Appendix A

Euler-Bernoulli Beam

$[\bar{N}(x)]$ is the shape function matrix for mechanical degrees of freedom and are expressed as

$$\bar{N}_{11} = 1 - \frac{x}{L}, \quad \bar{N}_{12} = -2\,\frac{x\beta_e}{L^2} + 2\,\frac{\beta_e\,x^2}{L^3}, \quad \bar{N}_{13} = -\frac{x\beta_e}{L} + \frac{\beta_e\,x^2}{L^2},$$

$$\bar{N}_{14} = \frac{x}{L}, \quad \bar{N}_{15} = 2\,\frac{x\beta_e}{L^2} - 2\,\frac{\beta_e\,x^2}{L^3}, \quad \bar{N}_{16} = -\frac{x\beta_e}{L} + \frac{\beta_e\,x^2}{L^2},$$

$$\bar{N}_{21} = 0, \quad \bar{N}_{22} = 1 - 3\,\frac{x^2}{L^2} + 2\,\frac{x^3}{L^3}, \quad \bar{N}_{23} = x - 2\,\frac{x^2}{L} + \frac{x^3}{L^2},$$

$$\bar{N}_{24} = 0, \quad \bar{N}_{25} = 3\,\frac{x^2}{L^2} - 2\,\frac{x^3}{L^3}, \quad \bar{N}_{26} = -\frac{x^2}{L} + \frac{x^3}{L^2}, \qquad .$$

$$\bar{N}_{31} = 0, \quad \bar{N}_{32} = -6\,\frac{x}{L^2} + 6\,\frac{x^2}{L^3}, \quad \bar{N}_{33} = 1 - 4\,\frac{x}{L} + 3\,\frac{x^2}{L^2}, \qquad .$$

$$\bar{N}_{34} = 0, \quad \bar{N}_{35} = 6\,\frac{x}{L^2} - 6\,\frac{x^2}{L^3}, \quad \bar{N}_{36} = -2\,\frac{x}{L} + 3\,\frac{x^2}{L^2}. \qquad .$$

$[K]$ is the element stiffness matrix. $\{R_T\}$ and $\{R_I\}$ are element load vector coefficient due to temperature rise and current actuation respectively.

$$K^{\star}_{11} = \frac{A^{\star}_{11}}{L}, \qquad K^{\star}_{13} = -\frac{B^{\star}_{11}}{L}, \qquad K^{\star}_{14} = -\frac{A^{\star}_{11}}{L}, \qquad K^{\star}_{16} = \frac{B^{\star}_{11}}{L},$$

$$K^{\star}_{22} = 4/3\,\frac{\gamma + 9\,D^{\star}_{11}}{L^3}, \qquad K^{\star}_{23} = 2/3\,\frac{\gamma + 9\,D^{\star}_{11}}{L^2}, \qquad K^{\star}_{25} = -4/3\,\frac{\gamma + 9\,D^{\star}_{11}}{L^3},$$

$$K^{\star}_{26} = 2/3\,\frac{\gamma + 9\,D^{\star}_{11}}{L^2}, \qquad K^{\star}_{33} = 1/3\,\frac{\gamma + 12\,D^{\star}_{11}}{L}, \qquad K^{\star}_{34} = \frac{B^{\star}_{11}}{L},$$

$$K^{\star}_{35} = -2/3\,\frac{\gamma + 9\,D^{\star}_{11}}{L^2}, \qquad K^{\star}_{36} = 1/3\,\frac{\gamma + 6\,D^{\star}_{11}}{L},$$

$$K^{\star}_{44} = \frac{A^{\star}_{11}}{L}, \qquad K^{\star}_{46} = -\frac{B^{\star}_{11}}{L}, \qquad K^{\star}_{55} = 4/3\,\frac{\gamma + 9\,D^{\star}_{11}}{L^3},$$

$$K^{\star}_{56} = -2/3\,\frac{\gamma + 9\,D^{\star}_{11}}{L^2}, \qquad K^{\star}_{66} = 1/3\,\frac{\gamma + 12\,D^{\star}_{11}}{L}.$$

$$\{R\} = \sum_p \begin{bmatrix} -A^{e}_{11p}\,r \\ 0 \\ r B^{e}_{11p} \\ A^{e}_{11p}\,r \\ 0 \\ -r B^{e}_{11p} \end{bmatrix} In_p + \begin{bmatrix} A^{\alpha\star}_{11} \\ 0 \\ -B^{\alpha\star}_{11} \\ -A^{\alpha\star}_{11} \\ 0 \\ B^{\alpha\star}_{11} \end{bmatrix} (\Delta T); \{R_U\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} In_p + \begin{bmatrix} A^{\alpha}_{11} \\ 0 \\ -B^{\alpha}_{11} \\ -A^{\alpha}_{11} \\ 0 \\ B^{\alpha}_{11} \end{bmatrix} (\Delta T).$$

Where r is ratio between two permeabilities ($\mu^{\sigma}/\mu^{\epsilon}$) and $\gamma = \beta_e^2 A^{\star}_{11} + 6\beta_e B^{\star}_{11} = 9\beta_e B^{\star}_{11}$. The nonzero elements of the mass matrix are

$$M_{11} = 1/3\,L I_0, \qquad M_{12} = -1/6\,I_0\,\beta_e + 1/2\,I_1, \qquad M_{13} = -1/12\,L I_0\,\beta_e - 1/12\,L I_1,$$

$$M_{14} = 1/6\,L I_0, \qquad M_{15} = 1/6\,I_0\,\beta_e - 1/2\,I_1, \qquad M_{16} = -1/12\,L I_0\,\beta_e + 1/12\,L I_1,$$

$$M_{22} = \frac{1}{105}\,\frac{39\,L^2 I_0 + 14\,I_0\,{\beta_e}^2 - 84\,I_1\,\beta_e + 126\,I_2}{L},$$

$$M_{23} = \frac{11}{210}\,L^2 I_0 + 1/15\,I_0\,{\beta_e}^2 - \frac{7}{30}\,I_1\,\beta_e + 1/10\,I_2,$$

$$M_{24} = -1/6\,I_0\,\beta_e + 1/2\,I_1, \quad M_{25} = -\frac{1}{210}\,\frac{-27\,L^2 I_0 + 28\,I_0\,{\beta_e}^2 - 168\,I_1\,\beta_e + 252\,I_2}{L},$$

$$M_{26} = -\frac{13}{420}\,L^2 I_0 + 1/15\,I_0\,{\beta_e}^2 - \frac{7}{30}\,I_1\,\beta_e + 1/10\,I_2,$$

$$M_{33} = \frac{1}{210}\,L\left(2\,L^2 I_0 + 7\,I_0\,{\beta_e}^2 - 7\,I_1\,\beta_e + 28\,I_2\right), \quad M_{34} = -1/12\,L I_0\,\beta_e + 1/12\,L I_1,$$

$$M_{35} = \frac{13}{420}\,L^2 I_0 - 1/15\,I_0\,{\beta_e}^2 + \frac{7}{30}\,I_1\,\beta_e - 1/10\,I_2,$$

$$M_{36} = \frac{1}{420}\,L\left(-3\,L^2 I_0 + 14\,I_0\,{\beta_e}^2 - 14\,I_1\,\beta_e - 14\,I_2\right),$$

$$M_{44} = 1/3\,L I_0, \quad M_{45} = 1/6\,I_0\,\beta_e - 1/2\,I_1, \quad M_{46} = -1/12\,L I_0\,\beta_e - 1/12\,L I_1,$$

$$M_{55} = \frac{1}{105}\,\frac{39\,L^2 I_0 + 14\,I_0\,{\beta_e}^2 - 84\,I_1\,\beta_e + 126\,I_2}{L},$$

$$M_{56} = -\frac{11}{210}\,L^2 I_0 - 1/15\,I_0\,{\beta_e}^2 + \frac{7}{30}\,I_1\,\beta_e - 1/10\,I_2,$$

$$M_{66} = \frac{1}{210}\,L\left(2\,L^2 I_0 + 7\,I_0\,{\beta_e}^2 - 7\,I_1\,\beta_e + 28\,I_2\right).$$

Appendix B

Timoshenko Beam

Matrices for superconvergent Timoshenko beam element can be calculated considering α and β_f as:

$$\alpha = \frac{A_{55}B_{11}^{\star}}{(D_{11}^{\star}A_{11}^{\star} - B_{11}^{\star}B_{11}^{\star})}, \qquad \beta_f = \frac{A_{11}^{\star}A_{55}}{(A_{11}^{\star}D_{11}^{\star} - B_{11}^{\star}B_{11}^{\star})}. \tag{B.1}$$

Shape function matrix for superconvergent Timoshenko beam element is given by

$$\bar{N}_{11} = 1 - \frac{x}{L}, \quad \bar{N}_{12} = -6\,\frac{x\alpha}{12 + L^2\beta_f} + 6\,\frac{x^2\alpha}{L\,(12 + L^2\beta_f)},$$

$$\bar{N}_{13} = -3\,\frac{xL\alpha}{12 + L^2\beta_f} - 1/2\,\frac{x^2\alpha\,\left(L^2\beta_f + 6\right)}{12 + L^2\beta_f} + 1/2\,x^2\alpha, \quad \bar{N}_{14} = \frac{x}{L},$$

$$\bar{N}_{15} = 6\,\frac{x\alpha}{12 + L^2\beta_f} - 6\,\frac{x^2\alpha}{L\,(12 + L^2\beta_f)}, \quad \bar{N}_{16} = -3\,\frac{xL\alpha}{12 + L^2\beta_f} + 3\,\frac{x^2\alpha}{12 + L^2\beta_f},$$

$$\bar{N}_{21} = 0, \quad \bar{N}_{22} = 1 - 12\,\frac{x - 1/6\,x^3\beta_f}{L\,(12 + L^2\beta_f)} - 3\,\frac{x^2\beta_f}{12 + L^2\beta_f},$$

$$\bar{N}_{23} = \frac{(x - 1/6\,x^3\beta_f)\,(L^2\beta_f + 6)}{12 + L^2\beta_f} + 1/6\,x^3\beta_f - 2\,\frac{x^2\,(3 + L^2\beta_f)}{L\,(12 + L^2\beta_f)}, \quad \bar{N}_{24} = 0,$$

$$\bar{N}_{25} = 12\,\frac{x - 1/6\,x^3\beta_f}{L\,(12 + L^2\beta_f)} + 3\,\frac{x^2\beta_f}{12 + L^2\beta_f}, \quad \bar{N}_{26} = -6\,\frac{x - 1/6\,x^3\beta_f}{12 + L^2\beta_f} - \frac{x^2\,(-6 + L^2\beta_f)}{L\,(12 + L^2\beta_f)},$$

$$\bar{N}_{31} = 0, \quad \bar{N}_{32} = 6\,\frac{x^2\beta_f}{L\,(12 + L^2\beta_f)} - 6\,\frac{x\beta_f}{12 + L^2\beta_f},$$

$$\bar{N}_{33} = 1 - 1/2\,\frac{x^2\beta_f\,(L^2\beta_f+6)}{12+L^2\beta_f} + 1/2\,x^2\beta_f - 4\,\frac{x\,(3+L^2\beta_f)}{L\,(12+L^2\beta_f)}, \quad \bar{N}_{34}=0,$$

$$\bar{N}_{35} = -6\,\frac{x^2\beta_f}{L\,(12+L^2\beta_f)} + 6\,\frac{x\beta_f}{12+L^2\beta_f}, \quad \bar{N}_{36} = 3\,\frac{x^2\beta_f}{12+L^2\beta_f} - 2\,\frac{x\,(-6+L^2\beta_f)}{L\,(12+L^2\beta_f)},$$

Nonzero entries of stiffness matrix. $[K^\star]$ of Timoshenko beam is given by:

$$K^\star_{11} = \frac{A^\star_{11}}{L}, \quad K^\star_{12}=0, \quad K^\star_{13} = -\frac{B^\star_{11}}{L}, \quad K^\star_{14} = -\frac{A^\star_{11}}{L}, \quad K^\star_{15}=0, \quad K^\star_{16} = \frac{B^\star_{11}}{L},$$

$$K^\star_{22} = 12\,\frac{A_{55}\,\psi}{L}, \quad K^\star_{23} = 6\,A_{55}\,\psi, \quad K^\star_{24}=0, \quad K^\star_{25} = -12\,\frac{A_{55}\,\psi}{L}, \quad K^\star_{26} = 6\,A_{55}\,\psi,$$

$$K^\star_{33} = \frac{D^\star_{11} + 3\,L^2\psi\,A_{55}}{L}, \quad K^\star_{34} = \frac{B^\star_{11}}{L}, \quad K^\star_{35} = -6\,A_{55}\,\psi, \quad K^\star_{36} = \frac{-D^\star_{11} + 3\,L^2\psi\,A_{55}}{L},$$

$$K^\star_{44} = \frac{A^\star_{11}}{L}, \quad K^\star_{45}=0, \quad K^\star_{46} = -\frac{B^\star_{11}}{L}, \quad K^\star_{55} = 12\,\frac{A_{55}\,\psi}{L}, \quad K^\star_{56} = -6\,A_{55}\,\psi,$$

$$K^\star_{66} = \frac{D^\star_{11} + 3\,L^2\psi\,A_{55}}{L}.$$

where $\psi = 1/(12+L^2\beta_f)$. Nonzero entries of mass matrix $[M]$ of a Timoshenko beam is given by

$$M_{11} = 1/3\,LI_0, \quad M_{12} = -1/2\,I_0\,L^2\alpha\,\psi + 1/2\,I_1 - 6\,I_1\,\psi,$$

$$M_{13} = -1/12\,L\left(3\,I_0\,L^2\alpha\,\psi + I_1 + 36\,I_1\,\psi\right), \quad M_{14} = 1/6\,LI_0,$$

$$M_{15} = 1/2\,I_0\,L^2\alpha\,\psi - 1/2\,I_1 + 6\,I_1\,\psi, \quad M_{16} = -1/12\,L\left(3\,I_0\,L^2\alpha\,\psi - I_1 + 36\,I_1\,\psi\right),$$

$$M_{22} = \frac{L\psi^2}{35}(13\,{\beta_f}^2 I_0\,L^4 + 42\,\alpha^2 I_0\,L^2 + 294\,\beta_f\,I_0\,L^2 + 1680\,I_0 - 84\,\beta_f\,\alpha\,I_1\,L^2 + 42\,{\beta_f}^2 I_2\,L^2),$$

$$M_{23} = \frac{11}{210}\,I_0\,L^2 - \frac{11}{70}\,I_0\,L^2\psi + \frac{12}{35}\,I_0\,\psi^2 L^2 + 3/5\,\alpha^2 I_0\,L^4\psi^2 - \frac{7}{10}\,\alpha\,I_1\,\psi\,L^2 + \frac{72}{5}\,\alpha\,I_1\,\psi^2 L^2 + 1/10\,I_2 - \frac{42}{5}\,I_2\,\psi + \frac{432}{5}\,I_2\,\psi^2,$$

$$M_{24} = -1/2\, I_0\, L^2 \alpha\, \psi + 1/2\, I_1 - 6\, I_1\, \psi,$$

$$M_{25} = -\frac{3L\psi^2}{70}(-3\,{\beta_f}^2 I_0\, L^4 + 28\,\alpha^2 I_0\, L^2 -84\,\beta_f\, I_0\, L^2 - 560\, I_0 - 56\,\beta_f\,\alpha\, I_1\, L^2 + 28\,{\beta_f}^2 I_2\, L^2),$$

$$M_{26} = 3/5\,\alpha^2 I_0\, L^4 \psi^2 + \frac{72}{5}\,\alpha\, I_1\,\psi^2 L^2 - \frac{42}{5}\, I_2\,\psi + \frac{432}{5}\, I_2\,\psi^2 - \frac{7}{10}\,\alpha\, I_1\,\psi\, L^2 -\frac{11}{70}\, I_0\, L^2\psi + \frac{12}{35}\, I_0\,\psi^2 L^2 + 1/10\, I_2 - \frac{13}{420}\, I_0\, L^2,$$

$$M_{33} = \frac{1}{105}\, I_0\, L^3 - 1/35\, I_0\, L^3\psi + \frac{6}{35}\, I_0\,\psi^2 L^3 + 3/10\,\alpha^2 I_0\, L^5\psi^2 - 1/10\,\alpha\, I_1\,\psi\, L^3 +\frac{36}{5}\,\alpha\, I_1\,\psi^2 L^3 + 2/15\, I_2 L - 6/5\, I_2 L\,\psi + \frac{216}{5}\, I_2\,\psi^2 L,$$

$$M_{34} = -1/12\, L\left(3\, I_0\, L^2\alpha\,\psi - I_1 + 36\, I_1\,\psi\right),$$

$$M_{35} = -3/5\,\alpha^2 I_0\, L^4\psi^2 - \frac{72}{5}\,\alpha\, I_1\,\psi^2 L^2 + \frac{42}{5}\, I_2\,\psi - \frac{432}{5}\, I_2\,\psi^2 + \frac{7}{10}\,\alpha\, I_1\,\psi\, L^2 +\frac{11}{70}\, I_0\, L^2\psi - \frac{12}{35}\, I_0\,\psi^2 L^2 - 1/10\, I_2 + \frac{13}{420}\, I_0\, L^2,$$

$$M_{36} = -\frac{1}{140}\, I_0\, L^3 - 1/35\, I_0\, L^3\psi + \frac{6}{35}\, I_0\,\psi^2 L^3 + 3/10\,\alpha^2 I_0\, L^5\psi^2 - 1/10\,\alpha\, I_1\,\psi\, L^3 +\frac{36}{5}\,\alpha\, I_1\,\psi^2 L^3 - 1/30\, I_2 L - 6/5\, I_2\,\psi L + \frac{216}{5}\, I_2\,\psi^2 L,$$

$$M_{44} = 1/3\, L I_0, \quad M_{45} = 1/2\, I_0\, L^2\alpha\,\psi - 1/2\, I_1 + 6\, I_1\,\psi,$$

$$M_{46} = -1/12\, L\left(3\, I_0\, L^2\alpha\,\psi + I_1 + 36\, I_1\,\psi\right),$$

$$M_{55} = 1/35\, L(13\,{\beta_f}^2 I_0\, L^4 + 42\,\alpha^2 I_0\, L^2 + 294\,\beta_f\, I_0\, L^2 + 1680\, I_0 -84\,\beta_f\,\alpha\, I_1\, L^2 + 42\,{\beta_f}^2 I_2\, L^2)\psi^2,$$

$$M_{56} = -\frac{11}{210}\, I_0\, L^2 + \frac{11}{70}\, I_0\, L^2\psi - \frac{12}{35}\, I_0\,\psi^2 L^2 - 3/5\,\alpha^2 I_0\, L^4\psi^2 + \frac{7}{10}\,\alpha\, I_1\,\psi\, L^2, -\frac{72}{5}\,\alpha\, I_1\,\psi^2 L^2 - 1/10\, I_2 + \frac{42}{5}\, I_2\,\psi - \frac{432}{5}\, I_2\,\psi^2,$$

$$M_{66} = \frac{1}{105}\beta_f{}^2 I_0\, L^7\psi^2 + 1/5\, I_0\, \beta_f\, L^5\psi^2 + 3/10\, I_0\, L^5\alpha^2\psi^2$$
$$+6/5\, I_0\, L^3\psi^2 - 1/10\,\beta_f\,\alpha\, I_1\, L^5\psi^2$$
$$+6\,\alpha\, I_1\, L^3\psi^2 + 2/15\,\beta_f{}^2 L^5 I_2\psi^2 + 2\, I_2\,\beta_f\, L^3\psi^2 + 48\, I_2 L\psi^2.$$

The elements of force vector for a Timoshenko beam is given by

$$\{R\} = \sum_p \begin{bmatrix} -A^e_{11p}\, r \\ 0 \\ r B^e_{11p} \\ A^e_{11p}\, r \\ 0 \\ -r B^e_{11p} \end{bmatrix} In_p + \begin{bmatrix} A^{\alpha\star}_{11} \\ 0 \\ -B^{\alpha\star}_{11} \\ -A^{\alpha\star}_{11} \\ 0 \\ B^{\alpha\star}_{11} \end{bmatrix} (\Delta T); \{R_U\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} In_p + \begin{bmatrix} A^{\alpha}_{11} \\ 0 \\ -B^{\alpha}_{11} \\ -A^{\alpha}_{11} \\ 0 \\ B^{\alpha}_{11} \end{bmatrix} (\Delta T).$$

Where r is ratio between two permeabilities.

Appendix C

Higher Order Theories

Elements of stiffness $[K]$, coupling stiffness $[K^m]$ and mass $[M]$ matrices, are written as

$$K_{u_1u_1} = \frac{1}{L}A_{11}^{[0]}; \quad K_{u_1u_{i1}} = \frac{1}{L}A_{11}^{[i]}; \quad K_{u_1w_1} = 0; \quad K_{u_1w_{i1}} = -\frac{1}{2}iA_{13}^{[i]};$$

$$K_{u_1u_2} = -\frac{1}{L}A_{11}^{[0]}; \quad K_{u_1u_{i2}} = -\frac{1}{L}A_{11}^{[i]}; \quad K_{u_1w_2} = 0; \quad K_{u_1w_{i2}} = -\frac{1}{2}iA_{13}^{[i]};$$

$$K_{u_{i1}u_{m1}} = \frac{1}{3L}\left(imA_{55}^{[i+m-2]}L^2 + 3A_{11}^{[i+m]}\right); \quad K_{u_{i1}w_1} = -\frac{i}{2}A_{55}^{[i-1]}; \quad K_{u_{i1}u_2} = -\frac{1}{L}A_{11}^{[i]};$$

$$K_{u_{i1}w_{m1}} = -\frac{1}{2}\left(mA_{13}^{[i+m-1]} + iA_{55}^{[i+m-1]}\right); \; K_{u_{i1}u_{m2}} = \frac{1}{6L}\left(imA_{55}^{[i+m-2]}L^2 - 6A_{11}^{[i+m]}\right);$$

$$K_{u_{i1}w_2} = \frac{i}{2}A_{55}^{[i-1]}; \quad K_{u_{i1}w_{m2}} = \frac{1}{2}\left(-mA_{13}^{[i+m-1]} + iA_{55}^{[i+m-1]}\right); \quad K_{w_1w_1} = \frac{1}{L}A_{55}^{[0]};$$

$$K_{w_1w_{i1}} = \frac{1}{L}A_{55}^{[i]}; \; K_{w_1u_2} = 0; \; K_{w_1u_{i2}} = -\frac{i}{2}A_{55}^{[i-1]}; \; K_{w_1w_2} = -\frac{1}{L}A_{55}^{[0]}; \; K_{w_1w_{i2}} = -\frac{1}{L}A_{55}^{[i]};$$

$$K_{w_{i1}w_{m1}} = \frac{1}{3L}\left(imA_{33}^{[i+m-2]}L^2 + 3A_{55}^{[i+m]}\right); \quad K_{w_{i1}u_{m2}} = \frac{1}{2}\left(mA_{13}^{[i+m-1]} - iA_{55}^{[i+m-1]}\right);$$

$$K_{w_{i1}u_2} = \frac{1}{2}iA_{13}^{[i]}; \quad K_{w_{i1}w_2} = -\frac{1}{L}A_{55}^{[i]}; \quad K_{w_{i1}w_{m2}} = \frac{1}{6L}\left(imA_{33}^{[i+m-2]}L^2 - 6A_{55}^{[i+m]}\right);$$

$$K_{u_2u_2} = \frac{1}{L}A_{11}^{[0]}; \; K_{u_2u_{i2}} = \frac{1}{L}A_{11}^{[i]}; \; K_{u_2w_2} = 0; \; K_{u_2w_{i2}} = \frac{1}{2}iA_{13}^{[i]}; K_{u_{i2}w_2} = \frac{i}{2}A_{55}^{[i-1]};$$

$$K_{u_{i2}u_{m2}} = \frac{1}{6L}\left(2imA_{55}^{[i+m-2]}L^2 + 6A_{11}^{[i+m]}\right);\; K_{u_{i2}w_{m2}} = \frac{1}{2}\left(mA_{13}^{[i+m-1]} + iA_{55}^{[i+m-1]}\right);$$

$$K_{w_2w_2} = \frac{1}{L}A_{55}^{[0]};\quad K_{w_2w_{i2}} = \frac{1}{L}A_{55}^{[i]};\quad K_{w_{i2}w_{m2}} = \frac{1}{6L}\left(imA_{33}^{[i+m-2]}L^2 + 6A_{55}^{[i+m]}\right);$$

$$K_{u_1u_1}^m = \frac{1}{A_\epsilon^\mu}\left(\frac{1}{L}A_{11e}^{[0]}A_{11e}^{[0]}\right);\qquad K_{u_1u_{i1}}^m = \frac{1}{A_\epsilon^\mu}\left(\frac{1}{L}A_{11e}^{[0]}A_{11e}^{[i]} - \frac{i}{2}A_{11e}^{[0]}A_{15e}^{[i-1]}\right);$$

$$K_{u_1w_1}^m = \frac{1}{A_\epsilon^\mu}\left(\frac{1}{L}A_{11e}^{[0]}A_{15e}^{[0]}\right);\qquad K_{u_1w_{i1}}^m = \frac{1}{A_\epsilon^\mu}\left(\frac{1}{L}A_{11e}^{[0]}A_{15e}^{[i]} - \frac{i}{2}A_{11e}^{[0]}A_{13e}^{[i-1]}\right);$$

$$K_{u_1u_2}^m = -\frac{1}{A_\epsilon^\mu}\left(\frac{1}{L}A_{11e}^{[0]}A_{11e}^{[0]}\right);\qquad K_{u_1u_{i2}}^m = \frac{1}{A_\epsilon^\mu}\left(-\frac{1}{L}A_{11e}^{[0]}A_{11e}^{[i]} - \frac{i}{2}A_{11e}^{[0]}A_{15e}^{[i-1]}\right);$$

$$K_{u_1w_2}^m = -\frac{1}{A_\epsilon^\mu}\left(\frac{1}{L}A_{11e}^{[0]}A_{15e}^{[0]}\right);\qquad K_{u_1w_{i2}}^m = \frac{1}{A_\epsilon^\mu}\left(-\frac{1}{L}A_{11e}^{[0]}A_{15e}^{[i]} - \frac{i}{2}A_{11e}^{[0]}A_{13e}^{[i-1]}\right);$$

$$K_{u_{i1}u_{m1}}^m = \frac{1}{A_\epsilon^\mu}\left(\frac{1}{L}A_{11e}^{[m]}A_{11e}^{[i]} - \frac{i}{2}A_{11e}^{[m]}A_{15e}^{[i-1]} - \frac{mL}{2}A_{11e}^{[i]}A_{15e}^{[m-1]} + \frac{imL}{3}A_{15e}^{[i-1]}A_{15e}^{[m-1]}\right);$$

$$K_{u_{m1}w_1}^m = \frac{1}{A_\epsilon^\mu}\left(\frac{1}{L}A_{11e}^{[m]}A_{15e}^{[0]} - \frac{m}{2}A_{15e}^{[0]}A_{15e}^{[m-1]}\right);$$

$$K_{u_{m1}w_{i1}}^m = \frac{1}{A_\epsilon^\mu}\left(\frac{1}{L}A_{13e}^{[m]}A_{15e}^{[i]} - \frac{i}{2}A_{11e}^{[m]}A_{13e}^{[i-1]} - \frac{m}{2}A_{15e}^{[i]}A_{15e}^{[m-1]} + \frac{imL}{3}A_{15e}^{[m-1]}A_{13e}^{[i-1]}\right);$$

$$K_{u_{m1}u_2}^m = \frac{1}{A_\epsilon^\mu}\left(-\frac{1}{L}A_{11e}^{[m]}A_{11e}^{[0]} + \frac{m}{2}A_{11e}^{[0]}A_{15e}^{[m-1]}\right);$$

$$K_{u_{m1}u_{i2}}^m = \frac{1}{A_\epsilon^\mu}\left(-\frac{1}{L}A_{11e}^{[m]}A_{11e}^{[i]} + \frac{m}{2}A_{11e}^{[i]}A_{15e}^{[m-1]} - \frac{i}{2}A_{11e}^{[m]}A_{15e}^{[i-1]} + \frac{imL}{6}A_{15e}^{[m-1]}A_{15e}^{[i-1]}\right);$$

$$K_{u_{m1}w_2}^m = \frac{1}{A_\epsilon^\mu}\left(-\frac{1}{L}A_{11e}^{[m]}A_{15e}^{[0]} + \frac{m}{2}A_{15e}^{[0]}A_{15e}^{[m-1]}\right);$$

$$K^m_{u_{m1}w_{i2}} = \frac{1}{A^\mu_\epsilon}\left(-\frac{1}{L}A^{[m]}_{11e}A^{[i]}_{15e} + \frac{m}{2}A^{[i]}_{15e}A^{[m-1]}_{15e} - \frac{i}{2}A^{[m]}_{11e}A^{[i-1]}_{13e} + \frac{imL}{6}A^{[m-1]}_{15e}A^{[i-1]}_{13e}\right);$$

$$K^m_{w_1w_1} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[0]}_{15e}A^{[0]}_{15e}\right); \qquad K^m_{w_1w_{m1}} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[m]}_{15e}A^{[0]}_{15e} - \frac{m}{2}A^{[0]}_{15e}A^{[m-1]}_{13e}\right);$$

$$K^m_{w_1u_2} = \frac{1}{A^\mu_\epsilon}\left(-\frac{1}{L}A^{[0]}_{11e}A^{[0]}_{15e}\right); \qquad K^m_{w_1u_{i2}} = \frac{1}{A^\mu_\epsilon}\left(-\frac{1}{L}A^{[i]}_{11e}A^{[0]}_{15e} - \frac{i}{2}A^{[0]}_{15e}A^{[i-1]}_{15e}\right);$$

$$K^m_{w_1w_2} = \frac{1}{A^\mu_\epsilon}\left(-\frac{1}{L}A^{[0]}_{15e}A^{[0]}_{15e}\right); \qquad K^m_{w_1w_{i2}} = \frac{1}{A^\mu_\epsilon}\left(-\frac{1}{L}A^{[i]}_{15e}A^{[0]}_{15e} - \frac{i}{2}A^{[0]}_{15e}A^{[i-1]}_{13e}\right);$$

$$K^m_{w_{i1}w_{m1}} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[m]}_{15e}A^{[i]}_{15e} - \frac{i}{2}A^{[m]}_{15e}A^{[i-1]}_{13e} - \frac{m}{2}A^{[i]}_{15e}A^{[m-1]}_{13e} + \frac{imL}{3}A^{[m-1]}_{13e}A^{[i-1]}_{13e}\right);$$

$$K^m_{w_{m1}u_2} = \frac{1}{A^\mu_\epsilon}\left(-\frac{1}{L}A^{[0]}_{11e}A^{[m]}_{15e} + \frac{m}{2}A^{[0]}_{11e}A^{[m-1]}_{13e}\right);$$

$$K^m_{w_{m1}u_{i2}} = \frac{1}{A^\mu_\epsilon}\left(-\frac{1}{L}A^{[i]}_{11e}A^{[m]}_{15e} + \frac{m}{2}A^{[i]}_{11e}A^{[m-1]}_{13e} - \frac{i}{2}A^{[m]}_{15e}A^{[i-1]}_{15e} + \frac{imL}{6}A^{[i-1]}_{15e}A^{[m-1]}_{13e}\right);$$

$$K^m_{w_{m1}w_2} = \frac{1}{A^\mu_\epsilon}\left(-\frac{1}{L}A^{[0]}_{15e}A^{[m]}_{15e} + \frac{m}{2}A^{[0]}_{15e}A^{[m-1]}_{13e}\right);$$

$$K^m_{w_{m1}w_{i2}} = \frac{1}{A^\mu_\epsilon}\left(-\frac{1}{L}A^{[i]}_{15e}A^{[m]}_{15e} + \frac{m}{2}A^{[i]}_{15e}A^{[m-1]}_{13e} - \frac{i}{2}A^{[m]}_{15e}A^{[i-1]}_{13e} + \frac{imL}{6}A^{[i-1]}_{13e}A^{[m-1]}_{13e}\right);$$

$$K^m_{u_2u_2} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[0]}_{11e}A^{[0]}_{11e}\right); \qquad K^m_{u_2u_{m2}} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[0]}_{11e}A^{[m]}_{11e} + \frac{m}{2}A^{[0]}_{11e}A^{[m-1]}_{15e}\right);$$

$$K^m_{u_2w_2} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[0]}_{11e}A^{[0]}_{15e}\right); \qquad K^m_{u_2w_{i2}} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[0]}_{11e}A^{[i]}_{15e} + \frac{i}{2}A^{[0]}_{11e}A^{[i-1]}_{13e}\right);$$

$$K^m_{u_{i2}u_{m2}} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[i]}_{11e}A^{[m]}_{11e} + \frac{i}{2}A^{[m]}_{11e}A^{[i-1]}_{15e} + \frac{m}{2}A^{[i]}_{11e}A^{[m-1]}_{15e} + \frac{imL}{3}A^{[i-1]}_{15e}A^{[m-1]}_{15e}\right);$$

$$K^m_{u_{m2}w_2} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[m]}_{11e}A^{[0]}_{15e} + \frac{m}{2}A^{[0]}_{15e}A^{[m-1]}_{15e}\right);$$

$$K^m_{u_{m2}w_{i2}} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[m]}_{11e}A^{[i]}_{15e} + \frac{m}{2}A^{[i]}_{15e}A^{[m-1]}_{15e} + \frac{i}{2}A^{[m]}_{11e}A^{[i-1]}_{13e} + \frac{imL}{3}A^{[m-1]}_{15e}A^{[i-1]}_{13e}\right);$$

$$K^m_{w_2w_2} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[0]}_{15e}A^{[0]}_{15e}\right); \qquad K^m_{w_2w_{m2}} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[0]}_{15e}A^{[m]}_{15e} + \frac{m}{2}A^{[0]}_{15e}A^{[m-1]}_{13e}\right);$$

$$K^m_{w_{i2}w_{m2}} = \frac{1}{A^\mu_\epsilon}\left(\frac{1}{L}A^{[i]}_{15e}A^{[m]}_{15e} + \frac{i}{2}A^{[m]}_{15e}A^{[i-1]}_{13e} + \frac{m}{2}A^{[i]}_{15e}A^{[m-1]}_{13e} + \frac{imL}{3}A^{[i-1]}_{13e}A^{[m-1]}_{13e}\right);$$

$$M_{u_1u_1} = \frac{L}{3}I_{[0]}; \qquad M_{u_1u_{i1}} = \frac{L}{3}I_{[i]}; \qquad M_{u_1w_1} = 0; \qquad M_{u_1w_{i1}} = 0;$$

$$M_{u_1u_2} = \frac{L}{6}I_{[0]}; \qquad M_{u_1u_{i2}} = \frac{L}{6}I_{[i]}; \qquad M_{u_1w_2} = 0; \qquad M_{u_1w_{i2}} = 0;$$

$$M_{u_{i1}u_{m1}} = \frac{L}{3}I_{[i+m]}; \qquad M_{u_{i1}w_1} = 0; \qquad M_{u_{i1}w_{m1}} = 0; \qquad M_{u_{i1}u_2} = \frac{L}{6}I_{[i]};$$

$$M_{u_{i1}u_{m2}} = \frac{L}{6}I_{[i+m]}; \qquad M_{u_{i1}w_2} = 0; \qquad M_{u_{i1}w_{m2}} = 0;$$

$$M_{w_1w_1} = \frac{L}{3}I_{[0]}; \qquad M_{w_1w_{i1}} = \frac{L}{3}I_{[i]}; \qquad M_{w_1u_2} = 0; \qquad M_{w_1u_{i2}} = 0;$$

$$M_{w_1w_2} = \frac{L}{6}I_{[0]}; \qquad M_{w_1w_{i2}} = \frac{L}{6}I_{[i]}; \qquad .$$

$$M_{w_{i1}w_{m1}} = \frac{L}{3}I_{[i+m]}; \qquad M_{w_{i1}u_2} = 0; \qquad M_{w_{i1}u_{m2}} = 0; \qquad M_{w_{i1}w_2} = \frac{L}{6}I_{[i]};$$

$$M_{w_{i1}w_{m2}} = \frac{L}{6}I_{[i+m]}; \qquad .$$

$$M_{u_2u_2} = \frac{L}{3}I_{[0]}; \qquad M_{u_2u_{i2}} = \frac{L}{3}I_{[i]}; \qquad M_{u_2w_2} = 0; \qquad M_{u_2w_{i2}} = 0;$$

$$M_{u_{i2}u_{m2}} = \frac{L}{3}I_{[i+m]}; \qquad M_{u_{i2}w_2} = 0; \qquad M_{u_{i2}w_{m2}} = 0;$$

$$M_{w_2w_2} = \frac{L}{3}I_{[0]}; \qquad M_{w_2w_{i2}} = \frac{L}{3}I_{[i]};$$

$$M_{w_{i2}w_{m2}} = \frac{L}{3}I_{[i+m]};$$

Matrix entries of W are given as

$$\begin{aligned}
W_{u_0u_0} &= A_{11}^{[0]}k^2 - I_{[0]}\omega^2 + \left[A_{11e}^{[0]}A_{11e}^{[0]}k^2\right]/A_\mu^\epsilon \\
W_{u_0u_m} &= A_{11}^{[m]}k^2 - I_{[m]}\omega^2 + jmkA_{15}^{[m-1]} + \left[A_{11e}^{[0]}A_{11e}^{[m]}k^2 + mjkA_{15e}^{[m-1]}A_{11e}^{[0]}\right]/A_\mu^\epsilon \\
W_{u_iu_0} &= A_{11}^{[i]}k^2 - I_{[i]}\omega^2 - ijkA_{15}^{[i-1]} + \left[A_{11e}^{[i]}A_{11e}^{[0]}k^2 - ijkA_{11e}^{[0]}A_{15e}^{[i-1]}\right]/A_\mu^\epsilon \\
W_{u_iu_m} &= A_{11}^{[i+m]}k^2 + imA_{55}^{[i+m-2]} - I_{[i+m]}\omega^2 + (m-i)jkA_{15}^{[i+m-1]} \\
&\quad + \left[A_{11e}^{[i]}A_{11e}^{[m]}k^2 + imA_{15e}^{[i-1]}A_{15e}^{[m-1]} - ijkA_{11e}^{[m]}A_{15e}^{[i-1]} + mjkA_{15e}^{[m-1]}A_{11e}^{[i]}\right]/A_\mu^\epsilon
\end{aligned}$$

$$\begin{aligned}
W_{w_0w_0} &= A_{55}^{[0]}k^2 - I_{[0]}\omega^2 + \left[A_{15e}^{[0]}A_{15e}^{[0]}k^2\right]/A_\mu^\epsilon \\
W_{w_0w_m} &= A_{55}^{[m]}k^2 - I_{[m]}\omega^2 + mjkA_{35}^{[m-1]} + \left[A_{15e}^{[0]}A_{15e}^{[m]}k^2\right]/A_\mu^\epsilon \\
W_{w_iw_0} &= A_{55}^{[i]}k^2 - I_{[i]}\omega^2 - ijkA_{35}^{[i-1]} + \left[A_{15e}^{[i]}A_{15e}^{[0]}k^2\right]/A_\mu^\epsilon \\
W_{w_iw_m} &= imA_{33}^{[i+m-2]} + A_{55}^{[i+m]}k^2 - I_{[i+m]}\omega^2 + (m-i)jkA_{35}^{[i+m-1]} + \left[A_{15e}^{[i]}A_{15e}^{[m]}k^2\right]/A_\mu^\epsilon
\end{aligned}$$

$$\begin{aligned}
W_{u_0w_0} &= A_{15}^{[0]}k^2 + \left[A_{11e}^{[0]}A_{15e}^{[0]}k^2\right]/A_\mu^\epsilon \\
W_{u_0w_m} &= mkjA_{13}^{[m-1]} + A_{15}^{[m]}k^2 + \left[A_{11e}^{[0]}A_{15e}^{[m]}k^2\right]/A_\mu^\epsilon \\
W_{u_iw_0} &= -ijkA_{55}^{[i-1]} + A_{15}^{[i]}k^2 + \left[A_{11e}^{[i]}A_{15e}^{[0]}k^2 - ijkA_{15e}^{[0]}A_{15e}^{[i-1]}\right]/A_\mu^\epsilon \\
W_{u_iw_m} &= mkjA_{13}^{[i+m-1]} + imA_{35}^{[i+m-2]} - ijkA_{55}^{[i+m-1]} + A_{15}^{[i+m]}k^2 \\
&\quad + \left[A_{11e}^{[i]}A_{15e}^{[m]}k^2 - ijkA_{15e}^{[m]}A_{15e}^{[i-1]}\right]/A_\mu^\epsilon
\end{aligned}$$

$$\begin{aligned}
W_{w_0u_0} &= A_{15}^{[0]}k^2 + \left[A_{11e}^{[0]}A_{15e}^{[0]}k^2\right]/A_\mu^\epsilon \\
W_{w_0u_i} &= ikjA_{55}^{[i-1]} + A_{15}^{[i]}k^2 + \left[A_{11e}^{[i]}A_{15e}^{[0]}k^2 + ijkA_{15e}^{[0]}A_{15e}^{[i-1]}\right]/A_\mu^\epsilon \\
W_{w_mu_0} &= -mkjA_{13}^{[m-1]} + A_{15}^{[m]}k^2 + \left[A_{11e}^{[0]}A_{15e}^{[m]}k^2\right]/A_\mu^\epsilon \\
W_{w_mu_i} &= ikjA_{55}^{[i+m-1]} - mkjA_{13}^{[i+m-1]} + A_{15}^{[i+m]}k^2 + imA_{35}^{[i+m-2]} \\
&\quad + \left[A_{11e}^{[i]}A_{15e}^{[m]}k^2 + ijkA_{15e}^{[m]}A_{15e}^{[i-1]}\right]/A_\mu^\epsilon
\end{aligned}$$

Matrices, $P0_{DC}$, $P0_{AC}$, $P1$, and $P2$ are given as

$P0_{DC}(1+m,1+i_1) = -(-mi_1A_{55}^{[m+i_1-2]}A_\mu^\epsilon - mi_1A_{15e}^{[m-1]}A_{15e}^{[i_1-1]})/A_\mu^\epsilon;$

$P0_{DC}(1+m,2+Un+i_2) = mi_2A_{15e}^{[m-1]}A_{13e}^{[i_2-1]}/A_\mu^\epsilon;$

$P0_{DC}(2+Un+n,1+i_1) = ni_1A_{15e}^{[i_1-1]}A_{13e}^{[n-1]}/A_\mu^\epsilon;$

$P0_{DC}(2+Un+n,2+Un+i_2) = (ni_2A_{33}^{[n+i_2-2]}A_\mu^\epsilon + ni_2A_{13e}^{[n-1]}A_{13e}^{[i_2-1]})/A_\mu^\epsilon;$

$P1(1,1+i_1) = i_1A_{11e}^{[0]}A_{15e}^{[i_1-1]}/A_\mu^\epsilon;$.

$P1(1,2+Un+i_2) = i_2(A_{13}^{[i_2-1]}A_\mu^\epsilon + A_{13e}^{[i_2-1]}A_{11e}^{[0]})/A_\mu^\epsilon;$.

$P1(1+m,1) = -mA_{11e}^{[0]}A_{15e}^{[m-1]}/A_\mu^\epsilon;$.

$P1(1+m,1+i_1) = -(-i_1A_{11e}^{[m]}A_{15e}^{[i_1-1]} + mA_{11e}^{[i_1]}A_{15e}^{[m-1]})/A_\mu^\epsilon;$.

$P1(1+m,2+Un) = -m(A_{55}^{[m-1]}A_\mu^\epsilon + A_{15e}^{[0]}A_{15e}^{[m-1]})/A_\mu^\epsilon;$.

$P1(1+m,2+Un+i_2) = (i_2A_{13}^{[i_2+m-1]}A_\mu^\epsilon + i_2A_{11e}^{[m]}A_{13e}^{[i_2-1]} - mA_{55}^{[i_2+m-1]}A_\mu^\epsilon - mA_{15e}^{[i_2]}A_{15e}^{[m-1]})/A_\mu^\epsilon;$

$P1(2+Un,1+i_1) = i_1(A_{55}^{[i_1-1]}A_\mu^\epsilon + A_{15e}^{[0]}A_{15e}^{[i_1-1]})/A_\mu^\epsilon;$

$P1(2+Un,2+Un+i_2) = i_2A_{15e}^{[0]}A_{13e}^{[i_2-1]}/A_\mu^\epsilon;$

$P1(2+Un+n,1) = -n(A_{13}^{[n-1]}A_\mu^\epsilon + A_{13e}^{[n-1]}A_{11e}^{[0]})/A_\mu^\epsilon;$.

$P1(2+Un+n,1+i_1) = -(nA_{13}^{[i_1+n-1]}A_\mu^\epsilon + nA_{11e}^{[i_1]}A_{13e}^{[n-1]} - i_1A_{55}^{[i_1+n-1]}A_\mu^\epsilon - i_1A_{15e}^{[n]}A_{15e}^{[i_1-1]})/A_\mu^\epsilon;$

$P1(2+Un+n,2+Un) = -nA_{15e}^{[0]}A_{13e}^{[n-1]}/A_\mu^\epsilon;$

$$P1(2+Un+n,2+Un+i_2)=(-nA_{15e}^{[i_2]}A_{13e}^{[n-1]}+i_2A_{15e}^{[n]}A_{13e}^{[i_2-1]})/A_{\mu}^{\epsilon};$$

$$P2(1,1)=-(-A_{11e}^{[0]}A_{11e}^{[0]}-A_{11}^{[0]}A_{\mu}^{\epsilon})/A_{\mu}^{\epsilon};\quad P2(1,1+i_1)=-(-A_{11}^{[i_1]}A_{\mu}^{\epsilon}-A_{11e}^{[0]}A_{11e}^{[i_1]})/A_{\mu}^{\epsilon};$$

$$P2(1,2+Un)=A_{11e}^{[0]}A_{15e}^{[0]}/A_{\mu}^{\epsilon};\quad P2(1,2+Un+i_2)=A_{11e}^{[0]}A_{15e}^{[i_2]}/A_{\mu}^{\epsilon};$$

$$P2(1+m,1)=-(-A_{11}^{[m]}A_{\mu}^{\epsilon}-A_{11e}^{[0]}A_{11e}^{[m]})/A_{\mu}^{\epsilon};$$

$$P2(1+m,1+i_1)=-(-A_{11}^{[i_1+m]}A_{\mu}^{\epsilon}-A_{11e}^{[m]}A_{11e}^{[i_1]})/A_{\mu}^{\epsilon};$$

$$P2(1+m,2+Un)=A_{15e}^{[0]}A_{11e}^{[m]}/A_{\mu}^{\epsilon};\qquad P2(1+m,2+Un+i_2)=A_{11e}^{[m]}A_{15e}^{[i_2]}/A_{\mu}^{\epsilon};$$

$$P2(2+Un,1)=A_{11e}^{[0]}A_{15e}^{[0]}/A_{\mu}^{\epsilon};\qquad P2(2+Un,1+i_1)=A_{15e}^{[0]}A_{11e}^{[i_1]}/A_{\mu}^{\epsilon};$$

$$P2(2+Un,2+Un)=-(-A55A_{\mu}^{\epsilon}-A_{15e}^{[0]}A_{15e}^{[0]})/A_{\mu}^{\epsilon};$$

$$P2(2+Un,2+Un+i_2)=-(-A_{55}^{[i_2]}A_{\mu}^{\epsilon}-A_{15e}^{[0]}A_{15e}^{[i_2]})/A_{\mu}^{\epsilon};$$

$$P2(2+Un+n,1)=A_{11e}^{[0]}A_{15e}^{[n]}/A_{\mu}^{\epsilon};\qquad P2(2+Un+n,1+i_1)=A_{11e}^{[i_1]}A_{15e}^{[n]}/A_{\mu}^{\epsilon};$$

$$P2(2+Un+n,2+Un)=-(-A_{55}^{[n]}A_{\mu}^{\epsilon}-A_{15e}^{[0]}A_{15e}^{[n]})/A_{\mu}^{\epsilon};$$

$$P2(2+Un+n,2+Un+i_2)=-(-A_{15e}^{[n]}A_{15e}^{[i_2]}-A_{55}^{[n+i_2]}A_{\mu}^{\epsilon})/A_{\mu}^{\epsilon};$$

where, i_1 and m varies from from 1 to Un and i_2 and n varies from from 1 to Wn.

Expression for $I_s, I_a, I_b, I_c, a_a, a_b, b_a, b_b, c_a, d_a, c_b, d_b$ are given by

$$\begin{aligned}
I_s &= \left(I_0I_2-{I_1}^2\right);\quad I_a=-I_0{I_3}^2-{I_2}^3+I_0I_2I_4-{I_1}^2I_4+2I_1I_2I_3;\\
I_b &= -I_2I_0I_4+I_0{I_3}^2+{I_2}^3-2I_1I_2I_3+{I_1}^2I_4;\\
I_c &= -{I_2}^3-{I_1}^2I_4+I_0I_4I_2-I_0{I_3}^2+2I_1I_2I_3;
\end{aligned}$$

$$a_a = 4I_0I_2A_{33}^{[2]}+I_0A_{33}^{[0]}I_4-4{I_1}^2A_{33}^{[2]}-4I_0I_3A_{33}^{[1]}-{I_2}^2A_{33}^{[0]}+4I_1I_2A_{33}^{[1]};$$

$$\begin{aligned}
a_b \;=\;& I_2{}^4 A_{33}^{[0]}{}^2 + 16 A_{33}^{[2]}{}^2 I_1{}^4 - 8 I_2{}^3 A_{33}^{[0]} I_1 A_{33}^{[1]} + 8 I_2{}^2 A_{33}^{[0]} A_{33}^{[2]} I_1{}^2 + 16 I_1{}^2 A_{33}^{[1]}{}^2 I_2{}^2 \\
& - 8 I_0{}^2 I_2 A_{33}^{[2]} A_{33}^{[0]} I_4 - 32 I_0{}^2 I_2 A_{33}^{[2]} I_3 A_{33}^{[1]} + 32 I_0 I_2{}^2 A_{33}^{[2]} I_1 A_{33}^{[1]} + 8 I_0 A_{33}^{[0]} I_4 I_1{}^2 A_{33}^{[2]} \\
& - 8 I_0{}^2 A_{33}^{[0]} I_4 I_3 A_{33}^{[1]} + 8 I_0 A_{33}^{[0]} I_4 I_1 I_2 A_{33}^{[1]} + 32 I_1{}^2 A_{33}^{[2]} I_0 I_3 A_{33}^{[1]} + 8 I_0 I_3 A_{33}^{[1]} I_2{}^2 A_{33}^{[0]} \\
& - 32 I_1 I_2 I_3 I_0 A_{33}^{[0]} A_{33}^{[2]} - 32 I_0 I_2 A_{33}^{[2]}{}^2 I_1{}^2 + 8 I_0 I_2{}^3 A_{33}^{[2]} A_{33}^{[0]} - 2 I_0 A_{33}^{[0]}{}^2 I_4 I_2{}^2 \\
& - 32 I_1{}^3 A_{33}^{[2]} I_2 A_{33}^{[1]} + 16 I_0{}^2 I_3{}^2 A_{33}^{[0]} A_{33}^{[2]} + 16 I_0{}^2 I_2 I_4 A_{33}^{[1]}{}^2 - 16 I_1{}^2 I_4 I_0 A_{33}^{[1]}{}^2 \\
& + 16 I_0{}^2 I_2{}^2 A_{33}^{[2]}{}^2 + I_0{}^2 A_{33}^{[0]}{}^2 I_4{}^2 - 16 I_2{}^3 I_0 A_{33}^{[1]}{}^2
\end{aligned}$$

$$b_a = -4 I_2 I_0 A_{55}^{[2]} + 4 I_3 I_0 A_{55}^{[1]} + I_2{}^2 A_{55}^{[0]} - 4 I_1 A_{55}^{[1]} I_2 + 4 A_{55}^{[2]} I_1{}^2 - A_{55}^{[0]} I_0 I_4;$$

$$\begin{aligned}
b_b \;=\;& -16 I_2{}^3 I_0 A_{55}^{[1]}{}^2 + 16 I_2{}^2 I_0{}^2 A_{55}^{[2]}{}^2 + 16 I_1{}^2 A_{55}^{[1]}{}^2 I_2{}^2 + A_{55}^{[0]}{}^2 I_0{}^2 I_4{}^2 + I_2{}^4 A_{55}^{[0]}{}^2 \\
& + 16 A_{55}^{[2]}{}^2 I_1{}^4 - 32 I_2 I_0{}^2 A_{55}^{[2]} I_3 A_{55}^{[1]} + 32 I_2{}^2 I_0 A_{55}^{[2]} I_1 A_{55}^{[1]} - 8 I_2 I_0{}^2 A_{55}^{[2]} A_{55}^{[0]} I_4 \\
& + 8 I_3 I_0 A_{55}^{[1]} I_2{}^2 A_{55}^{[0]} + 32 I_3 I_0 A_{55}^{[1]} A_{55}^{[2]} I_1{}^2 - 8 I_3 I_0{}^2 A_{55}^{[1]} A_{55}^{[0]} I_4 + 8 I_1 A_{55}^{[1]} I_2 A_{55}^{[0]} I_0 I_4 \\
& + 8 A_{55}^{[2]} I_1{}^2 A_{55}^{[0]} I_0 I_4 - 32 I_1 I_2 I_3 I_0 A_{55}^{[0]} A_{55}^{[2]} + 8 I_2{}^3 I_0 A_{55}^{[2]} A_{55}^{[0]} - 32 I_2 I_0 A_{55}^{[2]}{}^2 I_1{}^2 \\
& - 8 I_2{}^3 A_{55}^{[0]} I_1 A_{55}^{[1]} + 8 I_2{}^2 A_{55}^{[0]} A_{55}^{[2]} I_1{}^2 - 2 I_2{}^2 A_{55}^{[0]}{}^2 I_0 I_4 - 32 I_1{}^3 A_{55}^{[1]} I_2 A_{55}^{[2]} \\
& + 16 I_2 I_0{}^2 I_4 A_{55}^{[1]}{}^2 + 16 I_0{}^2 I_3{}^2 A_{55}^{[0]} A_{55}^{[2]} - 16 I_1{}^2 I_4 I_0 A_{55}^{[1]}{}^2;
\end{aligned}$$

$$c_a = 4 I_0 I_2 A_{55}^{[2]} - 4 I_1{}^2 A_{55}^{[2]} + 4 I_1 I_2 A_{55}^{[1]} - I_2{}^2 A_{55}^{[0]} + I_0 I_4 A_{55}^{[0]} - 4 I_0 I_3 A_{55}^{[1]};$$

$$d_a = I_0 A_{33}^{[0]} I_4 - 4 I_0 I_3 A_{33}^{[1]} + 4 I_1 I_2 A_{33}^{[1]} - I_2{}^2 A_{33}^{[0]} + 4 I_0 I_2 A_{33}^{[2]} - 4 I_1{}^2 A_{33}^{[2]};$$

$$\begin{aligned}
c_b \;=\;& 16 I_0{}^2 I_2{}^2 A_{55}^{[2]}{}^2 - 16 I_2{}^3 A_{55}^{[1]}{}^2 I_0 + 16 I_1{}^4 A_{55}^{[2]}{}^2 - 32 I_0 I_2 A_{55}^{[2]}{}^2 I_1{}^2 - 2 I_0 I_4 A_{55}^{[0]}{}^2 I_2{}^2 \\
& - 32 I_1{}^3 I_2 A_{55}^{[1]} A_{55}^{[2]} - 8 I_2{}^3 A_{55}^{[0]} I_1 A_{55}^{[1]} + 8 I_2{}^2 A_{55}^{[0]} I_1{}^2 A_{55}^{[2]} - 8 I_0{}^2 I_4 A_{55}^{[0]} I_3 A_{55}^{[1]} \\
& + 8 I_0 I_4 A_{55}^{[0]} I_1 I_2 A_{55}^{[1]} + 8 I_0 I_3 A_{55}^{[1]} I_2{}^2 A_{55}^{[0]} + 32 I_0 I_3 A_{55}^{[1]} I_1{}^2 A_{55}^{[2]} \\
& + 16 I_1{}^2 I_2{}^2 A_{55}^{[1]}{}^2 - 32 I_0{}^2 I_2 A_{55}^{[2]} I_3 A_{55}^{[1]} + 32 I_0 I_2{}^2 A_{55}^{[2]} I_1 A_{55}^{[1]} \\
& + I_2{}^4 A_{55}^{[0]}{}^2 - 16 I_4 I_1{}^2 A_{55}^{[1]}{}^2 I_0 + 16 I_4 I_2 A_{55}^{[1]}{}^2 I_0{}^2 - 32 I_3 I_2 I_1 A_{55}^{[2]} A_{55}^{[0]} I_0 + 8 I_2{}^3 A_{55}^{[2]} A_{55}^{[0]} I_0 \\
& + 8 I_4 I_1{}^2 A_{55}^{[2]} A_{55}^{[0]} I_0 - 8 I_4 I_2 A_{55}^{[2]} A_{55}^{[0]} I_0{}^2 + 16 I_3{}^2 A_{55}^{[2]} A_{55}^{[0]} I_0{}^2 + I_0{}^2 I_4{}^2 A_{55}^{[0]}{}^2;
\end{aligned}$$

$$\begin{aligned}
d_b \;=\;& -32 A_{33}^{[2]} A_{33}^{[0]} I_1 I_2 I_3 I_0 + 16 A_{33}^{[2]} A_{33}^{[0]} I_3{}^2 I_0{}^2 + 8 A_{33}^{[2]} A_{33}^{[0]} I_4 I_0 I_1{}^2 - 16 A_{33}^{[1]}{}^2 I_4 I_0 I_1{}^2 \\
& - 16 A_{33}^{[1]}{}^2 I_2{}^3 I_0 + I_0{}^2 A_{33}^{[0]}{}^2 I_4{}^2 + 8 I_0 A_{33}^{[0]} I_4 I_1 I_2 A_{33}^{[1]} - 32 I_0 I_2 A_{33}^{[2]}{}^2 I_1{}^2 - 2 I_0 A_{33}^{[0]}{}^2 I_4 I_2{}^2 \\
& + 16 A_{33}^{[1]}{}^2 I_4 I_2 I_0{}^2 + 16 I_1{}^4 A_{33}^{[2]}{}^2 + 32 I_0 I_3 A_{33}^{[1]} I_1{}^2 A_{33}^{[2]} - 8 I_2{}^3 A_{33}^{[0]} I_1 A_{33}^{[1]} + 8 I_2{}^2 A_{33}^{[0]} I_1{}^2 A_{33}^{[2]} \\
& - 32 I_1{}^3 I_2 A_{33}^{[1]} A_{33}^{[2]} + I_2{}^4 A_{33}^{[0]}{}^2 + 16 I_1{}^2 I_2{}^2 A_{33}^{[1]}{}^2 + 16 I_0{}^2 I_2{}^2 A_{33}^{[2]}{}^2 - 8 A_{33}^{[2]} A_{33}^{[0]} I_4 I_2 I_0{}^2 \\
& + 8 I_0 I_3 A_{33}^{[1]} I_2{}^2 A_{33}^{[0]} + 8 A_{33}^{[2]} A_{33}^{[0]} I_2{}^3 I_0 - 8 I_0{}^2 A_{33}^{[0]} I_4 I_3 A_{33}^{[1]} - 32 I_0{}^2 I_2 A_{33}^{[2]} I_3 A_{33}^{[1]} \\
& + 32 I_0 I_2{}^2 A_{33}^{[2]} I_1 A_{33}^{[1]};
\end{aligned}$$

Nonzero entries of $[Q0]$ and $Q1$ are given as

$$Q0(1,1+i_1) = i_1 A_{11e}^{[0]} A_{15e}^{[i_1-1]}/A_\mu^\epsilon; \qquad Q0(1+m,1+i_1) = i_1 A_{11e}^{[m]} A_{15e}^{[i_1-1]}/A_\mu^\epsilon;$$
$$Q0(1,2+Un+i_2) = (i_2 A_{13}^{[i_2-1]} A_\mu^\epsilon + i_2 A_{13e}^{[i_2-1]} A_{11e}^{[0]})/A_\mu^\epsilon;$$
$$Q0(1+m,2+Un+i_2) = (i_2 A_{11e}^{[m]} A_{13e}^{[i_2-1]} + i_2 A_{13}^{[m+i_2-1]} A_\mu^\epsilon)/A_\mu^\epsilon;$$

$$Q0(2+Un,1+i_1) = (i_1 A_{55}^{[i_1-1]} A_\mu^\epsilon + i_1 A_{15e}^{[0]} A_{15e}^{[i_1-1]})/A_\mu^\epsilon;$$
$$Q0(2+Un,2+Un+i_2) = i_2 A_{15e}^{[0]} A_{13e}^{[i_2-1]}/A_\mu^\epsilon;$$
$$Q0(2+Un+n,1+i_1) = (i_1 A_{55}^{[i_1+n-1]} A_\mu^\epsilon + i_1 A_{15e}^{[n]} A_{15e}^{[i_1-1]})/A_\mu^\epsilon; \qquad .$$
$$Q0(2+Un+n,2+Un+i_2) = i_2 A_{15e}^{[n]} A_{13e}^{[i_2-1]}/A_\mu^\epsilon;$$

$$Q1(1,1) = (A_{11}^{[0]} A_\mu^\epsilon + A_{11e}^{[0]} A_{11e}^{[0]})/A_\mu^\epsilon; \qquad Q1(1,1+i_1) = (A_{11e}^{[0]} A_{11e}^{[i_1]} + A_{11}^{[i_1]} A_\mu^\epsilon)/A_\mu^\epsilon;$$
$$Q1(1,2+Un) = A_{11e}^{[0]} A_{15e}^{[0]}/A_\mu^\epsilon; \qquad Q1(1,2+Un+i_2) = A_{11e}^{[0]} A_{15e}^{[i_2]}/A_\mu^\epsilon;$$

$$Q1(1+m,1) = (A_{11e}^{[0]} A_{11e}^{[m]} + A_{11}^{[m]} A_\mu^\epsilon)/A_\mu^\epsilon;$$
$$Q1(1+m,1+i_1) = (A_{11}^{[i_1+m]} A_\mu^\epsilon + A_{11e}^{[m]} A_{11e}^{[i_1]})/A_\mu^\epsilon;$$
$$Q1(1+m,2+Un) = A_{15e}^{[0]} A_{11e}^{[m]}/A_\mu^\epsilon; \qquad Q1(1+m,2+Un+i_2) = A_{11e}^{[m]} A_{15e}^{[i_2]}/A_\mu^\epsilon;$$

$$Q1(2+Un,1) = A_{11e}^{[0]} A_{15e}^{[0]}/A_\mu^\epsilon; \qquad Q1(2+Un,1+i_1) = A_{15e}^{[0]} A_{11e}^{[i_1]}/A_\mu^\epsilon;$$
$$Q1(2+Un,2+Un) = (A_{15e}^{[0]} A_{15e}^{[0]} + A55 A_\mu^\epsilon)/A_\mu^\epsilon;$$
$$Q1(2+Un,2+Un+i_2) = (A_{55}^{[i_2]} A_\mu^\epsilon + A_{15e}^{[0]} A_{15e}^{[i_2]})/A_\mu^\epsilon;$$

$$Q1(2+Un+n,1) = A_{11e}^{[0]} A_{15e}^{[n]}/A_\mu^\epsilon; \qquad Q1(2+Un+n,1+i_1) = A_{11e}^{[i_1]} A_{15e}^{[n]}/A_\mu^\epsilon;$$
$$Q1(2+Un+n,2+Un) = (A_{55}^{[n]} A_\mu^\epsilon + A_{15e}^{[0]} A_{15e}^{[n]})/A_\mu^\epsilon;$$
$$Q1(2+Un+n,2+Un+i_2) = (A_{15e}^{[n]} A_{15e}^{[i_2]} + A_{55}^{[n+i_2]} A_\mu^\epsilon)/A_\mu^\epsilon.$$

where, i_1 and m varies from from 1 to Un and i_2 and n varies from from 1 to Wn.