"Calculus AB FIVES WORKSHOP PROBLEMS"

1. $f(x) = 5x^{4/3}$ $f'(8)$

$f'(x) = \frac{20}{3}x^{1/3}$

$f'(8) = \frac{20}{3}\sqrt[3]{8} = \boxed{\frac{40}{3}}$

2. $\lim_{x\to\infty} \frac{5x^2-3x+1}{4x^2+2x+5} = \boxed{\frac{5}{4}}$

3. $f(x) = \frac{3x^2+x}{3x^2-x}$

$f'(x) = \frac{(3x^2-x)[6x+1]-(3x^2+x)[6x-1]}{(3x^2-x)^2}$

$f'(x) = \frac{18x^3-6x^2+3x^2-x-18x^3-6x^2+3x^2+}{(3x^2-x)^2}$

$= -12x^2+6x^2 = \frac{-6x^2}{(3x^2-x)^2}$

$= \frac{-6x^2}{x^2(3x-1)^2} = \boxed{\frac{-6}{(3x-1)^2}}$

4. $f(x) = \frac{x^2-7x+12}{x-4} = \frac{(x-4)(x-3)}{x-4} = x-3$

$f(4) = 4-3 = \boxed{1}$ then $f(x)$ is cont' at $x=4$.

5. $x^2-2xy+3y^2=8$

$2x-\left[2x\frac{dy}{dx}+y(2)\right]+6y\frac{dy}{dx}=0$

$\frac{dy}{dx}(-2x+6y) = 2y-2x$

$\frac{dy}{dx} = \boxed{\frac{y-x}{-x+3y}}$

6. $\int_a^b (\text{top}-\text{bottom})dx$

a, b are ~~[illegible]~~

1 and 2 respectively

$\int_1^2 [5-(1+x^2)]dx = \boxed{\int_1^2 (4-x^2)dx}$

7. $f(x) = \sec x + \csc x$

$f'(x) = \boxed{\sec x \tan x + (-\csc x \cot x)}$

8. $y = \sqrt{(3x^2+2x)}$ @ $(2,4)$ normal line

$y' = \frac{6x+2}{2\sqrt{3x^2+2x}} \Rightarrow y'(2) = \frac{3(2)+1}{\sqrt{12+4}} = \frac{7}{4}$

$y = -\frac{4}{7}x+b \Rightarrow 4 = -\frac{4}{7}(2)+b \Rightarrow 4+1\frac{1}{7} = b$

$y = -\frac{4}{7}x+\frac{36}{7} \Rightarrow \boxed{4x+7y=36}$

9. $\int_{-1}^{1} \frac{4}{1+x^2}dx = 4\tan^{-1}x \Big|_{-1}^{1}$

$4\tan^{-1}(1) - 4\tan^{-1}(-1) = 4\left(\frac{\pi}{4}\right) - 4\left(\frac{-\pi}{4}\right)$

$= \pi + \pi = \boxed{2\pi}$

11. $f(x) = \frac{5}{x^2+1}$ $g(x) = 3x$ $g(f(2)) = g(1) = \boxed{3}$

10. $f(x) = \cos^2 x$ $f''(\pi)$?

$f'(x) = -2\cos x \sin x$

$f''(x) = -2\cos x(\cos x) + \sin x(2\sin x)$

$f''(\pi) = -2[\cos(\pi)]^2 + 2[\sin(\pi)]^2$

$= -2(-1)^2 + 2(0) = \boxed{-2}$

12. $\int x\sqrt{5x^2-4}\,dx \Rightarrow \frac{1}{10}\int u^{1/2}du$

$u = 5x^2-4$

$du = 10x\,dx$

$\frac{1}{10}du = x\,dx$

$= \frac{1}{10}\cdot\frac{2}{3}u^{3/2}+c$

$= \boxed{\frac{1}{15}(5x^2-4)^{3/2}+c}$

13. $3x^2+5\ln y = 12$ slope of line @ $(2,1)$

$6x + 5\cdot\frac{1}{y}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-6x}{5y}$

@ $(2,1) = \boxed{\frac{-12}{5}}$

14. $y = 2-3\sin\frac{\pi}{4}(x-1)$ period $\frac{2\pi}{\frac{\pi}{4}} = \boxed{8}$

15. $f(x) = \begin{cases} x^2+5 & x<2 \\ 7x-5 & x\geq 2 \end{cases}$

cont' if $x^2+5 = 7x-5$ when $x=$:

$4+5 = 14-5$

$9=9$

$\therefore$ cont' $\forall x$.

$f'(x)$ exists if $2x=7$ @ $x=2$

$4 \neq 7$

$\therefore f'(x)$ doesn't exist $\forall x$

@ $x=2$ $f'(x) = 7$ $\therefore$ no min. @ $x=2$

so, only $\boxed{\text{I is true}}$

16. $f(x) = x^3 - 9x^2 - 120x + 6$ local min?

$f'(x) = 3x^2 - 18x - 120 = 0$

$x^2 - 6x - 40 = 0$

$(x-10)(x+4) = 0$

$x = 10, -4$

$+ \quad - \quad +$; -4 max ; 10 min

so, min at $\boxed{x = 10}$

17. $a(t) = 4t - 12 \quad v(0) = 10 \quad x(0) = 4$

when chging direction?

$v(t) = \int (4t-12)\,dt = 2t^2 - 12t + c \qquad 10 = 2(0) - 12(0) + c$

$10 = c$

$v(t) = 2t^2 - 12t + 10 = 0$

$t^2 - 6t + 5 = 0$

$(t-5)(t-1) = 0$

$t = 1, 5$

$+ \quad - \quad +$; 1 ; 5

$\boxed{\text{Changes @ } t = 1, 5}$

18. average value: $\frac{1}{5-1}\int_1^5 (x-1)^2\,dx$

[redacted] $\frac{1}{4}\left[\frac{1}{3}(x-1)^3\right]\Big|_1^5 = \frac{1}{12}(4)^3 - \frac{1}{12}(0)$

$= \boxed{\frac{16}{3}}$

19. $\int \left(e^{3\ln x} + e^{3x}\right) dx = \int \left(x^3 + e^{3x}\right) dx$

$\boxed{\text{Note: } e^{3\ln x} = e^{\ln x^3} = x^3}$

$= \boxed{\frac{1}{4}x^4 + \frac{1}{3}e^{3x} + C}$

20. $f(x) = \left(\sqrt{x^3 + 5x + 121}\right)(x^2 + x + 11) \quad f'(0) = ?$

$f'(x) = \left(\sqrt{x^3 + 5x + 121}\right)[2x+1] + (x^2 + x + 11)\left[\frac{3x^2 + 5}{2\sqrt{x^3 + 5x + 121}}\right]$

$f'(0) = 11(1) + (11)\left(\frac{5}{22}\right) = 11 + \frac{5}{2} = 13\tfrac{1}{2} = \boxed{\frac{27}{2}}$

21. $f(x) = 5^{3x}$

$\ln y = 3x \ln 5$

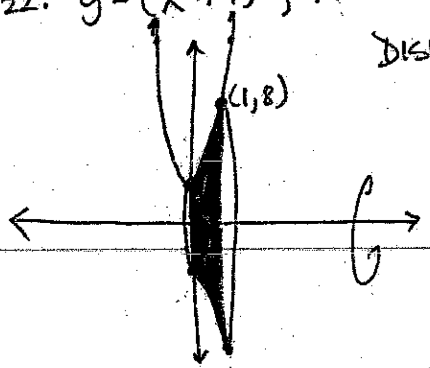
$\frac{1}{y}\frac{dy}{dx} = 3\ln 5$

$\frac{dy}{dx} = 3\ln 5 \cdot 5^{3x}$

$= (\ln 5^3) \cdot 5^{3x}$

$= \boxed{(\ln 125) 5^{3x}}$

22. $y = (x^2+1)^3, \ x = 1$ around x-axis



Disk Method $\pi \int_0^1 R^2\,dx$

$\pi \int_0^1 \left[(x^2+1)^3\right]^2 dx$

$\boxed{\pi \int_0^1 (x^2+1)^6\,dx}$

24. $\frac{dy}{dx} = \frac{3x^2 + 2}{y} \quad y = 4 \ x = 2$

find y when $x = 3$

$\int y\,dy = \int (3x^2 + 2)\,dx$

$\frac{1}{2}y^2 = x^3 + 2x + C$

$8 = 8 + 4 + C$

$-4 = C$

$\frac{1}{2}y^2 = x^3 + 2x - 4$

$\frac{1}{2}y^2 = 27 + 6 - 4$

$y^2 = 58 \Rightarrow y = \boxed{\sqrt{58}}$

23. $\lim_{x \to 0} 4\,\frac{\sin x \cos x - \sin x}{x^2} = \frac{0}{0}$ L'Hopital's

$\lim_{x \to 0} 4\,\frac{\sin x(\sin x) + \cos x(\cos x) - \cos x}{2x} = \frac{0}{0}$ L'Hopital's

$\lim_{x \to 0} 4\left(\frac{2\sin x \cos x + 2\cos x(-\sin x) + \sin x}{2}\right) = \frac{4}{2}(0) = \boxed{0}$

25. $\int \frac{dx}{9+x^2} =$ [redacted] $\boxed{\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C}$

27. $\int x\sqrt{x+3}\,dx$ Sometimes, it's easier to take the choices and work backwards to get the question.

A) Clearly not possible no $(x+3)$.

B) clearly not possible, the derivative will have no "x"

D) Almost same as B — crazy w/ same reasoning

E) maybe—try it: $\frac{4}{3}x^2(x+3)^{3/2}$

$\frac{4}{3}x^2\left[\frac{3}{2}(x+3)^{1/2}\right] + (x+3)^{3/2}\left[\frac{8}{3}x\right]$

$2x^2(x+3)^{1/2} + 8/3\,x(x+3)^{3/2}$ No

Must be "C" — Check.

$\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2}$

$(x+3)^{3/2} - 3(x+3)^{1/2}$

$(x+3)^{1/2}(x+3-3) = x(x+3)^{1/2}$ yes. $\boxed{C}$

26. $f(x) = \cos^3(x+1) \quad f'(\pi)$

$f'(x) = 3\cos^2(x+1) \cdot -\sin(x+1)$

$f'(\pi) = 3[\cos(\pi+1)]^2 (-\sin(\pi+1))$

$= \boxed{-3\cos^2(\pi+1)\sin(\pi+1)}$

28. $f(x) = \ln(\ln(1-x))$

$f'(x) = \frac{1}{\ln(1-x)} \cdot \frac{1}{1-x} \cdot -1 = \boxed{\frac{-1}{(1-x)\ln(1-x)}}$

Saturday Review Session "Calculus AB Fives Workshop Problems"

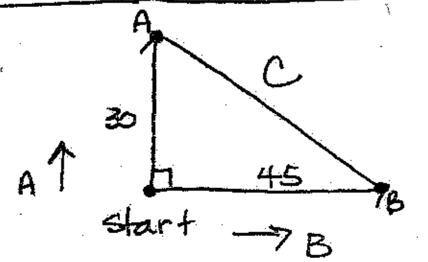
Section 1, Part B (The Calculator Section)

29. $\int_0^{\frac{\pi}{4}} \sin x\,dx + \int_{-\frac{\pi}{4}}^{0} \cos x\,dx =$

An easy question w/ or w/o a calculator. But, it's in the Calculator section so USE IT!

$\boxed{\text{"math" "9"}}$ fnInt(sinx, x, 0, (π/4)) + fnInt(cos x, x, $\left(\frac{-\pi}{4}\right)$, 0) = $\boxed{1}$

30.



$\frac{dA}{dt} = 12$ km/hr $\quad \frac{dB}{dt} = 18$ km/hr

2.5 hrs : A has traveled 30 kms
B " " 45 kms

when A = 30, B = 45 $\quad C = \sqrt{900 + 2025}$
$= \sqrt{2925}$

C = distance between A and B.

$A^2 + B^2 = C^2$

$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dC}{dt} \Rightarrow 30(12) + 45(18) = \sqrt{2925}\,\frac{dC}{dt}$

$\boxed{21.63 = \frac{dC}{dt}}$

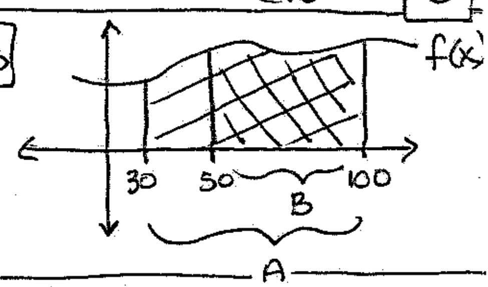
31. $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{6} + h\right) - \tan\left(\frac{\pi}{6}\right)}{h}$ Newton's Method

$f(x) = \tan x \quad x = \frac{\pi}{6} \quad f'(x) = \sec^2 x \quad f'\left(\frac{\pi}{6}\right) = \left[\sec\left(\frac{\pi}{6}\right)\right]^2 = \left[\frac{1}{\cos\left(\frac{\pi}{6}\right)}\right]^2$

$= \left(\frac{2}{\sqrt{3}}\right)^2 = \boxed{\frac{4}{3}}$

32. $\int_{30}^{100} f(x)dx = A \quad \int_{50}^{100} f(x)dx = B \quad \int_{30}^{50} f(x)\,dx = \boxed{A - B}$

(Not a calculator question)



33. Good Stuff! $f(x) = 3x^2 - x \quad g(x) = f^{-1}(x) \quad g'(10) = ?$

Rule:
if $f(a) = b$ then $f^{-1}(b) = a$
if $f'(a) = c$ then $(f^{-1})'(b) = \frac{1}{c}$

$f(x) = 10$ when $10 = 3x^2 - x \quad 0 = 3x^2 - x - 10$
$(3x+5)(x-2)$
$x = -5/3, 2$

$f(2) = 10 \quad \therefore g(10) = 2$

$f'(2) = 6(2) - 1 = 11$

$\therefore g'(10) = \boxed{\frac{1}{11}}$

33. The long way.

$f(x) = 3x^2 - x$

$y = 3x^2 - x$

$x = 3y^2 - y$

$x = \underbrace{3\left(y^2 - \frac{1}{3}y + \frac{1}{36}\right) - \frac{1}{12}}_{\text{complete the square!}}$

$x = 3\left(y - \frac{1}{6}\right)^2 - \frac{1}{12}$

$\frac{x + \frac{1}{12}}{3} = \left(y - \frac{1}{6}\right)^2$

$\frac{1}{6} + \sqrt{\frac{12x+1}{36}} = f^{-1}(x)$

$f^{-1}(x) = \frac{1 + \sqrt{12x+1}}{6} = g(x)$

$g'(x) = \frac{1}{6}\left(\frac{12}{2\sqrt{12x+1}}\right) = \frac{1}{\sqrt{12x+1}}$

$g'(10) = \frac{1}{\sqrt{120+1}} = \boxed{\frac{1}{11}}$

34. $y = x^3 - 5x^2 + 4x + 2$ local min?

$y' = 3x^2 - 10x + 4 = 0$ Calculator!

Graph y', use "calc" to find zeros.

.4648, 2.869

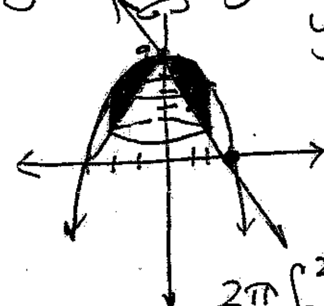
y' + − +

.465 max 2.87 min

$\boxed{(2.87, -4.06)}$

(you can tell from the graph of y' where to put the "+" and "−")

35. $y = 9 - x^2$ $y = 9 - 3x$ $0 \le x \le 2$ y-axis



you can find volume using washer method $\boxed{\pi\int(R^2 - r^2)\,dy}$ or Shell method $\boxed{2\pi\int rh\,dx}$

$2\pi\int_0^2 x\left(9 - x^2 - (9 - 3x)\right)dx$

$2\pi\int_0^2 x(3x - x^2)\,dx = \boxed{8\pi}$

36. average value: $\frac{1}{4-2}\int_2^4 \ln^2 x\,dx = \boxed{1.204}$

$\boxed{\text{"math" "9"}}$ $\frac{\text{fnInt}((\ln(x))^2, x, 2, 4)}{2}$

$(\ln(x))^2 \ne \ln(x^2)$!!

37. $\frac{d}{dx}\int_0^{3x} \cos(t)\,dt$

Fund. Thm. of Cal.: $\boxed{3\cos(3x)}$ ✱Not a Calc Ques.

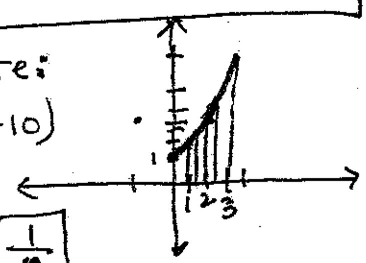
38. A Party! $\int_1^3 (x^2+1)\,dx = 10^2/_3$

x	$f(x)$
1	2
1.5	3.25
2	5
2.5	7.25
3	10

using trapezoids to approximate:

$\frac{1}{2}\left(\frac{1}{2}\right)\left(2 + 2(3.25) + 2(5) + 2(7.25) + 10\right)$

$= 10^3/_4$



amount of error $10^3/_4 - 10^2/_3 = \boxed{\frac{1}{12}}$

39. $y = ne^{kt}$

$10 = 4e^{2k}$

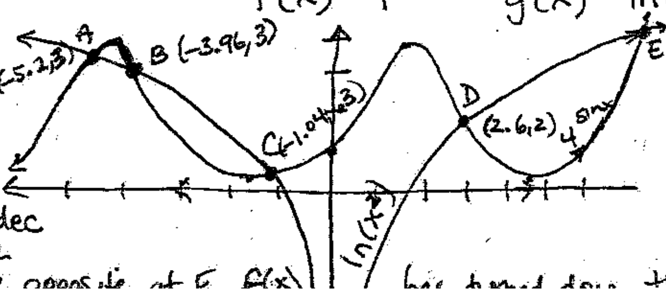
$\ln 2.5 = 2k$

$\frac{\ln 2.5}{2} = k$

... $\boxed{15.81}$

40. $f(x) = 4^{\sin x}$ $g(x) = \ln(x^2)$ Graph them!

@t which intersec. pts. will the slopes of the tangents be the same?



at pt A, f(x) is inc. while g(x) is dec.
at B, both are dec.
at C, g(x) is dec and f(x) is inc.
at D they are opposite, at E f(x) ...

41. $\int \ln(2x)\,dx \Rightarrow$ Integration by Parts

$u=\ln(2x)$ $\quad v=x$ $\qquad uv-\int v\,du$

$du=\frac{1}{x}dx$ $\quad dv=dx$

$x\ln(2x)-\int dx = \boxed{x\ln(2x)-x+c}$

42. $f(x)=\begin{cases} ax^3-6x & x\leq 1 \\ bx^2+4 & x>1 \end{cases}$

if cont' $\forall x$ then

$a(1)^3-6(1)=b(1)+4$

$a-6=b+4$

$f'(x)=\begin{cases} 3ax^2-6 & x\leq 1 \\ 2bx & x>1 \end{cases}$

if $f'(x)$ exists $\forall x$ then

$3ax^2-6=2bx$ @ $x=1$

$3a-6=2b$ $\quad$ Solve for a, b.

$a-6=b+4$

$3a-6=2b$

$-2a+12=-2b-8$

$a+6=-8$

$\boxed{a=-14}$

43. $y_1=\cos 2t$ $\quad y_2=4\sin t$

$y_1'=-2\sin 2t$ $\quad y_2'=4\cos t$

$y_1''=-4\cos 2t$ $\quad y_2''=-4\sin t$

$-4\cos 2t=-4\sin t$ $\quad 0<t<6$

$\cos 2t=\sin t$

$\cos 2t-\sin t=0$

for $t \ni 0<t<1$, $2<t<3$, $4<t<5$
$(.524)$ $\quad (2.62)$ $\quad (4.7)$

use "calc" to find the zeros between $t=0$ and $t=6$

there are $\boxed{3}$

44. distance traveled $0<t<4$ if $v(t)=7e^{-t^2}$

$=\int_0^4 |7e^{-t^2}|\,dt = \boxed{6.204}$

(absolute value not necessary in this case because $7e^{-t^2}>0$ $\forall t$)

45. $\int \tan^6 x\sec^2 x\,dx$

$u=\tan x$

$du=\sec^2 x\,dx$

$\int u^6\,du=\frac{1}{7}u^7+c=\boxed{\frac{1}{7}\tan^7 x+c}$