AP CALCULUS SPRING BREAK EXAM

SOLUTIONS

1. If $y=(x^3+1)^2$, then $\frac{dy}{dx}=$ $\boxed{2(x^3+1)^1\cdot(3x^2)=\mathbf{6x^2(x^3+1)}}$

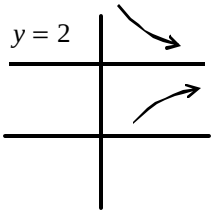
2. $\int_0^1 e^{-4x}\,dx=$

$g: e^{-4x}$

$c: -4e^{-4x}$

$a: -\frac{1}{4}e-4x\Big]_0=\left[-\frac{1}{4}e^{-4}\right]-\left[-\frac{1}{4}e^{0}\right]=\frac{\mathbf{-e^{-4}}}{\mathbf{4}}+\frac{\mathbf{1}}{\mathbf{4}}$

3. For $x\geq 0$, the horizontal line $y=2$ is an asymptote for the graph of the function f. Which of the following statements must be true?



$\therefore \boxed{\lim_{x\to\infty} f(x)=2}$

4. If $y=\frac{2x+3}{3x+2}$, then $\frac{dy}{dx}=$ $\boxed{\frac{2(3x+2)-(2x+3)(3)}{(3x+2)^2}=\frac{6x+4-6x-9}{(3x+2)^2}=\frac{-5}{(3x+2)^2}}$

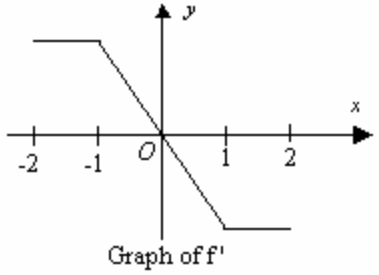
5. $\int_0^{\frac{p}{4}} \sin x\,dx=$

$g: \cos x$

$c: -\sin x$

$a: -\cos x\Big]_0^{p/4}=\left(-\cos\frac{p}{4}\right)-(-\cos 0)=\left(\frac{-\sqrt{2}}{2}\right)+1$

6. $\lim_{x\to\infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}=$ $\boxed{\lim_{x\to\infty} \frac{1-\frac{2}{x}+\frac{3}{x^2}-\frac{4}{x^3}}{4-\frac{3}{x}+\frac{2}{x^2}-\frac{1}{x^3}}=\frac{1}{4}}$



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

f ' is positive when $-2\leq x\leq 0$ so f (x) is increasing for $\mathbf{-2\leq x\leq 0}$

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8. $\int x^2\cos\left(x^3\right)dx =$

$g : \sin(x^3)$

$c : 3x^2\cos(x^3)$

$a : \frac{1}{3}\sin(x^3) + C$

9. If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0) =$

$$f'(x) = \left(\frac{1}{x + 4 + e^{-3x}}\right)\left(1 + 0 + -3e^{-3x}\right)$$

$$f'(0) = \left(\frac{1}{0 + 4 + e^{0}}\right)\left(1 + 0 + -3e^{0}\right) = \frac{-2}{5}$$

10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f ? $f(x)$ must be **decreasing** and **concave down**.

11. Using the substitution $u = 2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

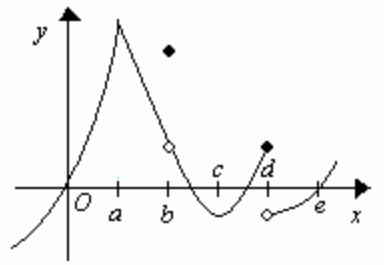
$du = 2dx$ so $dx = \frac{1}{2}du$ AND if $x = 0$, then $u = 2(0) + 1 = 1$

if $x = 2$, then $u = 2(2) + 1 = 5$

SO, $\int_{x=0}^{x=2} \sqrt{u}\,du \Rightarrow \int_{u=1}^{u=5} \sqrt{u} \times \frac{1}{2}du = \mathbf{\frac{1}{2}\int_1^5 \sqrt{u}\,du}$

12. The rate of change of the volume, *V*, of water in a tank with respect to time, *t*, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

$\frac{dv}{dt} = \mathbf{k\sqrt{volume}}$



13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

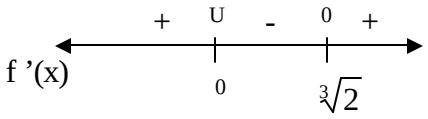
Answer = A

14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} = 2x(\sin 2x) + x^2(\cos 2x)2 = \mathbf{2x(\sin 2x + x\cos 2x)}$

15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

$f'(x) = x^2 - \frac{2}{x} = 0 \Rightarrow x^2 = \frac{2}{x} \Rightarrow x^3 = 2 \Rightarrow x = \sqrt[3]{2}$

$\therefore$ By looking at $f'(x)$ chart $\Rightarrow$ decreasing : $\mathbf{(0, \sqrt[3]{2}]}$



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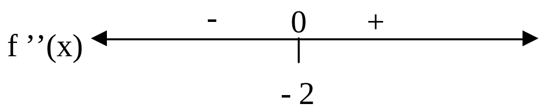
16. If the line tangent to the graph of the function f at point $(1,7)$ passes through the point $(-2,-2)$, then

$$f\ '(1) = \frac{-2-7}{-2-1} = \frac{-9}{-3} = \mathbf{3}$$

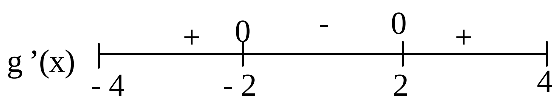
17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

$$f\ '(x) = 2e^x + 2xe^x = 0$$
$$f\ ''(x) = 2e^x + 2e^x + 2xe^x = 0$$
$$4e^x + 2xe^x = 0$$
$$2e^x(2+x) = 0$$
$$x = -2$$
$$\therefore \text{Concave Down when } \mathbf{x < -2}$$



x	-4	-3	-2	-1	0	1	2	3	4
g '(x)	2	3	0	-3	-2	-1	0	3	2

18. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?



Answer = A decreasing when -2 $\le$ x $\le$ 2

19. A curve has slope $2x+3$ at each point (x,y) on the curve. Which of the following is an equation for this curve if it passes through the point (1,2)?

$$\frac{dy}{dx} = 2x+3 \text{ so } dy = (2x+3)dx$$
$$y = \int (2x+3)dx = x^2 + 3x + C$$
$$2 = 1 + 3 + C$$
$$C = -2 \quad \therefore \quad \mathbf{y = x^2 + 3x - 2}$$

$$f(x) = \begin{cases} x+2 & , x \le 3 \\ 4x-7 & , x > 3 \end{cases}$$

20. Let f be the function given above. Which of the following statements are true about f ?

$$\lim_{x\to3^-} x+2 = 5 \quad \text{and} \quad \lim_{x\to3^+} 4x-7 = 5$$

I. **True** $\lim_{x\to3} f(x) = 5$

II. **True** $f(3) = \begin{cases} 3+2 = 5 \\ 4(3)-7 = 5 \end{cases}$

III. False $f\ '(x) = \begin{cases} 1 & x \le 3 \\ 4 & x > 3 \end{cases}$

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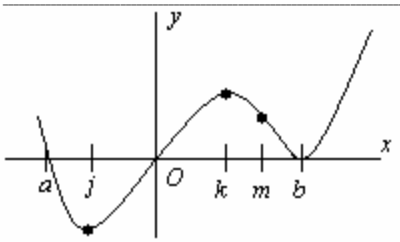
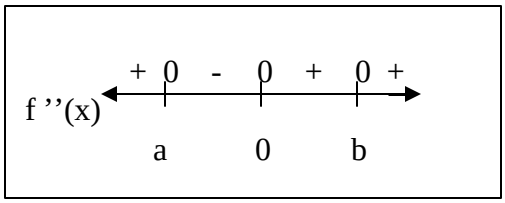
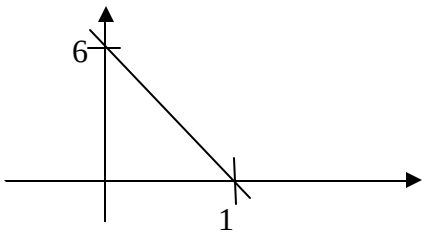


21. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?



$\therefore$ inflection point at $\mathbf{x = a \text{ and } x = 0}$



22. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

$$f'(1) = f(0) + f(1) = 5 + \int_0^1 f'(x) = 5 + \left(\tfrac{1}{2}\right)(1)(6) = \mathbf{8}$$

23 $\dfrac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt \right) =$ $\boxed{(2x)\sin\left(x^2\right)^3 = (\mathbf{2x})\mathbf{sin}\left(\mathbf{x^6}\right)}$

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where x = - 1?

$$f'(x) = 12x^2 - 5 \text{ and } f'(-1) = 12 - 5 = 7$$
$$f(-1) = 4(-1) - 5(-1) + 3 = -4 + 5 + 3 = 4$$

$\rightarrow$

$$y - 4 = 7(x+1)$$
$$y - 4 = 7x + 7$$
$$\mathbf{y = 7x + 11}$$

25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

$$v(t) = 6t^2 - 42t + 72 = 0$$
$$t^2 - 7t + 12 = 0$$
$$(t-4)(t-3) = 0$$
$$\mathbf{t = 4 \text{ or } t = 3}$$

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26. What is the slope of the tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point (3, 2)?

$$6y\left(\frac{dy}{dx}\right) - 4x = 0 + (-2)y + (-2x)\left(\frac{dy}{dx}\right)$$
$$12\left(\frac{dy}{dx}\right) - 12 = -4 + (-6)\left(\frac{dy}{dx}\right)$$
$$18\left(\frac{dy}{dx}\right) = 8 \qquad \therefore \frac{dy}{dx} = \frac{8}{18} = \mathbf{\frac{4}{9}}$$

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and g(2)=1, what is the value of g'(2)?

$$f[g(x)] = f[f^{-1}(x)]$$
$$f[g(x)] = x$$
$$\therefore f'[g(x)] \times g'(x) = 1$$
$$g'(x) = \frac{1}{f'[g(x)]} \rightarrow g'(2) = \frac{1}{f'[g(2)]} = \frac{1}{f'(1)} = \frac{1}{4}$$
$$f'(x) = 3x^2 + 1 \quad \text{so } f'(1) = 3 + 1 = \mathbf{4}$$

28. Let g be a twice-differentiable function with g'(x)>0 and g''(x)>0 for all real numbers x, such that g(4)=12 and g(5)=18. Of the following, which is a possible value for g(6)?

x	g(x)	g'(x)	g"(x)
4	12		
5	18	6	
6	□	Δ	Δ − 6

$\Delta - 6 > 0$ so that g"(x)>0

$\Delta > 6$

$\square - 18 = \Delta \therefore \square - 18 > 6$

$\square$ **>24** $\therefore$ **E**

76. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time t = 4?

$$a(t) = v'(t) = (-4.1)\sin(0.9t) \cdot (0.9)$$
$$a(4) = (-4.1)\sin(3.6) \cdot (0.9) = 1.633$$

CALCULATOR: MATH 8, 3+4.1cos(.9t), x, 4 = 1.633 **C**

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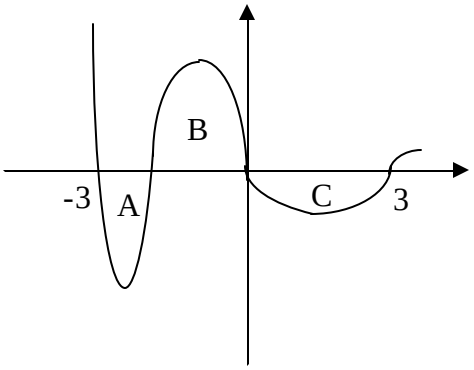


77. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis.

If the area of each region is 2, what is the value of $\int_{-3}^{3}(f(x)+1)dx$?

$$\int_{-3}^{3}(f(x)+1)dx=\int_{-3}^{3}f(x)+\int_{-3}^{3}1\,dx=-2+x\Big]_{-3}^{3}=-2+3-(-3)=\mathbf{4}$$

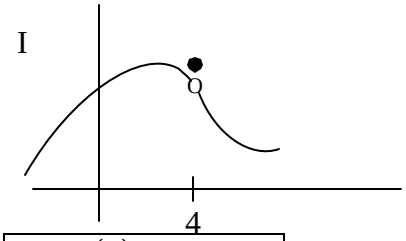
78. The radius of a circle is increasing a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is $20p$?

$C=2p\ r=20p$ $\qquad A=p\ r^2$

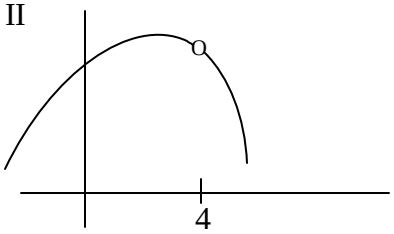
$r=10$ $\qquad \dfrac{dA}{dt}=2p\ r\dfrac{dr}{dt}=2p\ (10)(0.2)=\mathbf{4}p$

79. For which of the following does $\lim_{x\to 4} f(x)$ exist?

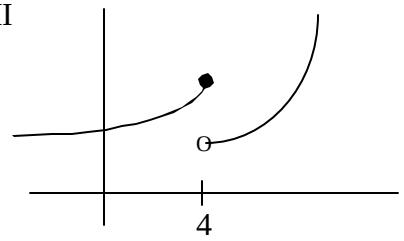
I



II



III



I. $\lim_{x\to 4} f(x)=3$

II. $\lim_{x\to 4} f(x)=4$

III. $\lim_{x\to 4} f(x)=\text{DNE}\ \therefore$

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80. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$.
If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A)There exists c, where $-2 < c < 1$, such that $f(c) = 0$. (true)

(B)There exists c, where $-2 < c < 1$, such that $f'(c) = 0$. (false)

(C)There exists c, where $-2 < c < 1$, such that $f(c) = 3$. (True)

(D)There exists c, where $-2 < c < 1$, such that $f'(c) = 3$. (True)

(E)There exists c, where $-2 \le x \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$. (True, Mean Value Theorem)

Slope = $\frac{4-(-5)}{1-(-2)} = \frac{9}{3} = 3$ (Mean Value Theorem)

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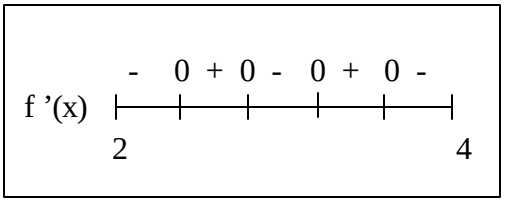
81. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval 2<x<4?



Graph f '(x) on calculator. ∴ f has 4 relative extrema

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82. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le x \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

Graph r(t). r(t) < 0 when 1.572 < t < 3.514 Therefore change in altitude = $\int_{1.572}^{3.514} r(t)dt$

---

83. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time t = 0 to time t = 3?

$\frac{1}{3}\int_0^3 e^t + te^t \approx \mathbf{20.086}$

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84. A pizza, heated to a temperature of 350 degrees Fahrenheit ($^\circ F$), is taken out of an oven and placed in a $75\,^\circ F$ room at time t = 0 minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time t = 5 minutes?

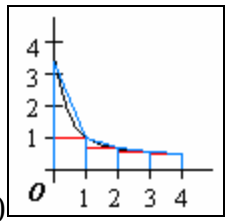
$$\frac{dp}{dt} = -110e^{-0.4T}$$

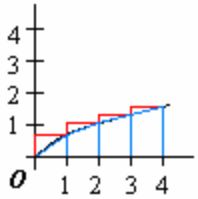
$$dp = -110e^{-0.4T}dt$$

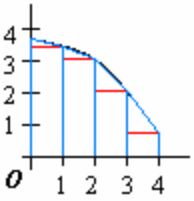
$$p = \int_0^5 -110e^{-0.4T}dt = -237.783$$

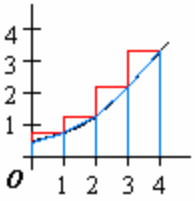
$$\therefore 350 - 237.783 \approx \mathbf{112}^\circ$$

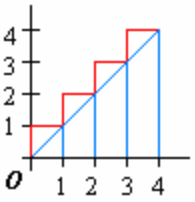
85. If a trapezoidal sum over-approximates $\int_0^4 f(x)\,dx$, and a right Riemann sum under-approximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

(A) 

(B) 

(C) 

(D) 

(E) 

86. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line y = 3, and the vertical line x = 1. For this solid, each cross section perpendicular to x-axis is a square. What is the volume of the solid?

$$\int_0^1 (\tan^{-1}x - 3)^2 = \mathbf{6.612}$$

87. The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

Graph f ''(x) by using Math 8, $\left(\dfrac{\sqrt{x}}{1+x+x^3}\right)$, x, x. Calculate zeros. Zero is at 0.473

Since inflection point is change of concavity, f ''(x) changes from positive to negative at x = **0.473**

88. On the closed interval [2,4], which of the following could be the graph of a function f with the property that $\dfrac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

This is the Average "y" Value Theorem. Which graph has an average "y" value of 1?

C is the correct answer because the area under the curve from 2 to 4 is:

$$1 \cdot 2 \cdot \frac{1}{2} + 1 \cdot 2 \cdot \frac{1}{2} = 2 = \int_2^4 f(t)dt \text{ and } \frac{1}{4-2} \times 2 = 1$$

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89. Let f be a differentiable function with $f(2)=3$ and $f'(2)=-5$, and let g be the function defined by $g(x)=xf(x)$. Which of the following is an equation of the line tangent to the graph of g at point where x = 2?

x	g(x)
2	2f(2)=6

$$g'(x)=f(x)+xf'(x)$$
$$g'(2)=f(2)+2f'(2)$$
$$=3+2(-5)=-7$$
$$\therefore \mathbf{y-6=-7(x-2)}$$

90. For all x in closed interval [2, 5], the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f ?

$f'(x)>0$ for only A and B. Now look t f ‘’(x) to determine if answer is A or B.

(A)

x	f(x)	f ’(x)	f ”(x)
2	7		
3	9	2	
4	12	3	1
5	16	4	1

(B)

x	f(x)	f ’(x)	f ”(x)
2	7		
3	11	4	
4	14	3	- 1
5	16	2	- 1

$\therefore$ B

91. A particle moves along the x-axis so that at any time t > 0, its acceleration is given by $a(t)=\ln(1+2^t)$. If the velocity of the particle is 2 at time t = 1, then velocity of the particle at time t = 2 is

$$v(t)=\int a(t)=\int \ln(1+2^t)$$
$$v(2)=v_1+\int_1^2 \ln(1+2^t)=2+1.346=3.346 \therefore \text{E}$$

92. Let g be the function given by $g(x)=\int_0^x \sin(t^2)dt$ for $-1\le x\le 3$. On which of the following intervals is g decreasing?

$$g'(x)=\sin(x^2)$$