Appendix A

Three dimensional intensity correlation function.

The speckle field generated by a stocastic sample is formed by speckles extending in both the orthogonal and parallel direction with respect to the direction of propagation of the wave. The intensity measured in a plane perpendicular to the direction of propagation varies as the plane is moved; small movements of the plane will give small variations in the intensities. As a matter of facts, the speckles appear and disappear as the plane is moved. This allows us to speack of the three dimensional appearence of the speckles. We will show that the speckles are elongated in the direction of the propagation of the light. If the diamer is α times a wave length λ, their length is α^2 times λ.

In the following sections, we will show that the three dimensional correlation function of the intensity of the scattered light gives more informations than the two dimensional one; in some cases it is possible to determine the sign of the field correlation function, thus determining it completely. Moreover, in analogy to the quadratic relation between the diameter and length of a speckle, the longitudinal frequencies should be related to the square root of the frequencies of the sample: measuring the longitudinal correlations should double the dynamic of the system.

A.1 Evolution equation of the field correlation.

For $q \ll k$, Eq. (3.5) can be approximated by:

$$E_z\left(\vec{q}\right) = E_0\left(\vec{q}\right) e^{ikz} e^{-i\frac{q^2}{2k}z} \tag{A.1}$$

In this approximation, neglecting the phase term $\exp\left(ikz\right)$, the field follows a Schröedinger equation:

$$i\frac{\partial}{\partial z}E\left(\vec{x},z\right) = -\frac{1}{2k}\nabla^2 E\left(\vec{x},z\right) \tag{A.2}$$

The three dimensional field correlation is defined as follows:

$$C_E(\Delta\vec{x},\Delta z)=\frac{1}{S}\int_S E(\vec{x},z)\,E(\vec{x}+\Delta\vec{x},z+\Delta z)\,\mathrm{d}\vec{x}\mathrm{d}z \tag{A.3}$$

In order to obtain an evolution equation for $C(\Delta\vec{x},\Delta z)$, as Δz increases, we evaluate the first derivative of the correlation function:

$$\frac{\partial}{\partial\Delta z}C_E(\Delta\vec{x},\Delta z)=\frac{1}{S}\int_S E(\vec{x},z)\,\frac{\partial}{\partial\Delta z}E(\vec{x}+\Delta\vec{x},z+\Delta z)\,\mathrm{d}\vec{x}\mathrm{d}z \tag{A.4}$$

Using eq. (A.2):

$$\frac{\partial}{\partial\Delta z}C_E(\Delta\vec{x},\Delta z)=\frac{1}{S}\int_S E(\vec{x},z)\,\frac{i}{2k}\nabla^2 E(\vec{x}+\Delta\vec{x},z+\Delta z)\,\mathrm{d}\vec{x}\mathrm{d}z \tag{A.5}$$

The operator ∇ acts on the first argument of $E(\vec{x},z)$, thus it can be considered as acting on $\Delta\vec{x}$:

$$\frac{\partial}{\partial\Delta z}C_E(\Delta\vec{x},\Delta z)=\frac{i}{2k}\nabla^2_{\Delta\vec{x}}\frac{1}{S}\int_S E(\vec{x},z)\,E(\vec{x}+\Delta\vec{x},z+\Delta z)\,\mathrm{d}\vec{x}\mathrm{d}z \tag{A.6}$$

This proves that the evolution equation for $(\Delta\vec{x},\Delta z)$, as Δz increases, is a Schröedinger equation:

$$i\frac{\partial}{\partial\Delta z}C_E(\Delta\vec{x},\Delta z)=-\frac{1}{2k}\nabla^2 C_E(\Delta\vec{x},\Delta z) \tag{A.7}$$

This equation can easily be solved in Fourier space:

$$C_E(\vec{q},z)=C_E(\vec{q},z=0)\,e^{-i\frac{q^2 z}{2k}} \tag{A.8}$$

We can now extend eq. (3.65) to the three dimensional case:

$$C_I(\Delta\vec{x},\Delta z)=\langle I(\vec{x},z)\,I(\vec{x}+\Delta\vec{x},z+\Delta z)\rangle=\langle I\rangle^2+|C_E(\Delta\vec{x},\Delta z)|^2 \tag{A.9}$$

A.2 Gaussian speckles.

In this section we consider gaussian speckles, and we evaluate their three dimensional correlation function.

Far field speckles are often generated by scattering a gaussian beam, so that the far field speckles have a gaussian correlation function. We consider gaussian speckles in near field, since the case is analitically solvable, and involves some calculations used in quantum mechanics.

The field correlation function of the scattered light, in the plane orthogonal to z, is gaussian:

$$C_E(\Delta\vec{x},\Delta z=0)=Ce^{-\frac{\Delta\vec{x}^2}{2\sigma^2}}. \tag{A.10}$$

In the Fourier space:

$$C_E\left(\vec{q}, \Delta z = 0\right) = 2\pi\sigma^2 C e^{-\frac{1}{2}\sigma^2 q^2}. \tag{A.11}$$

Using eq. (A.8):

$$C_E\left(\vec{q}, z\right) = 2\pi\sigma^2 C e^{-\frac{1}{2}\sigma^2 q^2 - i\frac{q^2 z}{2k}}. \tag{A.12}$$

Coming back to real space:

$$C_E\left(\vec{x}, z\right) = C\frac{\sigma^2}{\sigma^2 + iz/k} e^{-\frac{x^2}{2\left(\sigma^2 + iz/k\right)}}. \tag{A.13}$$

Now we evaluate the modulus of the field correlation function, the quantity needed in eq. (A.9) to determine the intensity correlation function:

$$\left|C_E\left(\vec{x}, z\right)\right|^2 = C^2\frac{\sigma^4}{\sigma^4 + z^2/k^2} e^{-\frac{x^2\sigma^2}{\sigma^4 + z^2/k^2}}. \tag{A.14}$$

We can now evaluate the intensity correlation function for $\vec{x} = 0$:

$$C_I\left(\vec{x} = 0, z\right) = C^2\left(1 + \frac{\sigma^4}{\sigma^4 + z^2/k^2}\right), \tag{A.15}$$

and for $z = 0$:

$$C_I\left(\vec{x}, z = 0\right) = C^2\left(1 + e^{\frac{x^2}{\sigma^2}}\right). \tag{A.16}$$

While the transverse correlation function follows a gaussian law, the longitudinal one is a Lorentzian, The diameter of the speckles is about σ, while their length is $\sigma^2 k$.

A.3 Determination of the sign of the field correlation function.

The power spectrum, that is $C_E\left(\vec{q}, z = 0\right)$, is real. If the sample is isotropic, it is symetric with respect to the origin, and then the correlation function $C_E\left(\vec{x}, z = 0\right)$ is real. The knowledge of the intensity correlation function with $\Delta z = 0$ gives the absolute value of the field correlation function. The sign of the field correlation function does not affect the intensity correlation function with $\Delta z = 0$, but it can affect its value for $\Delta z \neq 0$.

In figure A.1 and A.2 we see an example of this effect. The figures show the graphs of the square correlation functions. The first is such that $C_E\left(x, \Delta z = 0\right) =$

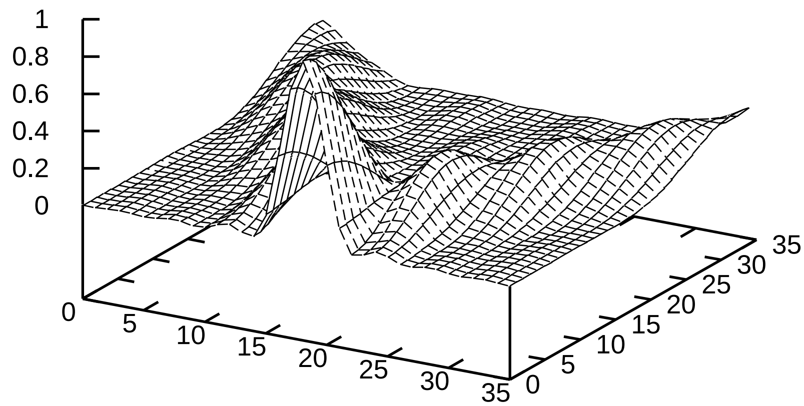

Figure A.1: Comparison between two square correlation functions. $C_E(x, \Delta z = 0) = \sin(x)/x$

$\sin(x)/x$; in the second, the correlation function has the same absolute value, but alwais positive sign, for $\Delta z = 0$. For $\Delta z = 0$ the square correlation functions are equal; their evolution for other values of Δz are different. We can explain this fact considering the evolution of the positive and negative parts of the correlation function. The two parts evolve, and overlap, as Δz increases. The interference of the two parts depends on the initial phase.

The sign of the correlation function is alwais possible, in principle. The presence of errors could limit this possibility.

A.4 Longitudinal correlation.

We want to derive the field correlation along the z axis. We consider its Fourier transform:

$$C_E(\Delta\vec{x} = 0, q_z) = \frac{1}{(2\pi)^2}\int C_E(\vec{q}, z)\, e^{-iq_z z}\mathrm{d}\vec{q}\mathrm{d}z \tag{A.17}$$

Using eq. (A.8):

$$C_E(\Delta\vec{x} = 0, q_z) = \frac{1}{(2\pi)^2}\int C_E(\vec{q}, z = 0)\, e^{-i\frac{q^2 z}{2k} - iq_z z}\mathrm{d}\vec{q}\mathrm{d}z \tag{A.18}$$

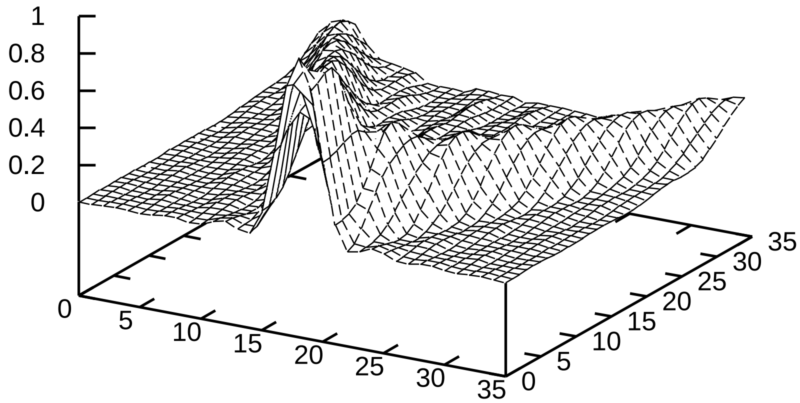

Figure A.2: Comparison between two square correlation functions. $C_E(x, \Delta z = 0) = |\sin(x)/x|$

The integration over z gives a Dirac delta:

$$C_E\left(\Delta\vec{x}=0,q_z\right)=\int C_E\left(\vec{q},z=0\right)\delta\left(\frac{q^2}{2k}+iq_z\right)\mathrm{d}\vec{q} \tag{A.19}$$

In radial coordinates:

$$C_E\left(\Delta\vec{x}=0,q_z\right)=\int C_E\left(q,\varphi,z=0\right)q\delta\left(\frac{q^2}{2k}+iq_z\right)\mathrm{d}q\mathrm{d}\varphi \tag{A.20}$$

If the sample is isotropic:

$$C_E\left(\Delta\vec{x}=0,q_z\right)=2\pi\int C_E\left(q,z=0\right)q\delta\left(\frac{q^2}{2k}+iq_z\right)\mathrm{d}q \tag{A.21}$$

The integral can be evaluated:

$$C_E\left(\Delta\vec{x}=0,q_z\right)=2\pi C_E\left(\sqrt{2kq_z},z=0\right)\sqrt{2kq_z} \tag{A.22}$$

The dynamic of an instrument measuring the longitudinal correlation function is twice that obtained with the transversal one. This facts closely mirrors the quadratic relation between the diameter and the length of the speckles.

Appendix B

Definitions

Fourier transform:

$$f\left(q\right)=\int f\left(x\right)e^{-iqx}\mathrm{d}x \tag{B.1}$$

Inverse Fourier transform:

$$f\left(x\right)=\frac{1}{\left(2\pi\right)^{n}}\int f\left(q\right)e^{iqx}\mathrm{d}q \tag{B.2}$$

Convolution:

$$\begin{aligned} g\left(x\right)&=\left|f\left(x\right)\right|^{2}\\ g\left(q\right)&=\tfrac{1}{\left(2\pi\right)^{n}}\int f\left(q'\right)f^{*}\left(q'-q\right)\mathrm{d}q' \end{aligned} \tag{B.3}$$

NFS Near Field Scattering
ONFS hOmodyne Near Field Scattering
ENFS hEterodyne Near Field Scattering
SNFS Schlieren-like Near Field Scattering
LS Light Scattering
SALS Small Angle Light Scattering
IFS Intensity Fluctuation Spectroscopy
CCD Charge Coupled Device