CALCULUS AB FIVES WORKSHOP PROBLEMS

SECTION I, PART A

Part A consists of 28 questions. Calculators are **not** permitted for this part of the exam. Time given for Part A — **55 minutes.**

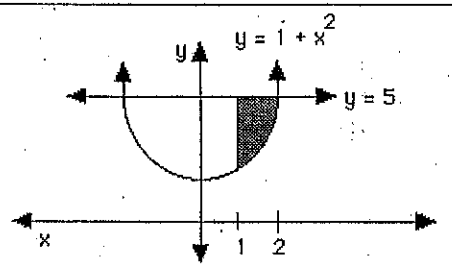
1. If $f(x) = 5x^{4/3}$, then $f'(8) =$ A) 10 •B) $\frac{40}{3}$ C) 40 D) 80 E) $\frac{160}{3}$

2. $\lim_{x\to\infty} \frac{5x^2-3x+1}{4x^2+2x+5}$ is A) 0 B) $\frac{4}{5}$ C) $\frac{3}{11}$ •D) $\frac{5}{4}$

3. If $f(x) = \frac{3x^2+x}{3x^2-x}$ then $f'(x)$ is
A) 1 B) $\frac{6x^2+1}{6x^2-1}$ •C) $\frac{-6}{(3x-1)^2}$ D) $\frac{-2x^2}{(x^2-x)^2}$ E) $\frac{36x^3-2x}{(x^2-x)^2}$

4. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-7x+12}{x-4}$ when $x \neq 4$, then $f(4) =$ •A) 1 B) $\frac{8}{7}$ C) –1 D) 0 E) undefined

5. If $x^2-2xy+3y^2 = 8$, then $\frac{dy}{dx} =$
A) $\frac{8+2y-2x}{6y-2x}$ B) $\frac{3y-x}{y-x}$ C) $\frac{2x-2y}{6y-2x}$ D) $\frac{1}{3}$ •E) $\frac{y-x}{3y-x}$



6. Which of the following integrals correctly corresponds to the area of the shaded region in the figure above?
A) $\int_1^2 (x^2-4)\,dx$ •B) $\int_1^2 (4-x^2)\,dx$ C) $\int_1^5 (x^2-4)\,dx$ D) $\int_1^5 (x^2+4)\,dx$ E) $\int_1^5 (4-x^2)\,dx$

7. If $f(x) = \sec x + \csc x$, then $f'(x) =$
A) 0 B) $\sec^2 x + \csc^2 x$ C) $\csc x - \sec x$ D) $\sec x \tan x + \csc x \cot x$
•E) $\sec x \tan x - \csc x \cot x$

8. An equation of the line normal to the graph of $y = \sqrt{(3x^2+2x)}$ at (2,4) is
A) $-4x + y + 20$ B) $4x + 7y = 20$ C) $-7x + 4y = 2$ D) $7x + 4y = 30$ •E) $4x + 7y = 36$

9. $\int_{-1}^{1} \frac{4}{1+x^2}\,dx =$ A) 0 B) π C) 1 •D) 2π E) 2

10. If $f(x) = \cos^2 x$, then $f''(\pi) =$ •A) –2 B) 0 C) 1 D) 2 E) 2π

11. If $f(x) = \frac{5}{x^2+1}$ and $g(x) = 3x$, then $g(f(2)) =$ A) –3 B) $\frac{5}{37}$ •C) 3 D) 5 E) $\frac{37}{5}$

12. $\int x\sqrt{5x^2-4}\,dx =$ A) $\frac{1}{10}(5x^2-4)^{3/2} + C$ •B) $\frac{1}{15}(5x^2-4)^{3/2} + C$
C) $-\frac{1}{5}(5x^2-4)^{-1/2} + C$ D) $\frac{20}{3}(5x^2-4)^{3/2} + C$ E) $\frac{3}{20}(5x^2-4)^{3/2} + C$

13. The slope of the line tangent to the graph of $3x^2 + 5 \ln y = 12$ at (2,1) is
•A) $-\frac{12}{5}$ B) $\frac{12}{5}$ C) $\frac{5}{12}$ D) 12 E) –7

14. The equation $y = 2 - 3\sin\frac{\pi}{4}(x-1)$ has a fundamental period of
A) $\frac{1}{8}$ B) $\frac{\pi}{4}$ C) $\frac{4}{\pi}$ •D) 8 E) 2π

15. If $f(x) = \begin{cases} x^2+5 \text{ if } x < 2 \\ 7x-5 \text{ if } x \geq 2 \end{cases}$, for all real numbers x, which of the following must be true?
I. f(x) is continuous everywhere.
II. f(x) is differentiable everywhere.
III. f(x) has a local minimum at x = 2.
•A) I only B) I and II only C) II and III only D) I and III only E) I, II, and III

16. For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum?
•A) 10 B) 4 C) 3 D) –4 E) –10

17. The acceleration of a particle moving along the x–axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when t = 0 and the position is 4 when t = 0, then the particle is changing direction at
A) t=1 B) t=3 C) t=5 •D) t=1 and t=5 E) t=1 and t=3 and t=5

18. The average value of the function $f(x) = (x-1)^2$ on the interval from x = 1 to x = 5 is
A) $-\frac{16}{3}$ •B) $\frac{16}{3}$ C) $\frac{64}{3}$ D) $\frac{66}{3}$ E) $\frac{256}{3}$

19. $\int (e^{3\ln x} + e^{3x})\,dx =$
A) $3 + \frac{e^{3x}}{3} + C$ B) $\frac{x^4}{4} + 3e^{3x} + C$ C) $\frac{e^{x^4}}{4} + 3e^{3x} + C$ D) $\frac{e^{x^4}}{4} + \frac{e^{3x}}{3} + C$ •E) $\frac{x^4}{4} + \frac{e^{3x}}{3} + C$

20. If $f(x) = \sqrt{(x^3 + 5x + 121)}\,(x^2 + x + 11)$ then $f'(0) =$
A) $\frac{5}{2}$ •B) $\frac{27}{2}$ C) 22 D) $22 + \frac{2}{\sqrt{5}}$ E) $\frac{247}{2}$

21. If $f(x) = 5^{3x}$ then $f'(x) =$
•A) $5^{3x}(\ln 125)$ B) $\frac{5^{3x}}{3\ln 5}$ C) $3(5^{2x})$ D) $3(5^{3x})$ E) $3x(5^{3x-1})$

22. A solid is generated when the region in the first quadrant enclosed by the graph of $y = (x^2 + 1)^3$, the line x = 1, the x–axis, and the y–axis is revolved about the x–axis. Its volume is found by evaluating which of the following integrals?
A) $\pi\int_1^8 (x^2+1)^3\,dx$ B) $\pi\int_1^8 (x^2+1)^6\,dx$ C) $\pi\int_0^1 (x^2+1)^3\,dx$
•D) $\pi\int_0^1 (x^2+1)^6\,dx$ E) $2\pi\int_0^1 (x^2+1)^6\,dx$

23. $\lim_{x\to 0} 4\frac{\sin x \cos x - \sin x}{x^2} =$ A) 2 B) $\frac{40}{3}$ C) ∞ •D) 0 E) undefined

24. If $\frac{dy}{dx} = \frac{(3x^2+2)}{y}$ and y = 4 when x = 2, then when x = 3, y =
A) 18 B) $\sqrt{66}$ C) 58 D) $\sqrt{74}$ •E) $\sqrt{58}$

25. $\int \frac{dx}{9+x^2} =$ A) $3\tan^{-1}\left(\frac{x}{3}\right) + C$ •B) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$ C) $\frac{1}{9}\tan^{-1}\left(\frac{x}{3}\right) + C$ D) $\frac{1}{3}\tan^{-1}(x) + C$ E) $\frac{1}{9}\tan^{-1}(x) + C$

26. If $f(x) = \cos^3(x+1)$ then $f'(\pi) =$ •A) $-3\cos^2(\pi+1)\sin(\pi+1)$ B) $3\cos^2(\pi+1)$
C) $3\cos^2(\pi+1)\sin(\pi+1)$ D) $3\pi\cos^2(\pi+1)$ E) 0

27. $\int x\sqrt{x+3}\,dx =$ A) $\frac{2}{3}(x)^{3/2} + 6(x)^{1/2} + C$ B) $\frac{2(x+3)^{3/2}}{3} + C$
•C) $\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$ D) $\frac{3(x+3)^{3/2}}{2} + C$ E) $\frac{4x^2(x+3)^{3/2}}{3} + C$

28. If $f(x) = \ln(\ln(1-x))$, then $f'(x) =$
A) $-\frac{1}{\ln(1-x)}$ B) $\frac{1}{(1-x)\ln(1-x)}$ C) $\frac{1}{(1-x)^2}$ •D) $-\frac{1}{(1-x)\ln(1-x)}$ E) $-\frac{1}{\ln(1-x)^2}$

SECTION I, PART B

Part B consists of 17 questions. Some questions in this part of the exam require the use of a graphing calculator. If the exact numerical value of the correct answer is not listed as a choice, select the choice that is closest to the exact numerical answer.
Time for Part B — **50 minutes**

29. $\int_0^{\frac{\pi}{4}} \sin x\,dx + \int_{-\frac{\pi}{4}}^{0} \cos x\,dx =$ A) $-\sqrt{2}$ B) -1 C) 0 •D) 1 E) $\sqrt{2}$

30. Boats A and B leave the same place at the same time. Boat A heads due North at 12 km/hr. Boat B heads due East at 18 km/hr. After 2.5 hours, how fast is the distance between the boats increasing (in km/hr)?
•A) 21.63 B) 31.20 C) 75.00 D) 9.84 E) 54.08

31. $\lim_{h\to 0} \frac{\tan\left(\frac{\pi}{6}+h\right) - \tan\left(\frac{\pi}{6}\right)}{h} =$ A) $\frac{\sqrt{3}}{3}$ •B) $\frac{4}{3}$ C) $\sqrt{3}$ D) 0 E) $\frac{3}{4}$

32. If $\int_{30}^{100} f(x)\,dx = A$ and $\int_{50}^{100} f(x)\,dx = B$ then $\int_{30}^{50} f(x)\,dx =$
A) A + B •B) A − B C) 0 D) B − A E) 20

33. If $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be A) 59 B) $\frac{1}{59}$ C) $\frac{1}{10}$ D) 11 •E) $\frac{1}{11}$

34. The graph of $y = x^3 - 5x^2 + 4x + 2$ has a local minimum at
A) (0.46, 2.87) B) (0.46,0) •C) (2.87, −4.06) D) (4.06, 2.87) E) (1.66, −0.59)

35. The volume generated by revolving about the y-axis the region enclosed by the graphs $y = 9 - x^2$ and $y = 9 - 3x$, for $0 \le x \le 2$, is A) -8π B) 4π •C) 8π D) 24π E) 48π

36. The average value of the function $f(x) = \ln^2 x$ on the interval [2,4] is
A) −1.204 •B) 1.204 C) 2.159 D) 2.408 E) 8.636

37. $\frac{d}{dx}\int_0^{3x} \cos(t)\,dt =$ A) sin 3x B) −3sin 3x C) cos 3x D) 3sin 3x •E) 3cos 3x

38. If the definite integral $\int_1^3 (x^2+1)\,dx$ is approximated by using the Trapezoid Rule with n=4, the error is A) 0 B) $\frac{7}{3}$ •C) $\frac{1}{12}$ D) $\frac{65}{6}$ E) $\frac{97}{3}$

39. The radius of a sphere is increasing at a rate proportional to its radius. If the radius is 4 initially, and the radius is 10 after two seconds, what will the radius be after three seconds?
A) 62.50 B) 13.00 •C) 15.81 D) 16.00 E) 25.00

40. At what point of intersection of $f(x) = 4^{\sin x}$ and $g(x) = \ln(x^2)$ do their derivatives have the same sign?
•A) −5.2 B) −4.0 C) −1.2 D) 2.6 E) 7.8

41. $\int \ln 2x\,dx =$ A) $\frac{\ln 2x}{x} + C$ B) $\frac{\ln 2x}{2x} + C$ C) $x\ln x - x + C$ •D) $x\ln 2x - x + C$ E) $2x\ln 2x - 2x + C$

42. If the function f(x) is continuous and differentiable $= \begin{cases} ax^3 - 6x; & \text{if } x \le 1 \\ bx^2 + 4; & x > 1 \end{cases}$ then a =
A) 0 B)1 •C) −14 D) −24 E) 26

43. Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the funtions $y_1 = \cos 2t$ and $y_2 = 4\sin t$ for $0 < t < 6$. For how many values of t do the particles have the same acceleration?
A) 0 B) 1 C) 2 •D) 3 E) 4

44. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t) = 7e^{-t^2}$; where t stands for time.
A) 0.976 •B) 6.204 C) 6.359 D) 12.720 E) 7.000

45. $\int \tan^6 x\sec^2 x\,dx =$ •A) $\frac{\tan^7}{7} + C$ B) $\frac{\tan^7 x}{7} + \frac{\sec^3 x}{3} + C$ C) $\frac{\tan^7 x \sec^3 x}{21} + C$ D) $7\tan^7 x\sec x + C$ E) $\frac{2}{7}\tan^7 x\sec x + C$