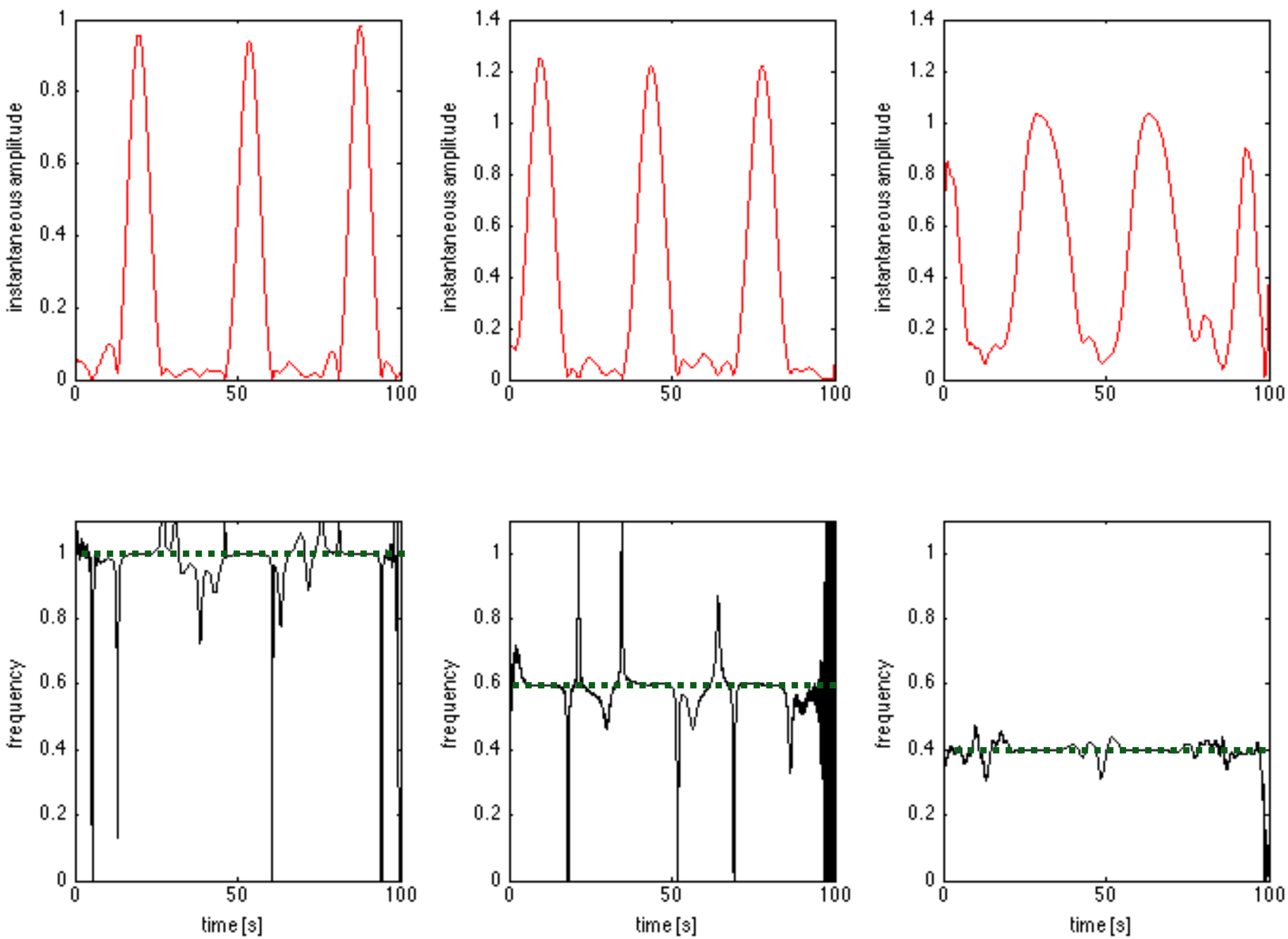


cut-off at
0.9Hz and 1.1Hz

cut-off at
0.5Hz and 0.7Hz

cut-off at
0.3Hz and 0.5Hz

1Hz band 0.6Hz band 0.4Hz band



(TF_19.m)

Konzept des Analytischen Signals wertvoll für

Bestimmung der Amplitudenmodulation,

aber **Vorsicht bei der Interpretation**

von instantanen Frequenzen