AP CALCULUS SPRING BREAK EXAM

Part #1: NO CALCULATOR, 28 QUESTIONS, 55 MINUTES

Date: _____ Time Began: _____ Time Ended: _____

1. If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$

(B) $2(x^3 + 1)$

(C) $2(3x^2 + 1)$

(D) $3x^2(x^3 + 1)$

(E) $6x^2(x^3 + 1)$

2. $\int_0^1 e^{-4x}\,dx =$

(A) $\frac{-e^{-4}}{4}$

(B) $-e^{-4}$

(C) $e^{-4} - 1$

(D) $\frac{1}{4} - \frac{e^{-4}}{4}$

(E) $4 - 4e^{-4}$

3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f.
Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x \to 2} f(x) = \infty$

(E) $\lim_{x \to \infty} f(x) = 2$

4. If $y = \frac{2x+3}{3x+2}$, then $\frac{dy}{dx} =$

(A) $\frac{12x+13}{(3x+2)^2}$

(B) $\frac{12x-13}{(3x+2)^2}$

(C) $\frac{5}{(3x+2)^2}$

(D) $\frac{-5}{(3x+2)^2}$

(E) $\frac{2}{3}$

5 $\int_0^{\frac{p}{4}} \sin x \, dx =$

(A) $\frac{-\sqrt{2}}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{-\sqrt{2}}{2} - 1$

(D) $\frac{-\sqrt{2}}{2} + 1$

(E) $\frac{\sqrt{2}}{2} - 1$

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6. $\lim_{x \to \infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

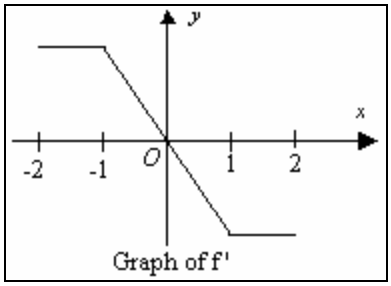
(A) 4

(B) 1

(C) $\frac{1}{4}$

(D) 0

(E) - 1



7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

(A) f is decreasing for $-1 \le x \le 1$

(B) f is increasing for $-2 \le x \le 0$

(C) f is increasing for $1 \le x \le 2$

(D) f has a local minimum at x = 0

(E) f is not differentiable at x = - 1 and x = 1.

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8. $\int x^2 \cos(x^3)\,dx =$

(A) $-\frac{1}{3}\sin(x^3)+C$

(B) $\frac{1}{3}\sin(x^3)+C$

(C) $-\frac{x^3}{3}\sin(x^3)+C$

(D) $\frac{x^3}{3}\sin(x^3)+C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

9. If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0) =$

(A) $-\frac{2}{5}$

(B) $\frac{1}{5}$

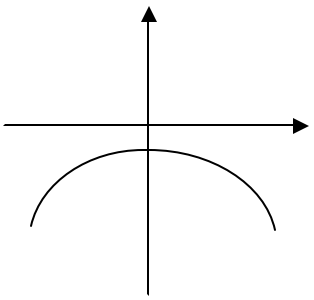
(C) $\frac{1}{4}$

(D) $\frac{2}{5}$

(E) Nonexistent

10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x.
Which of the following could be the graph of f ?

(A)



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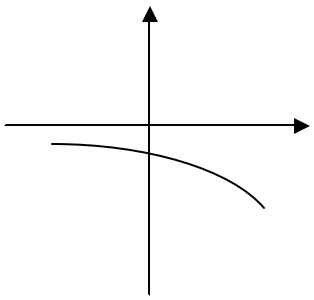

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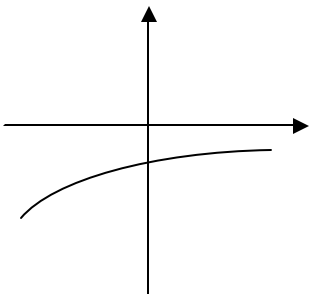

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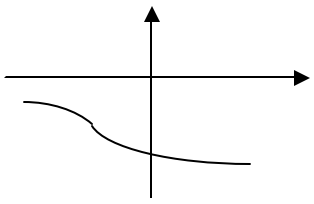

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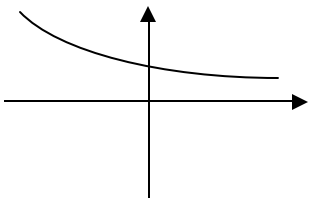

11. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{u}\,du$

(B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$

(C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$

(D) $\int_0^2 \sqrt{u}\,du$

(E) $\int_1^5 \sqrt{u}\,du$

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12. The rate of change of the volume, *V*, of water in a tank with respect to time, *t*, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

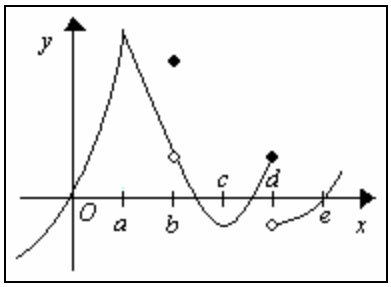
(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$



13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

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14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

(A) $2x\cos 2x$

(B) $4x\cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x\cos 2x)$

(E) $2x(\sin 2x + x\cos 2x)$

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15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only
(B) $(-\infty, 0)$
(C) $[-1, 0)$ only
(D) $(0, \sqrt[3]{2}]$
(E) $[\sqrt[3]{2}, \infty)$

16. If the line tangent to the graph of the function f at point $(1,7)$ passes through the point $(-2,-2)$, then $f'(1) =$

(A) - 5
(B) 1
(C) 3
(D) 7
(E) Undefined

17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$
(B) $x > -2$
(C) $x < -1$
(D) $x > -1$
(E) $x < 0$

x	- 4	- 3	- 2	- 1	0	1	2	3	4
g'(x)	2	3	0	- 3	- 2	- 1	0	3	2

18. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

(A) $-2 \le x \le 2$ only
(B) $-1 \le x \le 1$ only
(C) $x \ge -2$
(D) $x \ge 2$ only
(E) $x \le -2$ or $x \ge 2$

19. A curve has slope $2x + 3$ at each point (x,y) on the curve. Which of the following is an equation for this curve if it passes through the point (1,2)?

(A) $y = 5x - 3$
(B) $y = x^2 + 1$
(C) $y = x^2 + 3x$
(D) $y = x^2 + 3x - 2$
(E) $y = 2x^2 + 3x - 3$

$$f(x)=\begin{cases} x+2 \ , \ x\le 3 \\ 4x-7 \ , \ x>3 \end{cases}$$

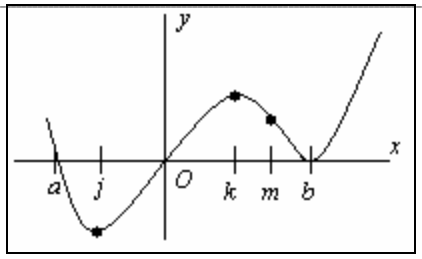
20. Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x\to 3} f(x)$ exists.

II. f is continuous at x = 3

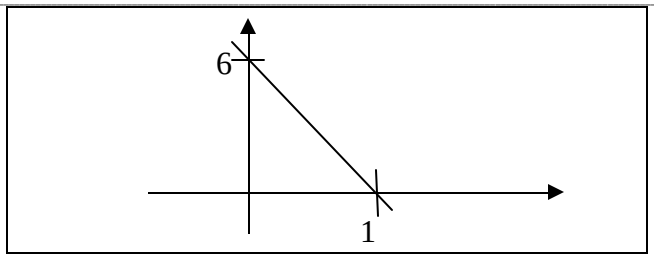
III. f is differentiable at x = 3.

(A) none
(B) I only
(C) II only
(D) I and II only
(E) I, II and III



21. The second derivative of the function f is given by $f''(x)=x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only
(B) 0 and m only
(C) b and j only
(D) 0, a, and b
(E) b, j, and k



22. The graph of f ', the derivative of f , is the line shown in the figure above. If $f(0)=5,$ then $f(1)=$

(A) 0
(B) 3
(C) 6
(D) 8
(E) 11

23 $\dfrac{d}{dx}\left(\int_0^{x^2}\sin(t^3)dt\right)=$

(A) $-\cos(x^6)$

(B) $\sin(x^3)$

(C) $\sin(x^6)$

(D) $2x\sin(x^3)$

(E) $2x\sin(x^6)$

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where x = - 1?

(A) y = 7x - 3
(B) y = 7x + 7
(C) y = 7x + 11
(D) y = - 5x - 1
(E) y = - 5x - 5

25. A particle moves along the x- axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only
(B) $t = 3$ only
(C) $t = \frac{7}{2}$ only
(D) $t = 3$ and $t = \frac{7}{2}$
(E) $t = 3$ and $t = 4$

26. What is the slope of the tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point (3, 2)?

(A) 0
(B) $\frac{4}{9}$
(C) $\frac{7}{9}$
(D) $\frac{6}{7}$
(E) $\frac{5}{3}$

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$
(B) $\frac{1}{4}$
(C) $\frac{7}{4}$
(D) 4
(E) 13

28. Let g be a twice- differentiable function with g'(x) > 0 and g''(x) > 0 for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

(A) 15

(B) 18

(C) 21

(D) 24

(E) 27

(End of Part A – Remember to mark TIME ENDED for Part #1)

You may now begin Part B. You MAY now use a CALCULATOR.

PLEASE MARK TIME BEGAN FOR PART #2.

AP CALCULUS SPRING BREAK EXAM

Part #2: CALCULATOR, 17 QUESTIONS, 50 MINUTES

Date: _____ Time Began: _____ Time Ended: _____

76. A particle moves along the x- axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time t = 4?

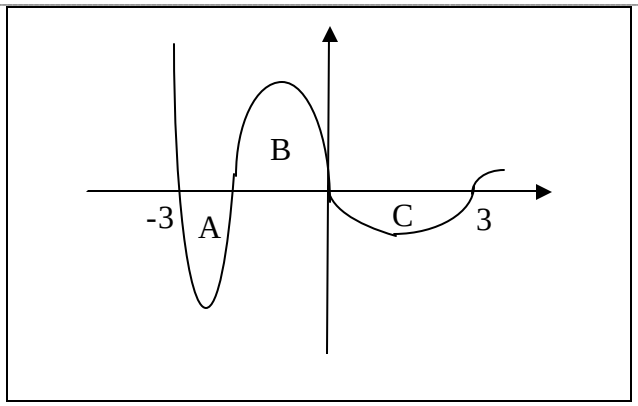
(A) - 2.016

(B) - 0.677

(C) 1.633

(D) 1.814

(E) 2.978



77. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x- axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x)+1)dx$?

(A) - 2

(B) - 1

(C) 4

(D) 7

(E) 12

78. The radius of a circle is increasing a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is $20p$?

(A) $0.04p$

(B) $0.4p$

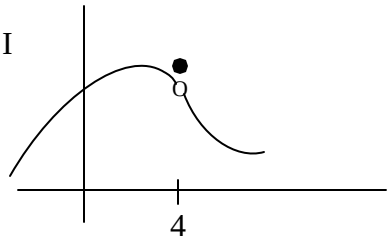
(C) $4p$

(D) $20p$

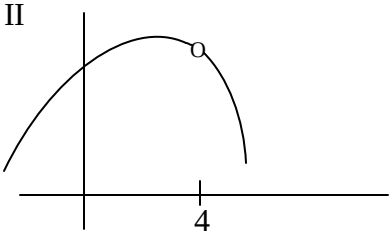
(E) $100p$

79. For which of the following does $\lim_{x\to 4} f(x)$ exist?

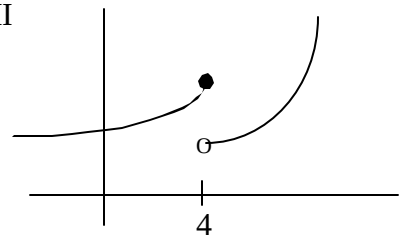
I



II



III



(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

80. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A)There exists c, where $-2 < c < 1$, such that $f(c) = 0$.

(B)There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.

(C)There exists c, where $-2 < c < 1$, such that $f(c) = 3$.

(D)There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.

(E)There exists c, where $-2 \le x \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

81. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One

(B) Two

(C) Three

(D) Four

(E) Five

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82. The rate of change of the altitude of a hot- air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le x \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)dt$

(B) $\int_{0}^{8} r(t)dt$

(C) $\int_{0}^{2.667} r(t)dt$

(D) $\int_{1.572}^{3.514} r'(t)dt$

(E) $\int_{0}^{2.667} r'(t)dt$

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83. The velocity, in ft/sec, of a particle moving along the x- axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 $\frac{\text{ft}}{\text{sec}}$

(B) 26.447 $\frac{\text{ft}}{\text{sec}}$

(C) 32.809 $\frac{\text{ft}}{\text{sec}}$

(D) 40.671 $\frac{\text{ft}}{\text{sec}}$

(E) 79.342 $\frac{\text{ft}}{\text{sec}}$

84. A pizza, heated to a temperature of 350 degrees Fahrenheit ($^\circ F$), is taken out of an oven and placed in a $75\,^\circ F$ room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) $112\,^\circ F$

(B) $119\,^\circ F$

(C) $147\,^\circ F$

(D) $238\,^\circ F$

(E) $335\,^\circ F$

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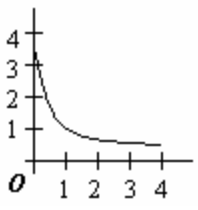
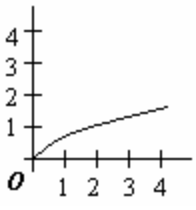
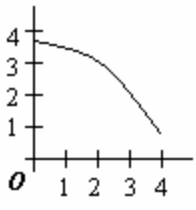
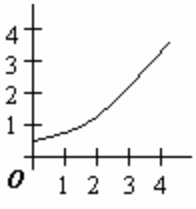
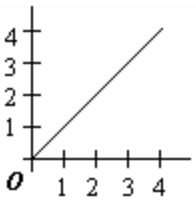
85. If a trapezoidal sum over- approximates $\int_0^4 f(x)\,dx$, and a right Riemann sum under- approximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of y = f(x) ?

(A) 

(B) 

(C) 

(D) 

(E) 

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86. The base of a solid is the region in the first quadrant bounded by the y- axis, the graph of $y = \tan^{-1} x$, the horizontal line y = 3, and the vertical line x = 1. For this solid, each cross section perpendicular to x- axis is a square. What is the volume of the solid?

(A) 2.561

(B) 6.612

(C) 8.046

(D) 8.755

(E) 20.773

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87. The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$. What is the x- coordinate of the inflection point of the graph of f ?

(A) 1.008

(B) 0.473

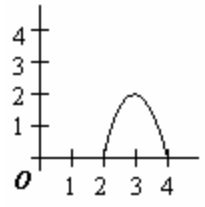
(C) 0

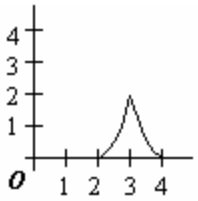
(D) - 0.278

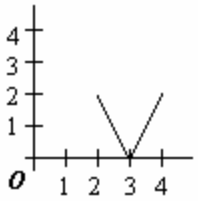
(E) The graph of f has no inflection point.

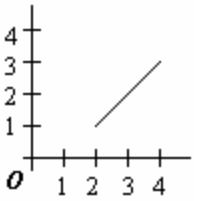
AP CALCULUS SPRING BREAK EXAM

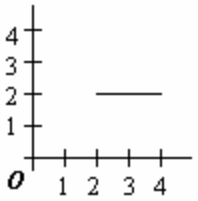
88. On the closed interval [2,4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

(A) 

(B) 

(C) 

(D) 

(E) 

89. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

(C) $y - 6 = -5(x - 2)$

(D) $y - 6 = -7(x - 2)$

(E) $y - 6 = -10(x - 2)$

90. For all x in closed interval [2, 5], the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

(A)

x	f(x)
2	7
3	9
4	12
5	16

(B)

x	f(x)
2	7
3	11
4	14
5	16

(C)

x	f(x)
2	16
3	12
4	9
5	7

(D)

x	f(x)
2	16
3	14
4	11
5	7

(E)

x	f(x)
2	16
3	13
4	10
5	7

91. A particle moves along the x- axis so that at any time t > 0, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time t = 1, then velocity of the particle at time t = 2 is

(A) 0.462

(B) 1.609

(C) 2.555

(D) 2.886

(E) 3.346

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92. Let g be the function given by $g(x) = \int_0^x \sin(t^2)dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(A) $-1 \le x \le 0$

(B) $0 \le x \le 1.772$

(C) $1.253 \le x \le 2.171$

(D) $1.772 \le x \le 2.507$

(E) $2.802 \le x \le 3$

END OF PART B
Mark Time Completed for Part B

Please proceed to Free Response Questions

For Questions 1- 3, A Calculator IS Allowed!!
You have 45 minutes for Questions 1- 3

STOP AFTER 45 MINUTES!!
(AND PUT AWAY YOUR CALCULATOR)

For Questions 4- 6, NO Calculator is Allowed!!
You have 45 minutes for Questions 4- 6

YOU MAY GO BACK TO 1- 3,
BUT YOU MAY NOT USE YOUR CALCULATOR