POTENCIJALNO STRUJANJE

Postupak rješavanja problema nestlačivog potencijalnog strujanja, odnosno određivanje polja brzine $\vec{V}$, polazi od rješavanja Laplaceove jednadžbe

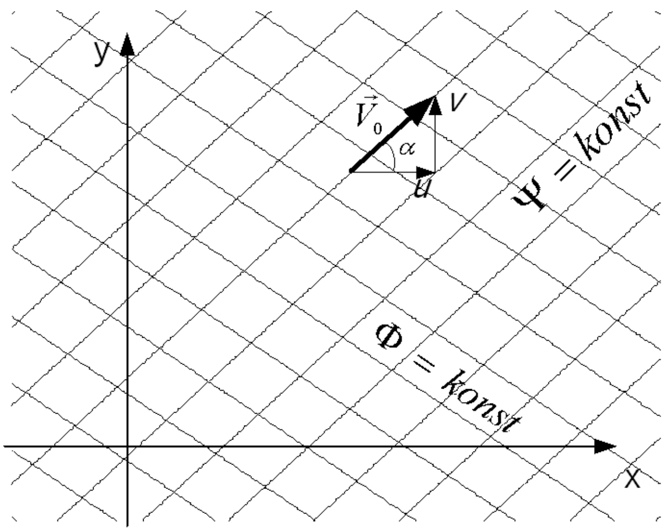
$$\nabla^2\Phi = 0 \quad \text{tj.} \quad \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0$$

gdje je Φ skalarni potencijal brzine, tj.

$$u = \frac{\partial\Phi}{\partial x}\ ; \qquad v = \frac{\partial\Phi}{\partial y}$$

Osnovna rješenja Laplaceove jednadžbe za ravninsko strujanje:

a) Paralelno strujanje



$$u = V_0 \cos\alpha$$
$$v = V_0 \sin\alpha$$

$$uvjet\ nevrtložnosti: \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$\Phi = (V_0\cos\alpha)x + (V_0\sin\alpha)y + C_1$$

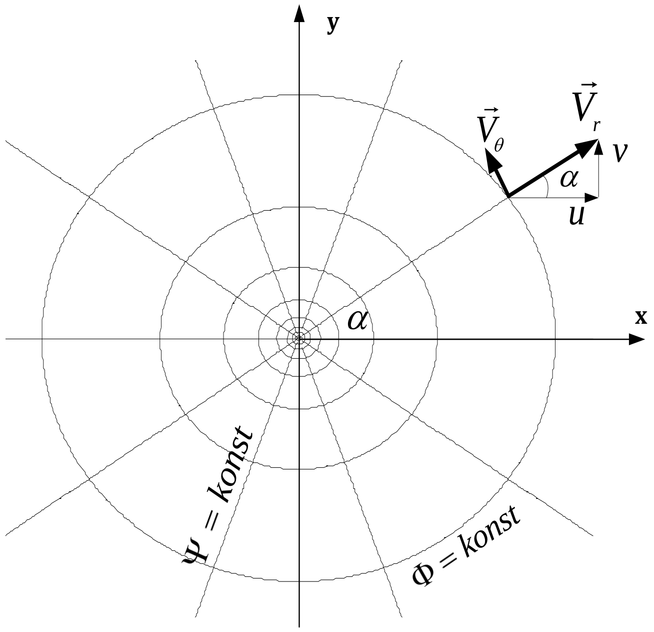
$$\Psi = -(V_0\sin\alpha)x + (V_0\cos\alpha)y + C_2$$

$$u = \frac{\partial\Phi}{\partial x} = \frac{\partial\Psi}{\partial y}; \qquad v = \frac{\partial\Phi}{\partial y} = -\frac{\partial\Psi}{\partial x}$$

$\Phi \ \rightarrow skalarni\ potencijal\ brzine$
$\Psi \ \rightarrow strujna\ funkcija\ (vektori\ brzine\ su\ tangente\ na\ \Psi = \text{konst})$

$$\left.\begin{aligned} \frac{\partial\Phi}{\partial x} &= \frac{\partial\Psi}{\partial y} \\ \frac{\partial\Phi}{\partial y} &= -\frac{\partial\Psi}{\partial x} \end{aligned}\right\} \quad Cauchy - Riemann - ovi\ \ uvjeti$$

b) Izvor ili ponor



$Q \;\rightarrow\;$ *kapacitet izvora po jedinici dužine okomit na ravninu z*

$$Q\left[\frac{m^3/s}{m}=\frac{m^2}{s}\right]$$

$$v_r=\frac{Q}{2r\pi};\qquad v_\theta=0$$

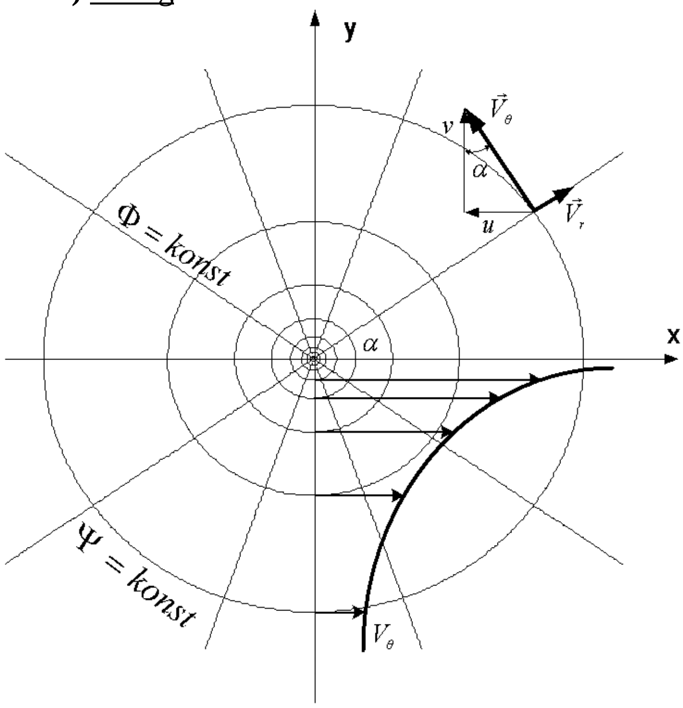
$$u=V_r\cos\alpha=\frac{Q}{2r\pi}\cdot\frac{x}{r}=\frac{Q}{2\pi}\cdot\frac{x}{x^2+y^2}$$

$$v=V_r\sin\alpha=\frac{Q}{2r\pi}\cdot\frac{y}{r}=\frac{Q}{2\pi}\cdot\frac{y}{x^2+y^2}$$

$$\Phi=\frac{Q}{4\pi}\ln(x^2+y^2)+C_3$$

$$\Psi=\frac{Q}{2\pi}\arctan\frac{y}{x}+C_4$$

c) Vrtlog



$$K=\frac{\Gamma_0}{2\pi}\;\rightarrow\;\textit{karakteristika strujanja}$$

$\Gamma_0 \;\rightarrow$ *cirkulacija* (*pozitivna u smjeru kazaljke na satu*)

$$v_r=0;\qquad v_\theta=\frac{K}{r}=\frac{\Gamma_0}{2r\pi}$$

$$u=-V_\theta\sin\alpha=-\frac{K}{r}\cdot\frac{y}{r}=-\frac{Ky}{x^2+y^2}$$

$$v=V_\theta\cos\alpha=\frac{K}{r}\cdot\frac{x}{r}=-\frac{Kx}{x^2+y^2}$$

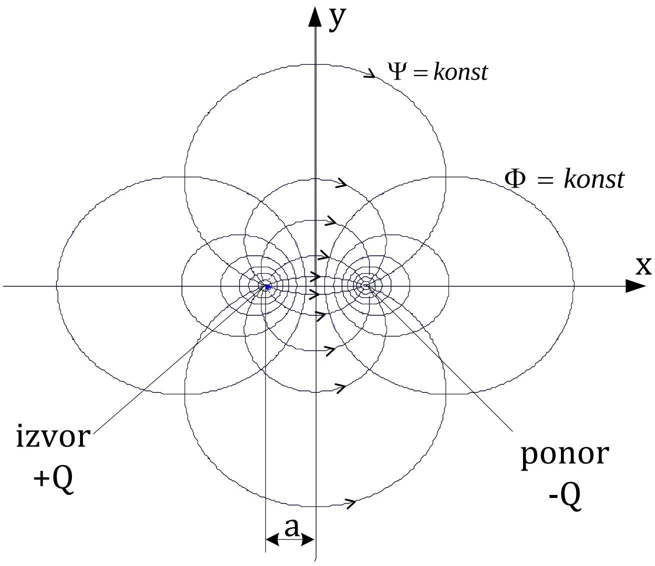
$$\Phi=\frac{\Gamma_0}{2\pi}\alpha+C_5$$

$$\Psi=-\frac{\Gamma_0}{2\pi}\ln r+C_6$$

Budući da funkcija potencijala i strujna funkcija zadovoljavaju Laplaceovu jednadžbu, zbrajanjem više funkcija potencijala dobiva se funkcija potencijala kombiniranog strujanja.

d) Superpozicija izvora i ponora istih kapaciteta

Zbrajanjem izraza za potencijal izvora kapaciteta Q u točki (-a,0) i izraza za potencijal ponora istog kapaciteta u točki (a,0) dobiva se potencijal kombinacije izvora i ponora.



$$\Phi = \Phi_{\text{izvor}} + \Phi_{\text{ponor}}$$

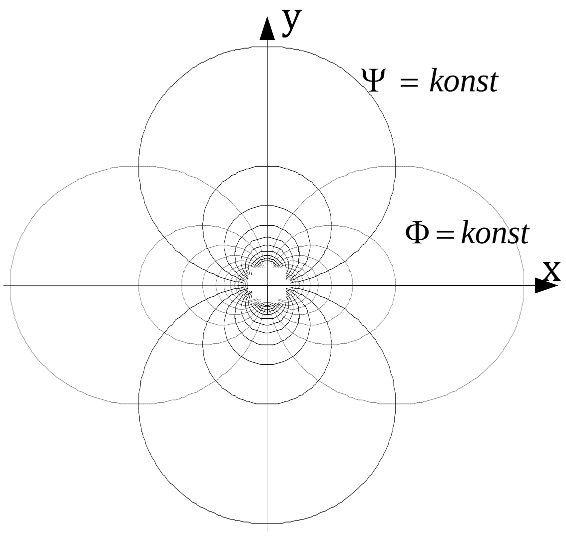
$$\Psi = \Psi_{\text{izvor}} + \Psi_{\text{ponor}}$$

$$\Phi = \frac{Q}{2\pi}\ln\sqrt{\frac{(x+a)^2+y^2}{(x-a)^2+y^2}} + C_7$$

$$\Psi = -\frac{Q}{2\pi}\arctan\frac{2ay}{x^2+y^2-a^2} + C_8$$

e) Dvopol

Potencijal dvopola nastaje na temelju potencijala kombinacije izvora i ponora primjenom graničnog procesa tako da se istovremeno izvor i ponor približavaju ishodištu $a \to 0$, kapaciteti teže beskonačnosti $Q \to \infty$, ali produkt članova teži konačnoj vrijednosti b, $2aQ \to b$.



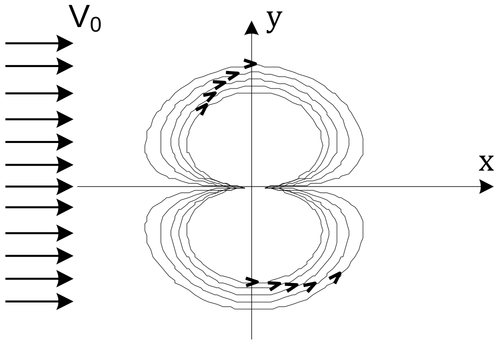
$$\begin{aligned} Q &\to \infty \\ a &\to 0 \\ 2aQ &\to b \end{aligned}$$

$$\Phi = \frac{b}{2\pi}\frac{\cos\alpha}{r} + C_9 = \frac{b}{2\pi}\frac{x}{x^2+y^2} + C_9$$

$$\Psi = -\frac{b}{2\pi}\frac{\sin\alpha}{r} + C_{10} = -\frac{b}{2\pi}\frac{y}{x^2+y^2} + C_{10}$$

f) Kombinacija paralelno strujanje + dvopol ≡ strujanje oko cilindra

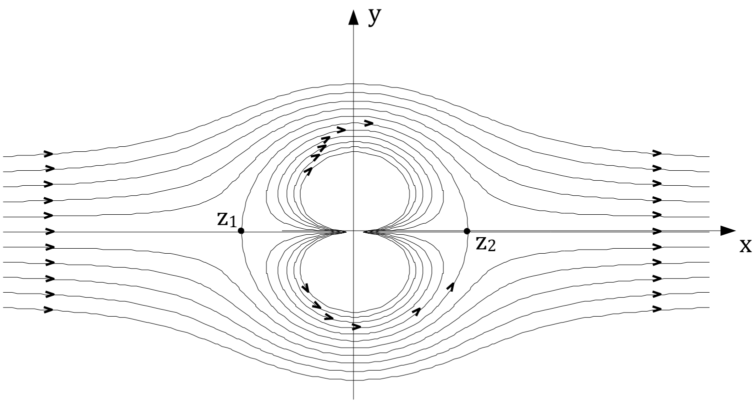
Zbrajanjem funkcija potencijala paralelnog strujanja i potencijala dvopola dobiva se nova funkcija potencijala koja predstavlja idealizirani slučaj strujanja oko cilindra radijusa r koji se nalazi u paralelnoj struji uniformnog rasporeda brzine u beskonačnosti V_0.



$$P.S.\quad \Psi = V_0 r \sin\theta + C_1$$

$$DV.\quad \Psi = -\frac{b}{2\pi r}\sin\theta + C_2$$

$$\Psi = \Psi_{\mathrm{PS}} + \Psi_{\mathrm{DV}} = V_0 r \sin\theta - \frac{b}{2\pi r}\sin\theta + C_3$$



$$\Psi = (V_0 r - \frac{b}{2\pi r})\sin\theta + C_3$$

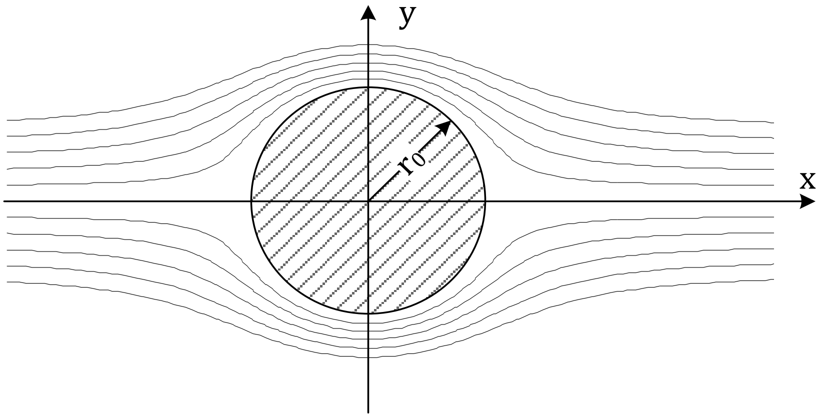
$$\Psi = konst = C \quad - strujne\ linije$$

$$C = (V_0 r - \frac{b}{2\pi r})\sin\theta$$

$\underline{C = 0}$

$$\sin\theta = 0 \quad \Rightarrow \quad \theta = 0, \pm\pi, \ldots \quad (x - os)$$

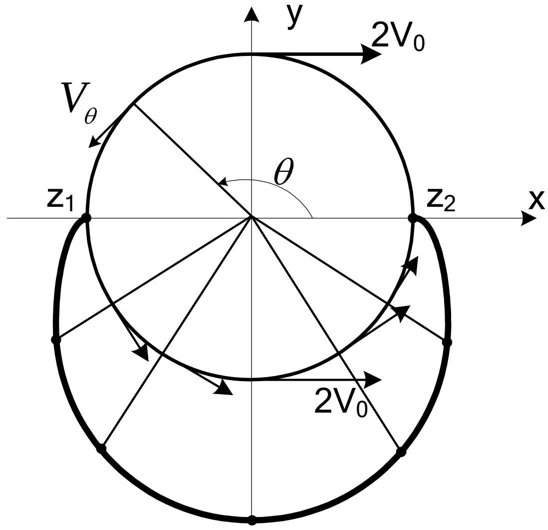
$$\left(V_0 r - \frac{b}{2\pi r}\right) = C \quad \Rightarrow \quad r = \sqrt{\frac{b}{2\pi V_0}} = r_0 \quad \rightarrow b = {r_0}^2 \cdot 2\pi V_0 \qquad (2aQ \ \rightarrow b)$$



$$\Psi = (V_0 r - \frac{{r_0}^2 \cdot 2\pi V_0}{2\pi r})\sin\theta + C_3$$

$$\Psi = V_0\left(r - \frac{{r_0}^2}{r}\right)\sin\theta + C_3$$

$$\left.\begin{aligned} v_r &= \frac{1}{r}\frac{\partial \Psi}{\partial \theta} = V_0\left(1 - \frac{{r_0}^2}{r^2}\right)\cos\theta \\ v_\theta &= -\frac{\partial \Psi}{\partial r} = -V_0\left(1 + \frac{{r_0}^2}{r^2}\right)\sin\theta \end{aligned}\right\} \; r = r_0 \;\; \Longrightarrow \qquad \left.\begin{aligned} v_r &= 0 \\ v_\theta &= -2V_0\sin\theta \end{aligned}\right\} \; \textit{brzina na cilindru } r = r_0$$



$$\left.\begin{aligned} \frac{\partial \Phi}{\partial r} &= v_r \\ \frac{1}{r}\frac{\partial \Phi}{\partial \theta} &= v_\theta \end{aligned}\right\} \; integriranjem \quad \Phi = V_0\left(r + \frac{{r_0}^2}{r}\right)\cos\theta$$

<u>Sila tlaka na cilindar</u>

$$B.J. \qquad p_0 + \frac{1}{2}\rho {V_0}^2 + \rho g z_0 = p_c + \frac{1}{2}\rho {V_c}^2 + \rho g z_c$$

$$p_c = p_0 + \frac{1}{2}\rho {V_0}^2\left(1 - \frac{{V_c}^2}{{V_0}^2}\right)$$

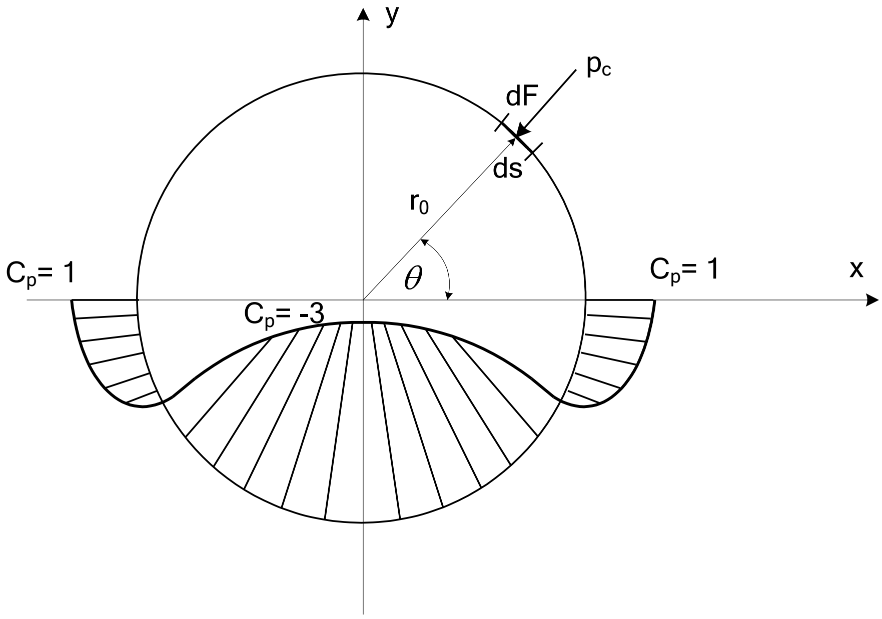
$$V_c = v_\theta = -2V_0\sin\theta$$

$$p_c = p_0 + \frac{1}{2}\rho {V_0}^2\left(1 - \frac{4{V_0}^2\sin^2\theta}{{V_0}^2}\right)$$

$$p_c = p_0 + \frac{1}{2}\rho {V_0}^2(1 - 4\sin^2\theta)$$

$$p_c - p_0 = \frac{1}{2}\rho {V_0}^2(1 - 4\sin^2\theta)$$

$$C_p = \frac{p_c - p_0}{\frac{1}{2}\rho {V_0}^2} = (1 - 4\sin^2\theta)$$



$$dF = p_c ds = p_c r d\theta$$

$$\left.\begin{aligned} dF_x &= -dF\cos\theta = -p_c r\cos\theta\, d\theta \\ dF_y &= -dF\sin\theta = -p_c r\sin\theta\, d\theta \end{aligned}\right\} \; integriranje \qquad \begin{aligned} F_x &= 0 \\ F_y &= 0 \end{aligned} \quad \rightarrow \quad D'Alambertov\ paradoks$$

g) Kombinacija paralelno strujanje + dvopol +vrtlog ≡ strujanje oko cilindra s cirkulacijom

$$\Psi = V_0 r \sin\theta - \frac{b}{2\pi r}\sin\theta - \frac{\Gamma_0}{2\pi}\ln r$$

$F_x = 0$

$F_y = \Gamma_0 \rho V_0$ teorem Kutta i Žukovskog